Chem 111

UNCERTAINTY IN MEASUREMENTS

PURPOSE:

This lab study packet deals with various errors in experiments, avoidable (systematic) and unavoidable (random), and how they affect your experimental results. In many of the Chem 111 experiments you will be asked to give some indication to the validity of your results you report. You need to know how much effect an error in one or more bits of data will have on your result and how to report it.

A MEASUREMENT is a result of taking a reading from a piece of equipment such as a balance, a ruler, a buret,....etc. These measuring devices are used in the lab to obtain measurements. A measurement always has two parts, a number and a unit. For example, a 43.20 ml read from a buret is a measurement.

SYSTEMATIC ERROR is a consistent error that arises from a flaw in equipment or the design of an experiment. For example, a balance that reads 0.5 g lower all the time will show negative systematic error. A systematic error can be identified and corrected if you compare your result to that made on a similar device but well calibrated. A systematic error may be negative always or positive always. On the contrary, a RANDOM ERROR has equal chance of being positive or negative. It is always present and cannot be corrected. There is random error associated with reading any scale. In this packet we will discuss only uncertainties due to RANDOM ERRORS

UNCERTAINTY OF MEASUREMENT (Absolute uncertainty and percent uncertainty)

1. ***ABSOLUTE UNCERTAINTY AND PERCENT UNCERTAINTY OF A SINGLE READING***

Every experiment has some uncertainty caused by limitations in the equipment you use. This is unavoidable RANDOM ERROR and does not reflect on your lab technique. The table given below lists the absolute uncertainties for some equipment used in the Chemistry lab. The absolute uncertainty expresses the margin of uncertainty associated with a reading, a measurement, or a calculation involving several readings.

EQUIPMENT	TYPICAL UNCERTAINTY
top loading balance	0.05 g
Analytical balance	0.0002 g
1000 ml graduated cylinder	2 ml
500 ml graduated cylinder	1 ml
100 ml graduated cylinder	0.4 ml
10 ml graduated cylinder	0.08 ml
50 ml buret	0.10 ml
Thermometer with 1° C graduations	0.5 ° C
Thermometer with 0.2° C graduations	0.1 ° C
Barometer	0.1 torr

The above table shows that a weighing of 23.25 g made on a top loading balance should be reported as 23.25 g ±0.05 g. Such an item of data means that the correct reading lies

between 23.20 g and 23.30 g
The uncertainty in a measurement can be expressed in two useful ways:
a. as the absolute uncertainty in the last digit written
b. as the percent uncertainty calculated as follows

$$\% \text{ uncertainty} = \frac{\text{absolute uncertainty}}{\text{measurement}} \times 100$$

$$\% \text{ uncertainty} = \frac{0.05 \text{ g}}{23.25 \text{ g}} \times 100 = 0.2\ \%$$

The answer may be reported as:

absolute uncertainty : 23.25 g ± 0.05 g
percent uncertainty : 23.25 g ± 0.2 %

Exercise

*ABSOLUTE UNCERTAINTY AND PERCENT UNCERTAINTY F IN A **SINGLE** READING*:
Use the uncertainties in the table above to calculate the % uncertainty in each of the following readings:

a. A barometer reading of 723.5 torr.

Setup:

__________%

b. 2.75 g weighed on a top loading balance.

Setup:

__________%

c. 2.7413 g weighed on an analytical balance.

Setup:

__________%

d. A temperature reading of 75.6 °C on a thermometer graduated to the nearest degree.

Setup:

__________%

e. 18.6 ml measured in 100 ml graduated cylinder.

Setup:

__________%

f. 43.7 ml measured in 100 ml graduated cylinder.

Setup:

__________%

If you compare (e) and (f) you will notice that a large volume (43.7 ml) has a smaller % uncertainty than a small volume (18.6 ml). *KEEP YOUR LAB MEASUREMENTS AS LARGE AS POSSIBLE*.

2. ***PROPAGATION OF UNCERTAINTY IN MULTIPLE MEASUREMENTS***

Uncertainty is based on how well we can read an instrument. Consider the unavoidable measurement errors that are usually random. Some will tend to make the answer too high while others will tend to make it too low. In many experiments, it is necessary to perform arithmetic operations on several numbers, each of which has an associated random error. The most likely uncertainty in the result is not simply the sum of the individual errors, because some of these are likely to be positive and some negative. Hence, we expect some cancellation of errors.
There are two different rules you must learn and apply to these *random* errors:

Rule 1: Addition and subtraction

When the measured quantities are added or subtracted, the absolute uncertainty in the answer is calculated from the absolute uncertainties of all separate measurements. Each measurement must be in the same unit, before you can add or subtract.
The following is an illustrative example on how to calculate the absolute uncertainty and percent uncertainty:

$$
\begin{array}{rrcl}
 & 1.54 & \pm & 0.02\text{ g} \\
 & +2.11 & \pm & 0.03\text{ g} \\
 & -0.78 & \pm & 0.02\text{ g} \\
\hline
= & 2.87 & \pm & (u)
\end{array}
$$

