Chapter 1

Numbers, Linear Functions, Equations, Inequalities, and their Applications

Covered in this Chapter:

1.1 Sets of Numbers and Set Builder Notation.

1.2 Rectangular Coordinate System. Pythagorean, Distance and Midpoint Formulas.

1.3 Relations and Functions. First Degree Functions. Slope of a Line.

1.4 Equations of Lines.

1.5 Linear Equations in One Variable.

1.6 Linear Inequalities in One Variable.

1.7 Applications.

► Answers to Exercises.

1.1 Sets

When we have a collection of data of similar but distinct objects, we use set notation. We write the data between a left brace "{" and a right brace "}". Data is separated by a comma ",". The left brace "{" will mean that we are going to write a list of data in set notation. Don't forget to close the set by writing the right brace "}".

Example: The set {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday} represents a set that contains the days of the week.

A set has an infinite number of objects if we write the three dots (ellipsis) at the end of the list of objects.

Sets of Numbers:

The Set of Natural Numbers: $\mathbb{N} = \{1, 2, 3, 4, 5, \ldots\}$

The Set of Whole Numbers: $\mathbb{W} = \{0, 1, 2, 3, 4, 5, \ldots\}$

The Set of Integers: $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

The Set of Rational Numbers: $\mathbb{Q} = \left\{ \frac{p}{q} \mid p \text{ and } q \in \mathbb{Z};\ q \neq 0 \right\}$, i.e. all numbers p and q such that p and q exist in the set of all integers $\mathbb{Z}$ and q is not equal to zero.

Note: All terminating decimal numbers and repeating decimal numbers are rational numbers.

Examples of Rational Numbers: $0.25,\ 0.625,\ 0.\overline{3},\ 0.\overline{7}$

All of the numbers given in the example are rational numbers since each can be written in the form $\frac{p}{q}$.

These numbers can be written in a rational form respectively as follows: $\left\{ \frac{1}{4}, \frac{5}{8}, \frac{1}{3}, \frac{7}{9} \right\}$.

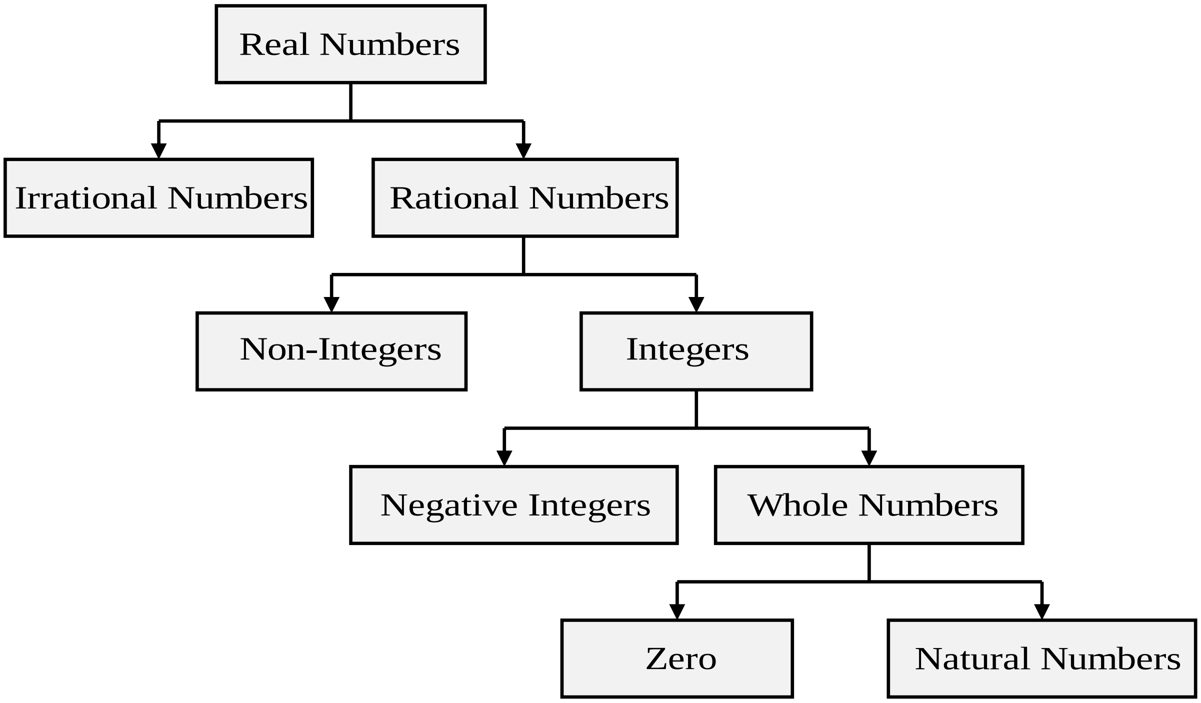
The Set of Irrational Numbers: $\mathbb{H} = \overline{\mathbb{Q}} = \{x \mid x \text{ is not a rational number}\}$, i.e. all numbers that cannot be written in rational form.

Examples of Irrational Numbers: $\pi,\ e,\ \sqrt{2}$.

In general, any number that cannot be written as a repeating or terminating decimal is an irrational number.

The Set of Real Numbers: $\mathbb{R} = \{x \mid x \text{ is a point on the number line}\}$, i.e. a real number can be irrational, rational, an integer, whole, or a natural number.

The figure on the top of the next page illustrates the relationships between all of the above sets of numbers. The block diagram shows the position of each set of numbers with respect to the other sets of numbers. All the lower sets of numbers are subsets of the highest set which is the set of all real numbers.



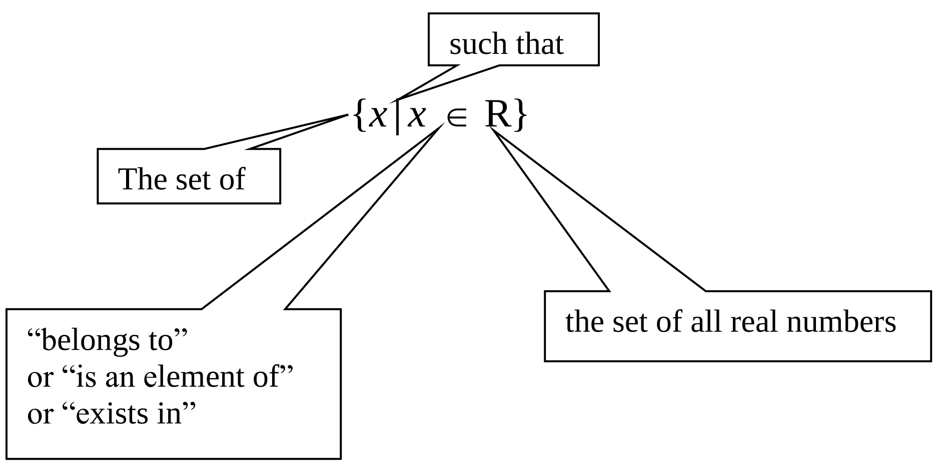
Set-Builder Notation:

We use the set-builder notation method (which is sometimes referred to as the roster notation method) when we can't write all elements in a set as in the set of all real numbers since it is infinite.

Example: $E = \{1, 3, 5, 7, 9, \ldots\} = \{x \mid x \text{ is a positive odd number}\}$.

The above set is read as "*The set* ***E*** *is the set of* ***x*** *such that* ***x*** *is a positive odd number*".

The illustration below shows how to read the set of all real numbers written in set-builder notation.



So, we read the set as follows: "The set of x such that x is an element of (or one of the other alternatives shown in the call-out) the set of all real numbers".

<u>The Real Number Line</u>:

Any real number can be represented as a point on a line called the **real number line**.

There is a one-to-one correspondence between real numbers and points on a line. That is, every real number corresponds to a point on the line, and each point on the line has a unique real number associated with it. Pick a point on the line somewhere in the center, and label it 0. This point, called the **origin**, corresponds to the real number 0 (see Figure 1). The point one unit to the right of 0 corresponds to the number 1. The distance between 0 and 1 determines the **scale** of the number line. For example, the point associated with the number 2 is twice as far from 0 as 1 is. Notice that an arrowhead on the right end of the line indicates the direction in which the numbers increase. Figure 1 also shows the points associated with the irrational numbers $\sqrt{2}$ and π. Points to the left of the origin correspond to the real numbers -1, -2, -3, and so on.

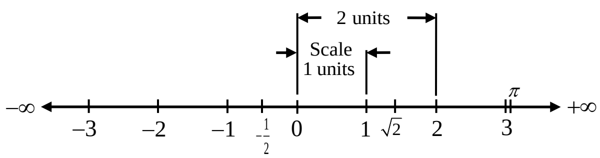


Figure 1

The real number associated with a point P is called the coordinate of P. The line whose points have been assigned coordinates is called the real number line.

The real number line consists of three classes of real numbers as shown in the figure 2 below.

1. The real number zero is the coordinate of the origin 0.

2. The negative real numbers are the coordinates of the points to the left of the origin 0.

3. The positive real numbers are the coordinates of the points to the right of the origin 0.

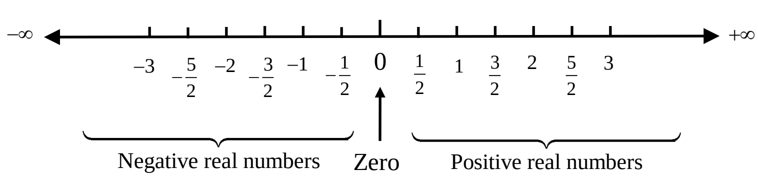


Figure 2

Exercises 1.1

In exercises 1 – 6, list all elements that belong to the set of natural numbers, whole numbers, integers, rational numbers, irrational numbers, and real numbers.

1. $\left\{-10, -\frac{8}{4}, 0, \sqrt{2}, e, 7\right\}$

2. $\left\{-\pi, \frac{1}{3}, \sqrt{4}, \frac{10}{6}, \sqrt{17}\right\}$

3. $\left\{-5.6, -2.5, 0.1, \sqrt{3}, 1.25\right\}$

4. $\left\{-0.\bar{3}, 1.\overline{234}, \frac{1}{7}, 0.7, \frac{14}{2}\right\}$

5. $\left\{-\sqrt{25}, -3, -1.25, \frac{5}{4}, \sqrt{7}\right\}$

6. $\left\{-4.25, -\frac{6}{2}, 0.125, 0.\bar{4}, 0.\overline{56}\right\}$

In exercises 7 – 23, plot the given points on a number line.

7) 0

8) -7

9) $\frac{8}{3}$

10) -1

11) 10

12) -4.75

13) $\frac{1}{2}$

14) -5

15) 2.25

16) -6

17) -3.5

18) $-\sqrt{2}$

19) $\frac{1}{4}$

20) $\sqrt{2}$

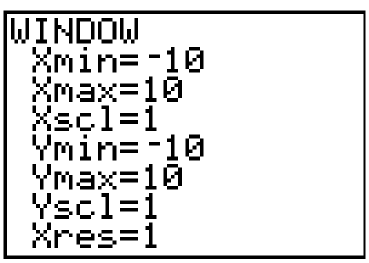
21) 2.5

22) 4

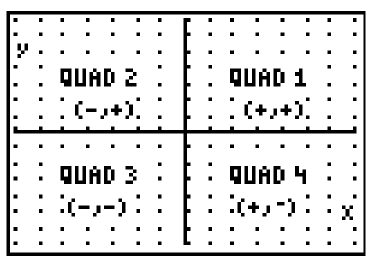
23) 5

1.2 Rectangular (Cartesian) Coordinate System

To graph a curve in two dimensions we need two number lines. The first number line is drawn horizontally and is called the *x*-axis. The second number line is perpendicular to the first number line at origin and is called the *y*-axis. From screen (b) below, you can see that the two number lines (axes) divide the plane into four quadrants. The signs in each quadrant are shown in screen (b) below. So to graph any point in the rectangular coordinates, we need two values. The first value tells us the horizontal distance from the *y*-axis. This value is called the *x*-coordinate. The second value tells us the vertical distance from the *x*-axis. This value is called the *y*-coordinate. The two values can be written together as an ordered pair. It is called an ordered pair because *x* and *y* coordinates must be written in order; the *x*-coordinate is written first followed by the *y* coordinate. We separate both coordinates by a comma "," and always enclose the two coordinates between a left and a right parenthesis. The general form of an ordered pair is (x, y). In screen (a) below, the standard window of the TI 83/84 is shown and you can see that *x*-values range from -10 to 10. Also, the *y*-values range from -10 to 10. `Xscl` and `Yscl` mean that the size of each unit in the *x* and *y* axes is one. Notice that the dimension of each dotted box is one by one. So, it should look like a square. Note: Since the width of the TI display is greater than its height (a rectangular display), each dotted box looks like a rectangle.



(a)



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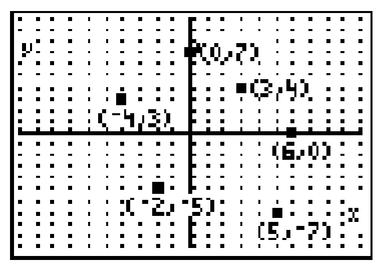
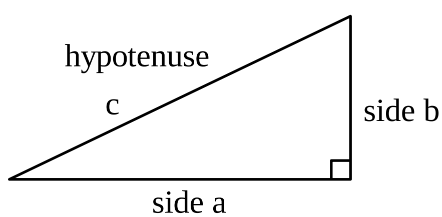


(c)

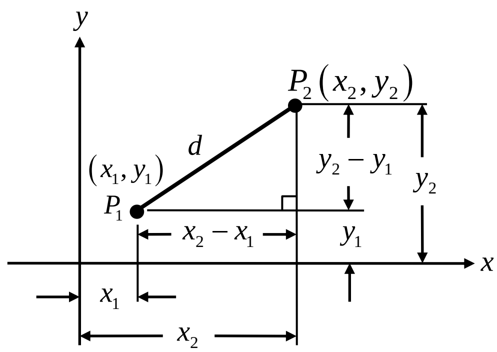
Pythagorean Theorem

●●● In a right triangle, the square of the hypotenuse is equal to the sum of the squares of its two sides.

$$\boxed{a^2+b^2=c^2}$$



Distance Between Two Points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$:



From the above figure, we see that $d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$.

By taking the square roots of each side, we get the **Distance Formula**:

$$\boxed{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

Note: When we find the distance between any two points, it doesn't matter which point you call point one or which point you call point two.

Example 1: Find the length of the line segment with end points (3, 5) and (7, 8).

Solution: $d = \sqrt{(7-3)^2 + (8-5)^2} = \sqrt{4^2 + 3^2} + \sqrt{16+9} = \sqrt{25} = 5$

Example 2: Find all points with *y*-coordinate equal to -3 whose distances from the point (2, 5) are 10.

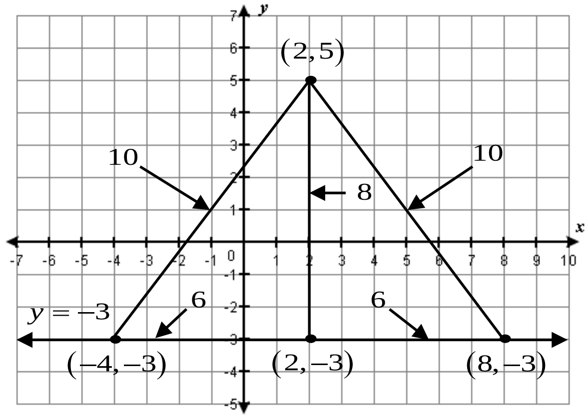
Solution: Let the points be $(x,\ y)$. Since the *y*-coordinate of these points is -3, the points are $(x, -3)$

The solution below shows how to solve the problem by using the distance formula.

$$d = 10 = \sqrt{(x-2)^2 + (-3-5)^2}$$
$$10 = \sqrt{(x-2)^2 + (-8)^2}$$
$$10 = \sqrt{(x-2)^2 + 64}, \text{ square both sides of the equation}$$
$$100 = (x-2)^2 + 64, \text{ subtract 64 from both sides of the equation}$$
$$36 = (x-2)^2, \text{ take the square root of each side}$$
$$\pm 6 = x - 2, \quad \therefore x = 2 \pm 6 \Rightarrow x = 2 - 6 \text{ or } x = 2 + 6$$
$$\therefore x = \{-4, 8\}$$
$$\boxed{\therefore \text{The points are } (-4, -3) \text{ and } (8, -3)}$$

Note: The symbol $\therefore$ means "therefore" or "then".

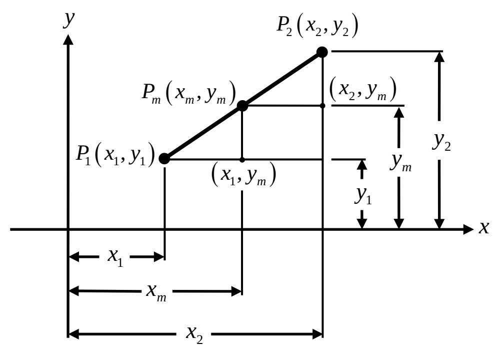
Another solution by graphing:



From the above sketch, we can see that points are (-4, -3) and (8,-3)

Midpoint Between Two Points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$:

$$x_{mid} = \frac{x_1 + x_2}{2}$$
$$y_{mid} = \frac{y_1 + y_2}{2}$$
$$M(x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$



Example 3: Find the coordinates of the midpoint of the line segment with endpoints (3, 5) and (7, 1).

Solution: $x_{mid} = \frac{3+7}{2} = \frac{10}{2} = 5,\ y_{mid} = \frac{5+1}{2} = \frac{6}{2} = 3,\quad \therefore M(x, y) = (5,3)$

Example 4: If the coordinates of the midpoint of the line segment with endpoints (*a*, 4) and (3, *b*) are (5, -2), find the values of *a* and *b*.