The experimental uncertainties in the 1st, 2nd, and 3rd measurements are 0.02, 0.03, and 0.02. These are designated by u_1, u_2, and u_3 respectively. The uncertainty, u, associated with this result is calculated from the absolute uncertainties of the individual terms as follows:

$$u = \sqrt{u_1^{\ 2} + u_2^{\ 2} + u_3^{\ 2}}$$

For the above problem, the uncertainty in the answer 2.87 g would be:

$$u = \sqrt{(0.02)^2 + (0.03)^2 + (0.02)^2} = 0.04$$

The absolute uncertainty, u, is ± 0.04 g, and we can write the answer as 2.87g ± 0.04g

$$\text{Percent uncertainty} = \frac{0.04\text{ g}}{2.87\text{g}} \times 100 = 1\%$$

<u>Answer</u> : 2.87 g (± 0.04 g) 2.87 g(± 1%)

Uncertainty in a measurement that is calculated as a difference in two readings

Often, what appears to be a single measurement as in grams of sample for example, is really a difference between two measurements. When you weigh by difference you have:

Sample weight = weight before- weight after

Remember to notice whether the item of data is a single measurement (for example an aluminum block weighed directly on the balance pan) or a difference between two readings (as temperature rise or volume change).

For example, if the temperature rise from 22.6˚C to 34.5 ˚C was measured on a thermometer accurate to ± 0.1 ˚C, what is the uncertainty in the rise in temperature?

Rise in temperature = 34.5 ˚C - 22.6˚C = 11.9 ˚C

There are two readings and each reading has an uncertainty of ± 0.1˚C

$$\text{Absolute uncertainty} = \sqrt{(0.1)^2 + (0.1)^2} = 0.1$$

$$\%\ \text{uncertainty} = \frac{\underline{\mathbf{0.1}}\ ^\circ\mathrm{C}}{11.9\ ^\circ\mathrm{C}} \times 100 = 1\ \%$$ **(reported as 1 sig. fig.)**

Answer

absolute uncertainty:	11.9 ˚C (± 0.1 ˚C)
% uncertainty:	11.9 ˚C (± 1%)

Exercise: Propagation of uncertainties (addition and subtraction)

1. The weights of three pieces of wood were 1.543 ± 0.003 g, 2.2233 ±0.0002 g, and 2.9342 ± 0.0005 g.

a. What is the absolute uncertainty in the total mass?

Setup:

± ________g

b. How should the total mass be reported?

Answer _____ ± ____ g

c. What is the percent uncertainty in the total mass?

Setup

_____%

2. The initial mass of $KClO_3$ is 3.456g and the final mass after it lost all its oxygen by heating is 2.579 g. The uncertainty of a reading on the balance used is ± 0.003 g.

a. What is the absolute uncertainty in mass of oxygen lost?

Setup:

a) ±_________g

b. Find the percent uncertainty in the mass of the oxygen.

Setup:

b) ________%

3. A student measures 23.4 ml of solution from his buret, as accurately as he can. What is the percent uncertainty of his data if he reads the buret to the nearest 0.1 ml? (Hint: He makes two buret readings, and the difference in the two buret readings is 23.4 ml)

Setup:

________%

4. A student measures 23.40 ml of solution from his buret, as accurately as he can.

a) What is the percent uncertainty of his data if he reads the buret to the nearest 0.02 ml ? (Hint: He makes two buret readings, and the difference in the two buret readings is 23.40 ml)

Setup:

_________%

b) Compare your results in 3 and 4; would you read the buret to 0.1 ml or 0.02 ml?Answer _____ ml.

Why ? ____

5. a. Consider the following setup to calculate the temperature drop:

$$\begin{array}{rl} & 87.4 \pm 0.2\ ^\circ C \\ - & 52.1 \pm 0.2\ ^\circ C \\ \hline \end{array}$$

_________°C

b. Calculate the absolute uncertainty in the temperature drop.
Setup:

±_________° C

c. How should the drop in temperature be reported?

_____° C ± _____° C

d. Calculate the percent uncertainty in the above temperature drop.
Setup:

_________%

6. Consider the following set up for calculating the total mass:

	5.7755	±	0.0001	g
+	8.2233	±	0.0001	g

a. Calculate the absolute uncertainty in the total mass.
Setup:

±_________g

b. How should the total mass be reported?

________g ± _______g

c. Calculate the percent uncertainty in the above total mass.