Solution: $\because x_{mid} = \frac{x_1 + x_2}{2},\quad \therefore 5 = \frac{a+3}{2},$ multiply both sides of the equation by 2

$\therefore 10 = a + 3,\quad \therefore \boxed{a = 7}$

$\because y_{mid} = \frac{y_1 + y_2}{2},\quad \therefore -2 = \frac{4+b}{2},$ multiply both sides of the equation by 2

$\therefore -4 = 4 + b,\quad \therefore \boxed{b = -8}$

<u>**Note:**</u> The symbol $\because$ means "since".

Exercises 1.2

In exercises 1 – 8, plot the given ordered pairs on a rectangular coordinate system.

1) $(-2,5)$ 2) $(4,0)$ 3) $(1,-3)$ 4) $(2,5)$

5) $(0,-2)$ 6) $(-2,-1)$ 7) $(-5,0)$ 8) $(0,4)$

9) Find the length of the line segment with endpoints $(-1,4)$ and $(5,-4)$.

10) Find the length of the line segment with endpoints $(4,-3)$ and $(9,9)$.

11) Find all points with *y*-coordinate equal to 5 whose distances from the point $(3,-1)$ are 10 units.

12) Find all points with *y*-coordinate equal to -4 whose distances from the point $(8,1)$ are 13 units.

13) Find all points with *x*-coordinate equal to -2 whose distances from the point $(1,-3)$ are 5 units.

14) Find all points with *x*-coordinate equal to 3 whose distances from the point $(-5,2)$ are 10 units.

15) Find the coordinates of the midpoint of the line segment with endpoints $(1,-3)$ and $(7,5)$.

16) Find the midpoint of the line segment with endpoints $(-5,0)$ and $(3,8)$.

17) If the coordinates of the midpoint of the line segment with endpoints $(a,3)$ and $(5,b)$ are $(8,-5)$, find the values of *a* and *b*.

18) If the coordinates of the midpoint of the line segment with endpoints $(a,-7)$ and $(-2,b)$ are $(3,-4)$, find the values of *a* and *b*.

1.3 Relations and Functions

Often, we want to associate two quantities or objects together. This association between the two quantities is called a relation. Therefore, a relation is a set of ordered pairs. When you go to the grocery store to buy oranges, the price you pay depends on the quantity you buy. Let's say that one pound of oranges costs 50 cents. A table of costs and quantities is given:

Number of Pounds	Cost
1	\$0.50
2	\$1.00
3	\$1.50
5	\$2.50
10	\$5.00

We can write the above table as a set of ordered pairs as follows:

$$\{(1, 0.50),\ (2, 1.00),\ (3, 1.50),\ (5, 2.50),\ (10, 5.00)\}$$

The set of all x-coordinates is called the **domain** of the relation. The set of all y-coordinates is called the **range** of the relation. Therefore, the domain of the relation is {1, 2, 3, 5, 10} and the range of the relation is {0.50, 1.00, 1.50, 2.50, 5.00}.

Functions:

A function is a special type of relation in which no two ordered pairs can have the same first coordinate and different second coordinate.

i.e. In a function, there is a unique y for each x.

When we represent functions, we can write the function as a set of ordered pairs, as data in a table, as an equation, or as a graph.

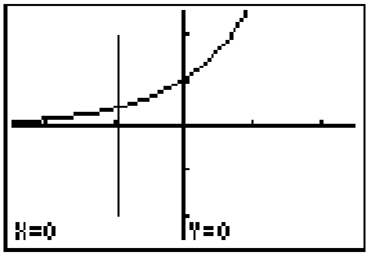
We should be able to tell if a given graph is a function or a relation. From the graph we can write the domain and range of the relation or the function.

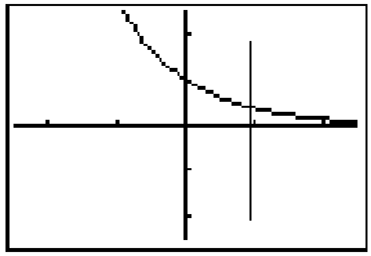
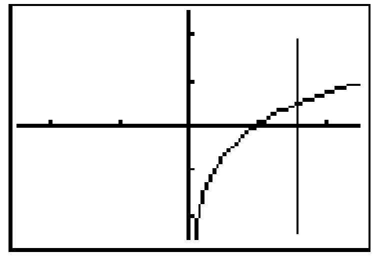
Vertical Line Test:

If any vertical line intersects a graph at no more than one point, then the graph is a graph of a function.

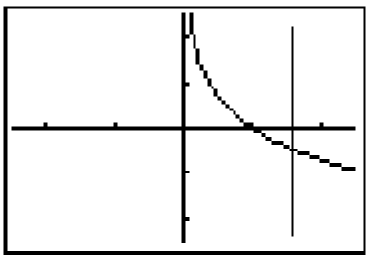
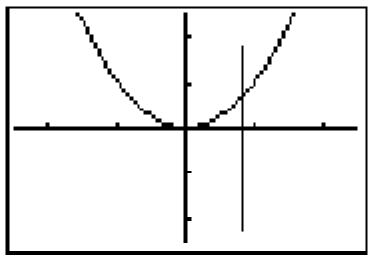
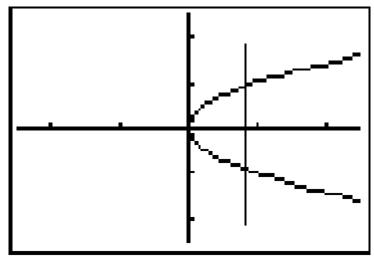
We will discuss the underlined words when we start covering one to one functions.

Example 1: Below are some graphs that we will be dealing with the rest of the course.

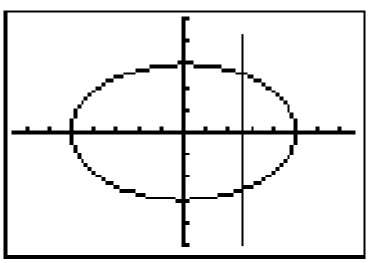
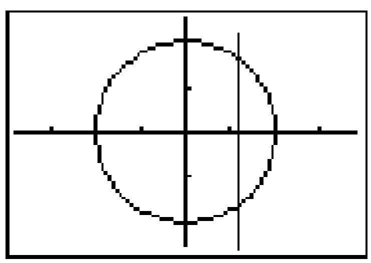
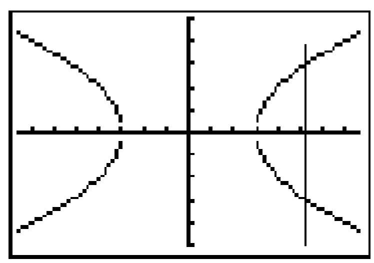


	Function	*Function*	*Function*
Domain:	$\mathbb{R}$	$\mathbb{R}$	$x > 0$
Range:	$y > 0$	$y > 0$	$\mathbb{R}$

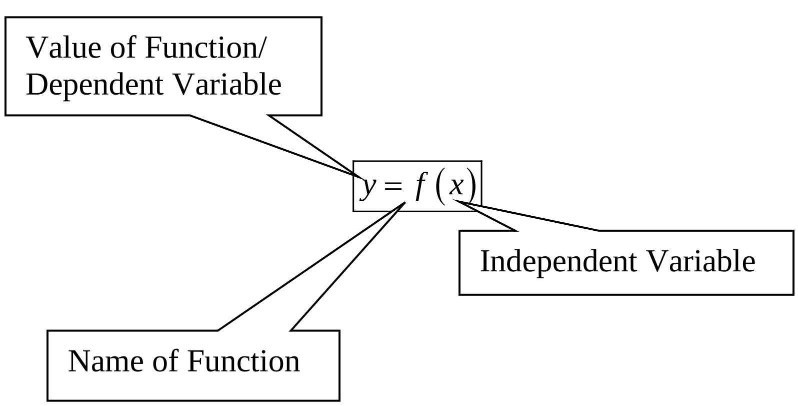




	Function	*Function*	*Not a Function*
Domain:	$x > 0$	$\mathbb{R}$	$x \geq 0$
Range:	$\mathbb{R}$	$y \geq 0$	$\mathbb{R}$

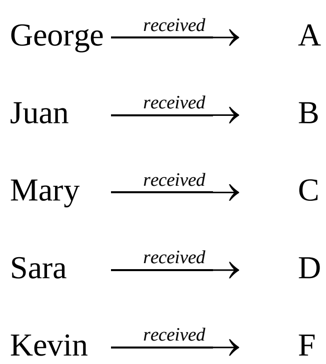




	Not a Function	*Not a Function*	*Not a Function*
Domain:	$-5 \leq x \leq 5$	$-2 \leq x \leq 2$	$x \leq -3 \cup x \geq 3$
Range:	$-3 \leq y \leq 3$	$-2 \leq y \leq 2$	$\mathbb{R}$

<u>Function Notation</u>:

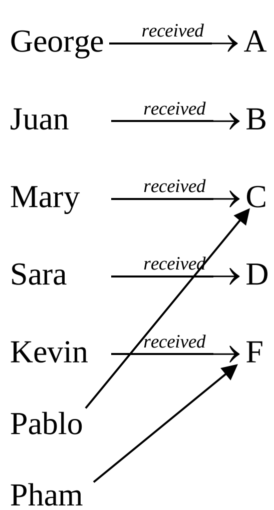


Example 2: Let's assume that there is a math class in your college with only five students. The instructor must assign a letter grade at the end of the course. The grade must be one out of these grades A, B, C, D, and F.