_______%

Rule 2: Multiplication and division

For multiplication and division, first convert all uncertainties to percent uncertainties. Then calculate the % uncertainty of the product as follows:

$$\%u = \sqrt{(\%u_1)^2 + (\%u_2)^2 + (\%u_3)^2 + \cdots}$$

Example: You calculate the density of a liquid by measuring its mass (2.22g ±0.05g) and volume (1.14 ± 0.04 ml). The density would be 1.947368 g/ml **(Do not round off yet until you calculate absolute uncertainty.)**

$$\text{Density} = \frac{2.22\text{ g }(\pm\ 0.05\text{ g})}{1.14 \pm (0.04\text{ ml})} = 1.947368\text{g/ml}$$

To calculate the uncertainty in the calculated density, first you need to calculate the percent uncertainty of the measured values as follows:

Percent uncertainty in mass = $\frac{0.05 \text{ g}}{2.22 \text{ g}} \times 100 = \mathbf{2}\ \%$

Percent uncertainty in volume = $\frac{0.04 \text{ ml}}{1.14 \text{ ml}} \times 100 = \mathbf{4}\ \%$

Density= $\frac{2.22 \text{ g } (\pm\ \mathbf{2}\ \%)}{1.14\ (\pm\ \mathbf{4}\ \%)} = 1.947368 \text{g/ml}$

The percent uncertainty in computed density, % u:

$$\%u = \sqrt{(2\%)^2 + (4\%)^2} = 4\ \%$$

The percent uncertainty in the density is ± **4** %.

But what is the ***absolute uncertainty*** *in the computed density and how many* ***significant figures*** *should be used in reporting the density?*

Absolute uncertainty in density = $\frac{\mathbf{4}}{100} \times 1.947368$ g/ml= **0.08** g/ml

(The answer is ± 0.08 g/ml; one sig fig.)

The answer is:

Density=1.95 g/ml ± 0.08 g/ml

NOTE:

1. The density is reported to the hundredth place (1.95 g/ml) because the absolute uncertainty is accurate only to the hundredth place(±0.08 g/ml).

2. When rounding off a calculated answer to the correct number of significant figures, you must consider the absolute uncertainty to determine to which digit the answer should be rounded.

Exercise 1: Propagation of uncertainties (multiplication and division)
A student weighed a 26.91 g block to the nearest 0.01 g and measured its volume as 25 ml to the nearest ml. What is the uncertainty in the calculated density?

$$\text{Density} = \frac{26.91\text{g} \pm 0.01\text{ g}}{25\text{ ml} \pm 1\text{ ml}} = 1.0764\text{ g/ml}$$

The density is 1.0764 g/ml. It should not be rounded off yet! Do you know why?
Answer: ______________________________________

a. Find % uncertainty in mass.
Setup:
______%

b. Find % uncertainty in volume.
Setup:
______%

c. Find % uncertainty in density.
Setup:
______%

d. Find absolute uncertainty in density.
Setup
±______g/ml

e. Now report the density to the correct number of significant figures?
Answer: Density= _______g/ml

f. Report the answer in the correct number of significant figures.
Density= ________g/ml ± _______g/ml

g. Why did you round off the density to the hundredth place?
____

Notice that the answer 1.08 g/ml has three significant figures although the denominator in the density set up (25 ml) has only two significant figures.

Exercise 2:
Consider the following operation:

$$\frac{1.85\text{ g}(\pm 0.02\text{ g})}{0.48\text{ cm}(\pm 0.02\text{ cm}) \times 1.67\text{ cm}(\pm 0.03\text{ cm})} = \underline{\qquad\qquad}$$

Follow the steps given below to find how the answer should be rounded off.

a. Give the answer without rounding off.
Setup:
______g/cm^2

b. Find % uncertainty in the reading, 1.85 g.
Setup:
______%

c. Find % uncertainty in the reading, 0.48 cm.
Setup:
______%

d. Find % uncertainty in the reading, 1.67 cm.
Setup:

______%

e. Find % uncertainty in the answer to the operation given above.
Setup:

______%

f. Find the absolute uncertainty in the answer to the operation given above.
Setup:

±_______g/cm^2

g. Express the answer to the above operation in the correct number of significant figures.

______g/cm^2 ±_______g/cm^2

h. Why did you round off your answer to the tenth place?

____

Exercise 3:

Perform the following operation and report your answer in the proper number of significant figures according to the uncertainty rules.

$$\frac{0.003427\ (\pm\ 0.000005\ \text{cg})}{0.03611\ (\pm\ 0.00003\ \mu\text{L})} = 0.094904459$$

a. Find % uncertainty in 0.003427 cg.
Setup:

_________ %

b. Find % uncertainty in 0.03611 µL.
Setup:

_________ %

c. Find % uncertainty in the answer to the operation given above.
Setup:

_________ %

d. Find the absolute uncertainty in the answer to the operation given above.
Setup:

±_______cg/ µL

e. Express the answer to the operation given above in the correct number of significant figures.

__________(±_______cg/ µL)

<u>**Conclusion:**</u>

1. Uncertainties in measurements (and not the rules of significant figures that you covered earlier) dictate how to round off the answer. That is, the rules of uncertainties prevail over the rules of significant figures in deciding on how to round off the final answer.

2. Propagation of uncertainty in <u>*addition and subtraction*</u> requires <u>**absolute uncertainties:**</u>

$$u = \sqrt{u_1^{\ 2} + u_2^{\ 2} + u_3^{\ 2} + \cdots}$$

3. Propagation of uncertainty in <u>*multiplication and division*</u> requires <u>**percent uncertainties:**</u>

$$\%u = \sqrt{(\%u_1)^2 + (\%u_2)^2 + (\%u_3)^2 + \cdots}$$