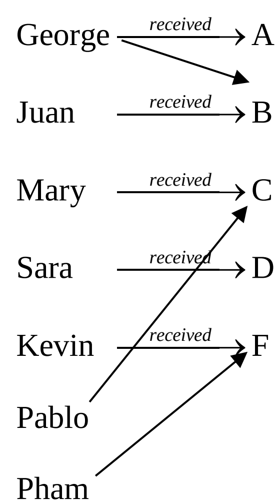
The above illustration is definitely a function since there is a unique grade for each student in class.

Now, let's change the scenario a little bit and assume that there are 7 students in class. Again, the instructor must assign a grade for each student.



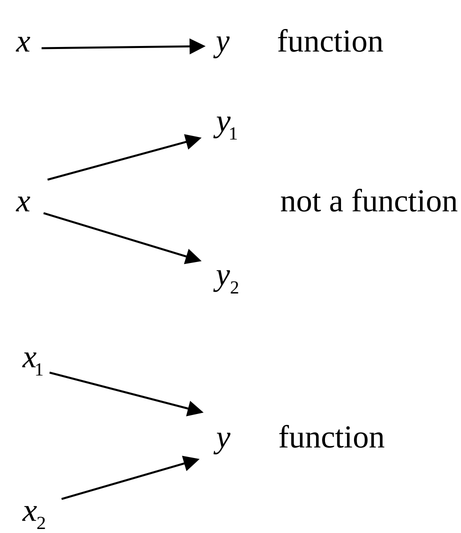
Do you think the above illustration is still a function? If you said yes, you are right since each student received a unique grade. It makes sense because more than one student can get an A in a certain course.

Now, let's change the example one more time and use the previous assumption.



Do you think the above is a function? Notice that George received A and B at the end of the course. Do you think this makes any sense? Can a student receive two different grades at the end of the course? Definitely not. Therefore the last example is NOT a function.

To summarize the definition of a function we give the following illustration:

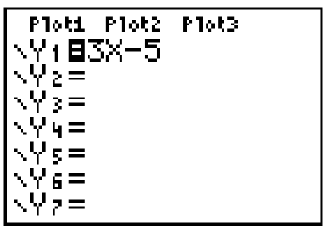


<u>**Note**</u>: When you enter a function in your TI, there is a fast and elegant way of generating a list of ordered pairs of that function. When you are given a function, you should be able to evaluate that function at any given value by simply substituting the given value into the function and evaluating the result. The TI 83/84 can do this in many ways as illustrated in the given example.

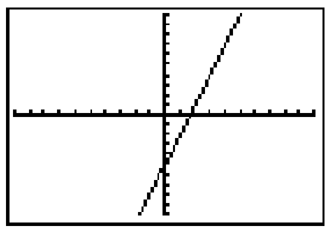
Example 3: Evaluate the function $f(x) = 3x - 5$ at $x = \{-2, 0, 4, 7, 10\}$.

Method 1: Press $\boxed{\text{Y =}}$ to go to the function editor. Let $Y_1 = f(x) = 3x - 5$ (see screen 'a').

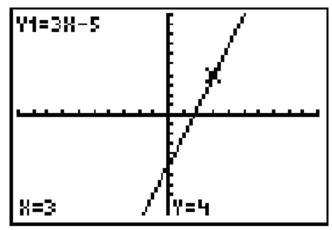
Press $\boxed{\text{GRAPH}}$ (see screen 'b'). Now, press $\boxed{\text{TRACE}}$ followed by the value of *x* at which you are evaluating the function. The corresponding *y*-value will appear at the bottom of the screen (see screen 'c'). Keep entering the rest of the *x*-values and record their corresponding *y*-values.



(a)



(b)

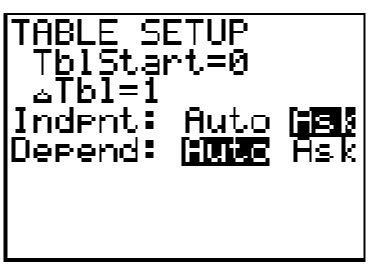


(c)

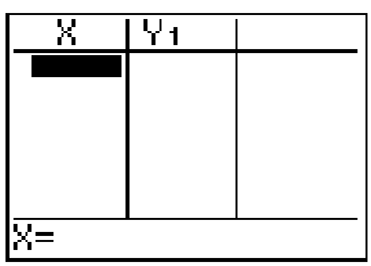
Method 2: Press $\boxed{\text{Y =}}$ to go to the function editor. Let $Y_1 = f(x) = 3x - 5$ (see screen 'a').

Press $\boxed{\text{2nd}}\boxed{\text{TBLSET}}$. In that screen you can customize your ordered pairs by setting the starting value for the independent value *x*. You can also set the increment by which *x* increases (if it is uniform).

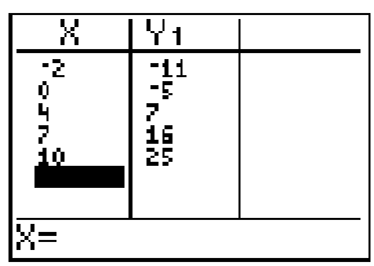
In our case, because we are concerned about certain values of the independent value of *x*, enter the following keys $\boxed{\texttt{Indpnt:}}\boxed{\texttt{Ask}}\boxed{\text{ENTER}}$. $\boxed{\texttt{Ask}}$ will be highlighted as shown in screen 'a' below. Now press $\boxed{\text{2nd}}\boxed{\text{TABLE}}$. You'll get screen 'b' below which shows an empty table. Every time you enter a value for *x*, the corresponding dependent value of *y* will automatically appear. Enter all values of *x* and write the 5 ordered pairs as they appear in screen 'c'.



(a)



(b)



(c)

Linear (First Degree) Functions:

General Form of a Linear (First Degree) Function
$\boxed{f(x) = ax + b}$
where a and b are real numbers

Notes:

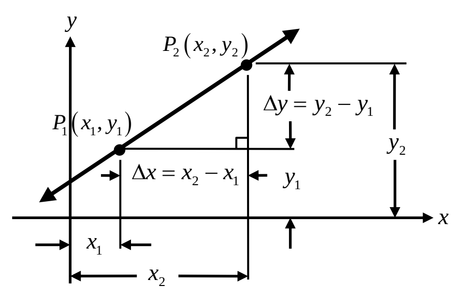
- The name **linear** comes from the fact that the graph of the function is always a non-vertical line.
- When we graph a line in rectangular coordinates, the line may intersect the *x*-axis, the *y*-axis, or it may intersect both axes.
- The *x*-intercept is where the line intersects the *x*-axis. To find the *x*-intercept, set *y* equal to zero and solve for *x*. The coordinate of *x*-intercept is $(A, 0)$.
- The *y*-intercept is where the line intersects the *y*-axis. To find the *y*-intercept, set *x* equal to zero and solve for *y*. The coordinate of *y*-intercept is $(0, b)$.
- The values at which the function is equal to zero are called the zeros of the function.
  i.e. If $f(c) = 0$, then *c* is called a **zero** of the function *f*.
- A zero of a function is the same as saying a **root** of the function which is also the same as saying an *x*-intercept of the function. i.e. $\boxed{\text{a zero} \equiv \text{a root} \equiv \text{an } x \text{ intercept}}$
- When we say find a complete graph using your TI, we mean find a window that shows all of the *x*-intercepts of the graph, the *y*-intercept of the graph, and all twists and turns of the graph.

Slope of a Line:

The slope of a line is the measure of its steepness (or inclination).

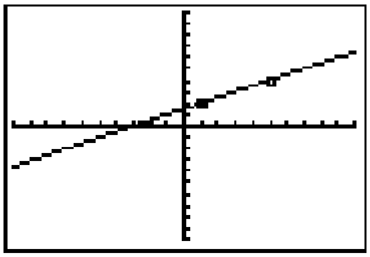
The slope of a line is denoted by *m* and is given by:

$$\boxed{\begin{aligned} slope &= \frac{rise}{run} \\ slope &= \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} \\ m &= \frac{y_2 - y_1}{x_2 - x_1} \end{aligned}}$$



Example 4: Find the slope of the line passing through the two points $(1,2)$ and $(5,4)$.

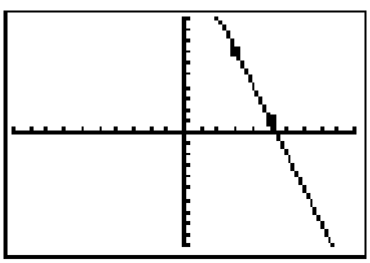
$$m=\frac{4-2}{5-1}=\frac{2}{4}=\frac{1}{2} \text{ (a positive slope)}$$



The line rises from left to right.

Example 5: Find the slope of the line passing through the two points $(3,7)$ and $(5,1)$.

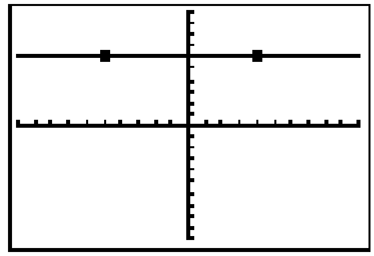
$$m=\frac{1-7}{5-3}=\frac{-6}{2}=-3 \text{ (a negative slope)}$$



The line falls from left to right.

Example 6: Find the slope of the line passing through the two points $(-5,6)$ and $(4,6)$.

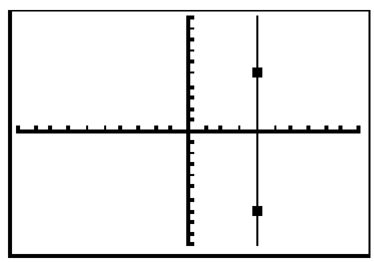
$$m=\frac{6-6}{4-(-5)}=\frac{0}{9}=0 \text{ (a zero slope)}$$



The line is a horizontal line.

Example 7: Find the slope of the line passing through the two points $(4,-7)$ and $(4,5)$.

$$m=\frac{5-(-7)}{4-4}=\frac{12}{0}=\text{(Undefined slope)}$$



The line is a vertical line.

Note 1: When you find the slope of a line, it doesn't matter which point you call point one or which point you call point two.

Note 2: A linear function is called a constant function if its slope is zero (i.e. its graph is a horizontal line).

Note 3: A vertical line is not a function. The slope of a vertical line is not defined.

Exercises 1.3

In exercises 1 – 18, decide whether the given relation is a function. State the domain and range of each relation. In any TI screen, assume that Xscl = Yscl = 1 and all boundary points are included.

1) $\{(-1,3),(0,2),(3,-4),(7,5),(10,10)\}$

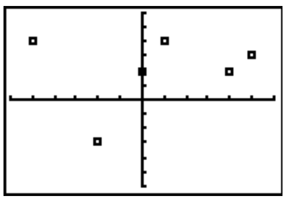
2) $\{(-2,1),(-1,2),(3,5),(5,4),(7,9)\}$

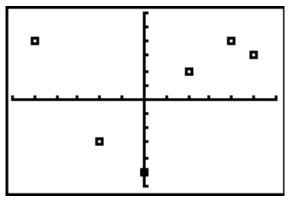
3) $\{(-5,2),(0,2),(1,2),(3,2),(6,2)\}$

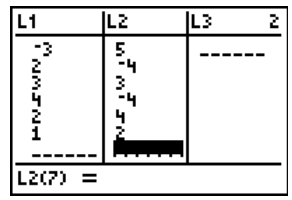
4) $\{(-3,1),(-1,1),(3,1),(5,1),(7,1)\}$

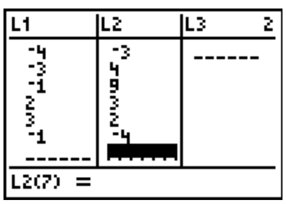
5) $\{(-6,-4),(-3,-1),(0,1),(-3,4),(5,6)\}$

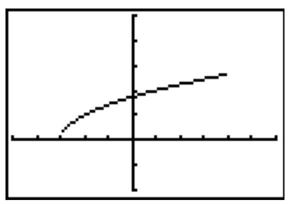
6) $\{(-4,-1),(1,2),(3,5),(1,4),(6,8)\}$

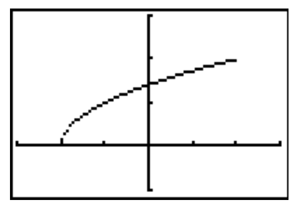
7) 

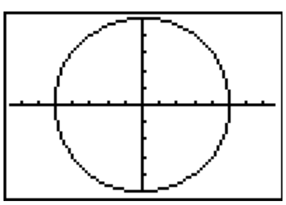
8) 

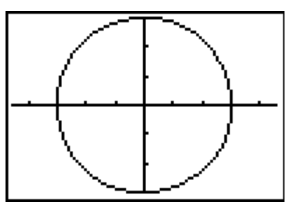
9) 

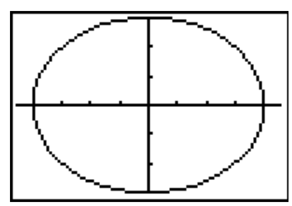
10) 

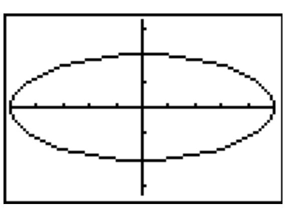
11) 

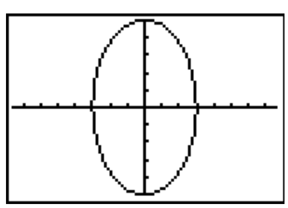
12) 

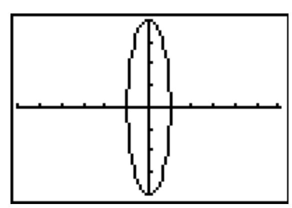
13) 

14) 

15) 

16) 

17) 

18) 

In exercises 19 – 27, evaluate the function at the given value of x.

19) $f(x) = 3x + 4, x = 2$

20) $f(x) = 2x - 5, x = -4$

21) $f(x) = x^2 - 2x + 3, x = 1$

22) $f(x) = 3x^2 + 4x - 10, x = -1$

23) $f(x) = \sqrt{2x^2 - 2}, x = 3$

24) $f(x) = \sqrt{3x^2 + 6}, x = 1$

25) $f(x) = |2x + 5| - 1, x = 2$

26) $f(x) = |3x + 4| - 2, x = 5$

27) $f(x) = x + |2x - 1|, x = 3$

In exercises 28 – 32, find the slope of the line passing through the given two points.

28) $(-2, 4), (5, 9)$

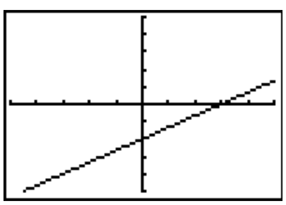
29) $(1, 5), (7, -2)$

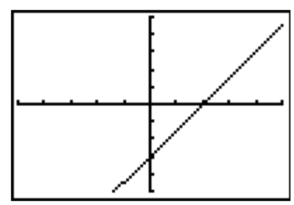
30) $(4, 6), (8, 6)$

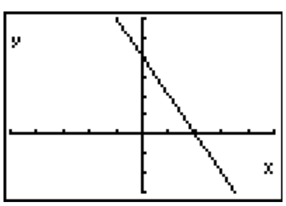
31) $(2, 3), (2, 5)$

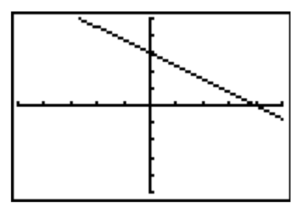
32) $(-1, -4), (5, 2)$

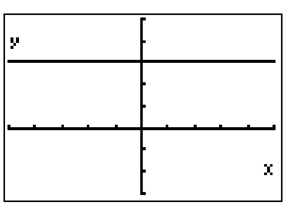
In exercises 33 – 40, find the slope of the given line. Assume that Xscl = Yscl = 1.

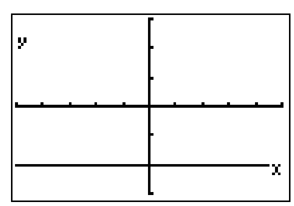
33) 

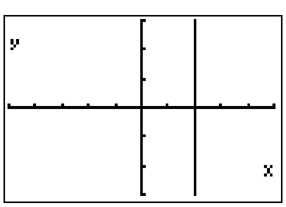
34) 

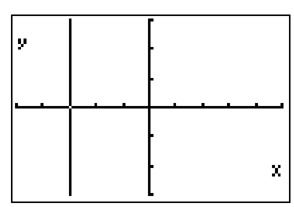
35) 

36) 

37) 

38) 

39) 

40) 

1.4 Equations of Lines

Example 1: Find the slope of the line passing through the two points (x_1, y_1) and (x, y).

Solution: $m = \dfrac{y - y_1}{x - x_1}$; cross multiply and you will get: $y - y_1 = m(x - x_1)$

Point - Slope Form of a Line

If a line passes through the point (x_1, y_1) with slope m, then its equation is given by

$$y - y_1 = m(x - x_1)$$

Example 2: Find the equation of the line passing through the point $(2,5)$ with slope 3.

Solution:

$$y - y_1 = m(x - x_1)$$
$$y - 5 = 3(x - 2)$$
$$y - 5 = 3x - 6$$
$$\therefore \boxed{y = 3x - 1}$$

Note: You have to solve the equation for y if you want to use your TI to graph it.

Example 3: Find the equation of the line passing through the point $(-3,5)$ with slope $\dfrac{2}{5}$.

Solution:

$$y - y_1 = m(x - x_1)$$
$$y - 5 = \frac{2}{5}(x - (-3))$$
$$y - 5 = \frac{2}{5}x + \frac{6}{5}$$
$$y = \frac{2}{5}x + \frac{6}{5} + 5$$
$$\therefore \boxed{y = \frac{2}{5}x + \frac{31}{5}}$$

Example 4: Find the equation of the line passing through the two points $(-2,3)$ and $(3,4)$.

Solution:

First we need to find the slope of the line

$$m = \frac{4-3}{3-(-2)} = \frac{1}{5}$$

Now, use the point-slope form to find the equation of the line.

$$y - y_1 = m(x - x_1)$$

Note: You can use either of the given two points and substitute it into the point-slope form.

$$y - 4 = \frac{1}{5}(x - 3)$$
$$y - 4 = \frac{1}{5}x - \frac{3}{5}$$
$$y = \frac{1}{5}x - \frac{3}{5} + 4$$

The equation of the line is:

$$\boxed{y = \frac{1}{5}x + \frac{17}{5}}$$

Example 5: Find the equation of the line passing through the point $(0,b)$ with slope m.

$$y-y_1=m(x-x_1),\ y-b=m(x-0)$$
$$y-b=mx-0,\ \therefore\ \boxed{y=mx+b}$$

or

$$m=\frac{y-b}{x-0}=\frac{y-b}{x},\ \text{cross multiply}$$
$$y-b=mx,\quad \therefore\ \boxed{y=mx+b}$$

<u>Slope y - intercept Form of a Line</u>

If a line passes through the point $(0,b)$ with a slope equal to m, then its equation is given by $y=mx+b$, where b is the value of the y-intercept at the point $(0,b)$.

Example 6: Find the equation of the line passing through the point (0, 3) with slope 2/5.

Solution: We are given the *y*-int (0, 3) so *b* = 3 and the slope *m* is equal to 2/5.

$$y=mx+b$$
$$\therefore\ \boxed{y=\frac{2}{5}x+3}$$

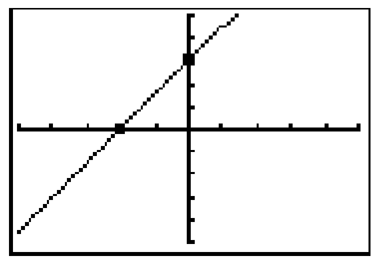
Example 7: Find the slope and *y*-intercept of the line $2x+3y=6$.

Solution: We need to solve for *y* to put the equation in the slope *y*-intercept form

$$y=mx+b$$

$$2x+3y=6$$
$$3y=-2x+6$$
$$y=-\frac{2}{3}x+2$$
$$\therefore\ \boxed{m=-\frac{2}{3},\ y_{\text{int}}=b=2}$$

Example 8: Find the equation of the line given in the TI screen below.



(Xscl=Yscl=1)

Solution: From the screen, we can see that

x-intercept is (-2, 0) and *y*-intercept = *b* = 3.

First, we need to find the slope of the line.

$$m=\frac{3-0}{0-(-2)}=\frac{3}{2}$$

Now use the slope *y*-intercept form

$$y=mx+b$$
$$\therefore\ \boxed{y=\frac{3}{2}x+3}$$

- **Standard Form of a Line**: $\boxed{ax+by=c}$
- Two lines are called parallel $(\|)$ iff (read as "<u>if and only if</u>") their slopes are equal: $l_1 \| l_2$ iff $m_{l1}=m_{l2}$
- Two lines are called perpendicular $(\perp)$ iff their slopes are the negative inverses of each other, in other words, the product of their slopes is equal to (-1): $l_1 \perp l_2$ iff $m_{l1}=-\frac{1}{m_{l2}}$; *i.e*: $(m_{l1})(m_{l2})=-1$

Example 9: Decide whether the following two lines are parallel, perpendicular, or neither:

$$2x+3y=6 \text{ and } 3x-2y=1$$

Solution: First, we need to solve each equation for y and then compare the two slopes.

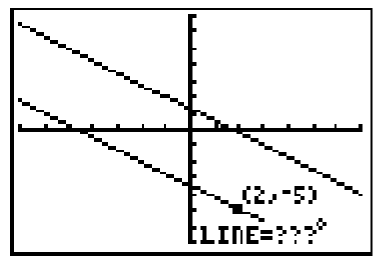
$$2x+3y=6 \Rightarrow 3y=-2x+6 \Rightarrow y=-\frac{2}{3}x+2 \Rightarrow \boxed{m_{l1}=-\frac{2}{3}}$$

$$3x-2y=1 \Rightarrow 2y=3x-1 \Rightarrow y=\frac{3}{2}x-\frac{1}{2} \Rightarrow \boxed{m_{l2}=\frac{3}{2}}$$

$$\boxed{\because m_{l1}\cdot m_{l2}=-1, \ \therefore \text{The lines are perpendicular to each other}}$$

Example 10: Find the equation of the line passing through the point (2, -5) and parallel to the line

$$3x+4y=5.$$



Solution: We need to find the slope of the given line.

$$3x+4y=5$$
$$4y=-3x+5$$
$$\therefore y=-\frac{3}{4}x+\frac{5}{4}, \text{ so } \boxed{m=-\frac{3}{4}}$$

<u>**Note**</u>**:** We can use the following short cut to find the slope of any line that is written in standard form:

$$m=-\frac{\text{coefficient of } x}{\text{coefficient of } y}=\boxed{-\frac{3}{4}}$$

If two lines are parallel, then their slopes are equal.

$\therefore$ The slope of the new line is also $-\frac{3}{4}$.

Now, use the given point (2, -5) and the slope and substitute into the point-slope form

$$y-y_1=m\left(x-x_1\right)$$

$$y+5=-\frac{3}{4}\left(x-2\right)$$

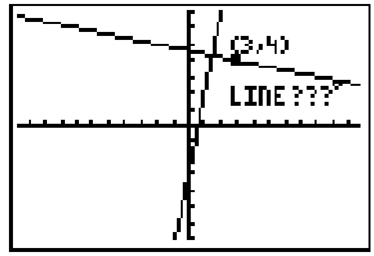
$$y+5=-\frac{3}{4}x+\frac{3}{2}$$

$$y=-\frac{3}{4}x+\frac{3}{2}-5$$

$$\therefore \boxed{y=-\frac{3}{4}x-\frac{7}{2}}$$

Example 11: Find the equation of the line passing through the point (3, 4) and perpendicular to the line

$5x - y = 3$.



Solution: We need to find the slope of the given line.

$5x - y = 3$

$y = 5x - 3$, so $\boxed{m = 5}$

Or use the short cut given in the previous example.

$$m = -\frac{coefficient\ of\ x}{coefficient\ of\ y}$$

$$\therefore \boxed{m = -\frac{5}{-1} = 5}$$

$\because$ If two lines are perpendicular, their slopes are the negative inverses of each other.

$\therefore$ The slope of the new line is $-\frac{1}{5}$.

Now, use the given point (3, 4) and the slope and substitute into the point-slope form

$$y - y_1 = m(x - x_1)$$

$$y - 4 = -\frac{1}{5}(x - 3)$$

$$y - 4 = -\frac{1}{5}x + \frac{3}{5}$$

$$y = -\frac{1}{5}x + \frac{3}{5} + 4$$

$$\therefore \boxed{y = -\frac{1}{5}x + \frac{23}{5}}$$

Exercises 1.4

Note: If possible, write the equation of each line in slope *y*-intercept form.

In exercises 1 – 4, find the equation of the line passing through the given point with the given slope.

1) $(-1,3); m=2$

2) $(3,-2); m=-5$

3) $(-2,5); m=\frac{2}{3}$

4) $(3,4); m=-\frac{2}{5}$

In exercises 5 – 8, find the equation of the line passing through the given two points.

5) $(-1,3)$ and $(2,-3)$

6) $(3,-2)$ and $(4,-1)$

7) $(-2,5)$ and $(4,-5)$

8) $(3,4)$ and $(1,3)$

9) Find the equation of the line passing through the point $(-2,5)$ with a zero slope.

10) Find the equation of the line passing through the point $(4,3)$ with a zero slope.

11) Find the equation of the line passing through the point $(5,2)$ with an undefined slope.

12) Find the equation of the line passing through the point $(3,-4)$ with an undefined slope.

In exercises 13 – 18, find the slope and *y*-intercept of the given line.

13) $2x+3y=6$

14) $5x+4y=8$

15) $3x-7y=7$

16) $5x-2y=4$

17) $x=31$

18) $y=-5$

In exercises 19 – 23, decide whether the given two lines are parallel, perpendicular, or neither.

19) $5x-3y=4$
$3x+5y=1$

20) $2x-3y=5$
$6x-4y=2$

21) $3x+6y=2$
$x+2y=5$

22) $2x-3y=4$
$4x-8y=3$

23) $3x+7y=1$
$7x+3y=-2$

In exercises 24 – 27, a linear function is defined by Y_1 and a table of ordered pairs is given. Use the points to find the slope of the line and *y*-intercept of the line. Then use the slope *y*-intercept form to write the equation of the line.

24)

X	Y1	
-3	5.25	
-2	4.5	
-1	3.75	
0	3	
1	2.25	
2	1.5	
3	.75	

X=3

25)

X	Y1	
-5	6	
-2	4.2	
-1	3.6	
1	2.4	
2	1.8	
3	1.2	
5	0	

X=5

26)

X	Y1	
-5	1	
-2	1.6	
-1	1.8	
1	2.2	
2	2.4	
3	2.6	
5	3	

X= -5

27)

X	Y1	
-4	0	
-2	1.5	
-1	2.25	
1	3.75	
2	4.5	
3	5.25	
4	6	

X=4

28) Find the equation of the line passing through the point $(-2,3)$ and parallel to the line $4x+5y=1$.

29) Find the equation of the line passing through the point $(1,-5)$ and parallel to the line $2x+7y=3$.

30) Find the equation of the line passing through the point $(5,2)$ and perpendicular to the line $3x-y=2$.

31) Find the equation of the line passing through the point $(-3,-1)$ and perpendicular to the line $4x-3y=5$.

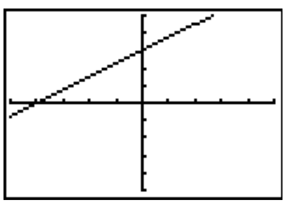
32) Find the equation of the line passing through the point $(3,-1)$ and the midpoint of the line segment with endpoints $(4,-3)$ and $(6,5)$.

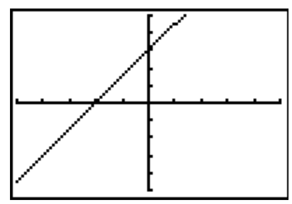
33) Find the equation of the line passing through the point $(1,-2)$ and the midpoint of the line segment with endpoints $(-5,-2)$ and $(3,6)$.

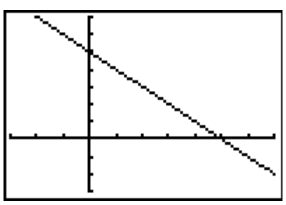
34) Find the equation of the line passing through the midpoint of the line segment with endpoints $(5,-4)$ and $(-3,-2)$, and perpendicular to the line $7x-2y=4$.

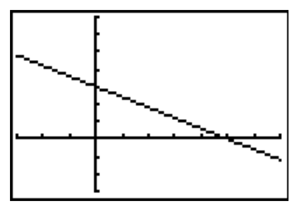
35) Find the equation of the line passing through the midpoint of the line segment with endpoints $(3,-7)$ and $(5,-3)$, and perpendicular to the line $5x+4y=3$.

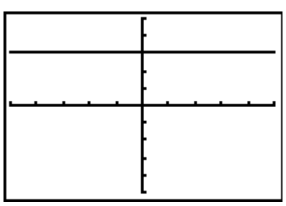
In exercises 36 – 43, use the given TI screen to find the equation of the line. Assume that Xscl = Yscl = 1.

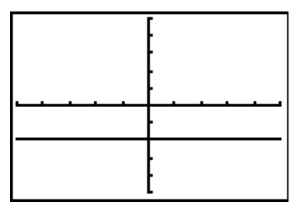
36) 

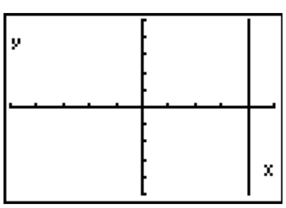
37) 

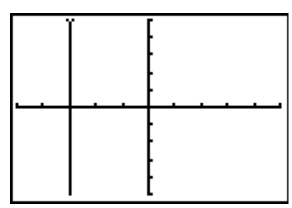
38) 

39) 

40) 

41) 

42) 

43) 

1.5 Linear (First Degree) Equations in One Variable

A linear equation is a statement of equality in which at least one side of the equation contains a first degree variable (i.e. a linear equation equates two first degree expressions).

The solution to a linear equation in one variable is the set of all values that make the equation a true statement.

Equations have the following two properties:

1. **Addition Property of Equality**: If $a = b$ then $a + c = b + c$ and $a - c = b - c$

   The above property simply says that if you add/subtract a number to/from one side of the equation, you must add/subtract the same number to/from the other side of the equation to keep the equation balanced.

2. **Multiplication Property of Equality**: If $a = b$ then $ac = bc$ and $\frac{a}{c} = \frac{b}{c}$ where $c \neq 0$

   The above property simply says that if you multiply or divide one side of the equation by a non-zero number, you must multiply or divide the other side of the equation by the same number to keep the equation balanced.

To solve a linear equation in one variable:

1. Clear all fractions from both sides of the equation by multiplying both sides by the least common denominator.
2. Simplify each side of the equation by using the distributive property and combining like terms.
3. Move all variable terms on one side of the equation and all constants on the other side. This can be done by using the addition property of equality.
4. Simplify each side of the equation by combining like terms.
5. Now, the equation has the form $ax = b$. Solve for the variable by multiplying each side of the equation by the reciprocal of the variable's coefficient.
6. Check your answer by substituting the solution into the original equation to verify that the solution will make the equation a true statement.

There are three types of equations:

1. **Conditional Equation:** This occurs when we have a countable number of values at which the equation is true.

2. **Identity Equation:** This occurs when we have infinitely many solutions at which the equation is true.

3. **Inconsistent (Contradiction) Equation:** This occurs when we get a false statement, or contradiction, when we solve the equation. We have no solution at which the equation is true.

Example 1: Solve the following equation: $6(2x-4)-3(x+1)=3(2x+3)-6$

Solution: Step 1: Simplify each side completely:

$$12x-24-3x-3=6x+9-6$$
$$9x-27=6x+3$$

Step 2: Use the addition property of equality to get all variable terms on one side of the equation and all constants on the other side of the equation. $\Rightarrow 3x=30$

Step 3: Use the multiplication property of equality to solve for the variable. Multiply both sides of the equation by 1/3 (which is the same as dividing both sides of the equation by 3). Remember that the division operation is a special case of multiplication.

$x=10,$ we have one solution $\Rightarrow \therefore$ The equation is conditional.

Note: Due to space limitation, we will not show the steps for checking solutions.

Example 2: Solve the following equation: $\frac{2}{3}(x+5)+\frac{1}{4}=\frac{5}{6}(2x-3)+\frac{1}{2}+\frac{7}{12}$

Solution: $\frac{2}{3}(x+5)+\frac{1}{4}=\frac{5}{6}(2x-3)+\frac{1}{2}+\frac{7}{12}$

We can see that the LCD is 12, so we multiply both sides of the equation by 12.

$$12\left[\frac{2}{3}(x+5)+\frac{1}{4}\right]=12\left[\frac{5}{6}(2x-3)+\frac{1}{2}+\frac{7}{12}\right]$$
$$8(x+5)+3=10(2x-3)+6+7$$
$$8x+40+3=20x-30+13$$
$$8x+43=20x-17$$
$$60=12x$$

$x=5,$ we have one solution $\Rightarrow \therefore$ The equation is conditional.

Example 3: Solve the following equation: $5(x-2)-3(2x+1)=3(x-3)-4(x+1)$

Solution: $5(x-2)-3(2x+1)=3(x-3)-4(x+1)$

$5x-10-6x-3=3x-9-4x-4$

$-x-13=-x-13$ (A true statement; we can stop here.)

$0=0$ (A true statement)

$\therefore$ The solution is the set of all real numbers which is a set of infinitely many real numbers.

$\therefore$ The equation is called an identity.

Example 4: Solve the following equation: $3(2x-5)+2(3x-1)=4(3x+1)+7$

Solution: $3(2x-5)+2(3x-1)=4(3x+1)+7$

$6x-15+6x-2=12x+4+7$

$12x-17=12x+11$

$-17=11$ (This is a false statement, so we can stop here.)

$0=28$ (false statement), $\therefore$ There is no solution to the given equation.

The equation is called inconsistent or contradiction.

<u>Solving equations using TI 83/83+, TI 84+</u>:

There are several methods to solve equations using graphing calculators. Here, we will demonstrate how to solve the equations using the TI 84+ calculator. When we mention the TI 84+, the same commands also apply to the TI 83, TI 83+, TI 83+ silver edition, and TI 84+ silver edition.

Example 5: Solve the following equation: $3(2x-5)+1=4(3-x)+4$

<u>Solution - Method 1: The Intersection Method.</u>

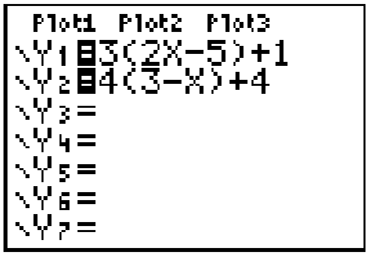
Let the left hand side of the equation equal to Y_1 and the right hand side of the equation equal to Y_2

$\therefore Y_1=3(2x-5)+1$ and $Y_2=4(3-x)+4$

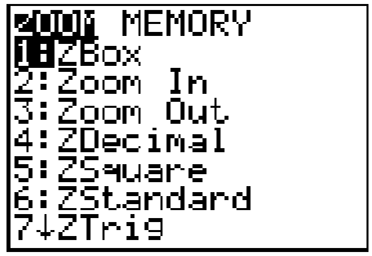
Enter the above two functions in your TI by pressing the following key: $\boxed{\text{Y =}}$. You'll get a screen called the function editor (screen (a) on the next page). Enter both functions as shown above.

Press $\boxed{\text{ZOOM}}$ $\boxed{\text{6: ZStandard}}$ (screen b). Now you will see screen (c) in your TI. Press $\boxed{\text{2nd}}$ $\boxed{\text{CALC}}$ $\boxed{\text{5: intersect}}$ 5 as shown in screen (d). The calculator will ask for the first curve (screen (e)). Press $\boxed{\text{ENTER}}$. Cursor will move to the second curve and the calculator will ask for second curve (screen (f)). Use the up/down arrow if you have more than two graphs. In our example you can simply press $\boxed{\text{ENTER}}$ and you will get screen (g). For the prompt "guess", simply press $\boxed{\text{ENTER}}$ one more time. Now you'll get screen (h). At the bottom of screen (h) you'll see the *x*-coordinate and *y*-coordinate of the point of intersection.

The *x*-value is the solution to the given equation since it is the value at which the two lines Y_1 and Y_2 are intersecting. Therefore, ignore the value of *Y*.



(a)



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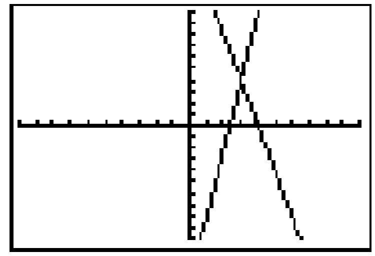
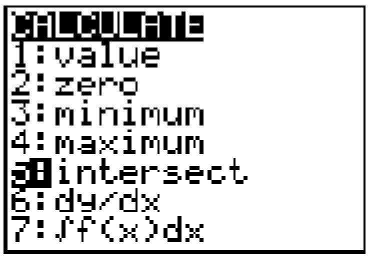
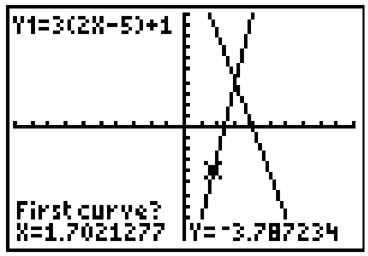


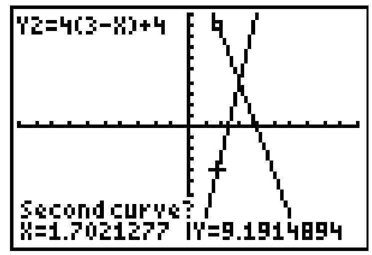
(c)



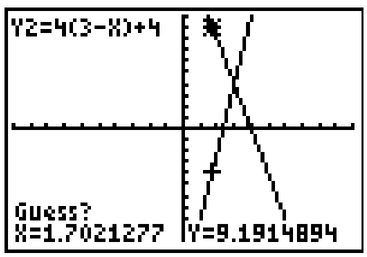
(d)



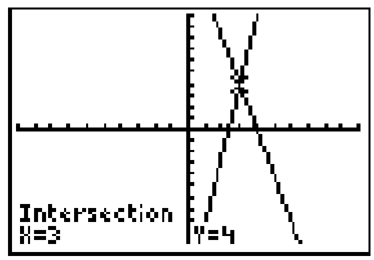
(e)



(f)



(g)

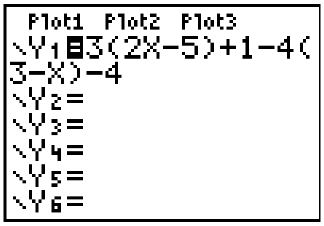


(h)

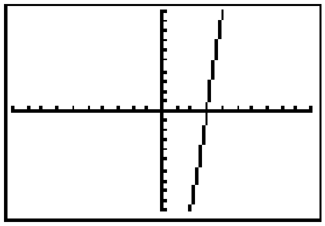
<u>Note</u>: If you have only two lines, simply press ENTER three times and you'll get the answer.

<u>Solution - Method 2: The *x*-intercept (Root or Zero) Method.</u>

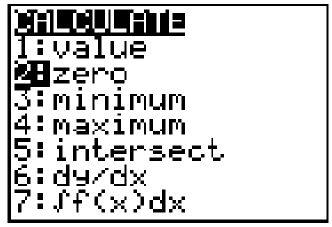
For the equation $3(2x-5)+1=4(3-x)+4$, first write the equation in standard form by simply having a zero on one side of the equation. Press $\boxed{Y=}$ and let $Y_1=3(2x-5)+1-4(3-x)-4$, (see screen (a)). Now, press GRAPH and you'll get the graph shown in screen (b). Press 2nd CALC `2:zero` as shown in screen (c). You'll get the prompt "Left bound?" shown in screen (d). Move the cursor anywhere to the left of the *x*-intercept and press ENTER .You'll get the prompt "Right Bound?" shown in screen (e). Move the cursor anywhere to the right of *x*-intercept and press ENTER one more time. For the prompt "guess" in screen (f), press ENTER or first move the cursor close to the *x*-intercept, then press ENTER . Now you'll get screen (g). The value of *x*-intercept is shown at the bottom of the screen. This is the solution to the given equation.



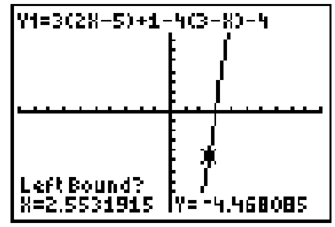
(a)



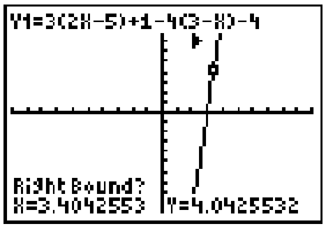
(b)



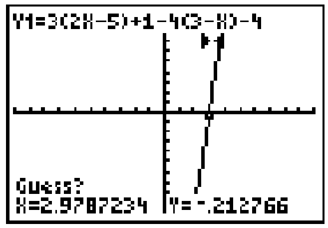
(c)



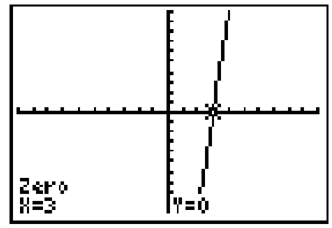
(d)



(e)



(f)



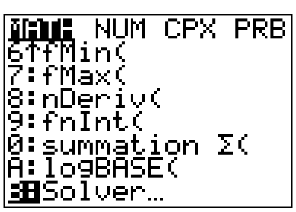
(g)

<u>Solution - Method 3: Using the Built-in Function "Solver".</u>

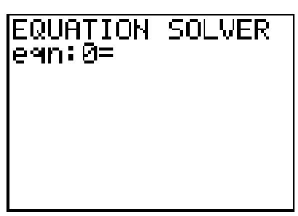
For the equation $3(2x-5)+1=4(3-x)+4$, first write the equation in standard form by simply having a zero on one side of the equation. Press $\boxed{\text{MATH}}$ $\boxed{\text{B: Solver}}$ $\boxed{\text{ENTER}}$, as shown in screen (a) below (or you can simply press $\boxed{\text{MATH}}$ $\boxed{\text{ALPHA}}$ $\boxed{\text{B}}$ directly). Now, you will get screen (b). If you have any previous equation in your TI solver, use the up arrow ▲ until you see the old equation and press clear. Enter the equation you want to solve in your TI solver, as shown in screen (c), and press $\boxed{\text{ENTER}}$. You'll get screen (d) as shown. Now, the cursor is blinking on an old value of *x*, either enter a guess for the solution to the equation (you'll do this if you have more than one solution to the equation and you have an idea about the solutions) or press the green buttons $\boxed{\text{ALPHA}}$ $\boxed{\text{SOLVE}}$. The SOLVE is above the $\boxed{\text{ENTER}}$ button. Now, you will get screen (e), on that screen, you will see that the cursor is blinking on the value next to *x*. This is the solution to the given equation.

<u>Note</u>: To find Solver using the TI-83 graphing calculator press $\boxed{\text{MATH}}$ $\boxed{\text{0: Solver}}$ $\boxed{\text{ENTER}}$

(a)



(b)



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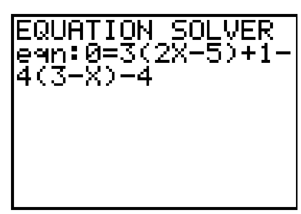

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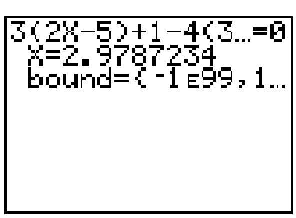
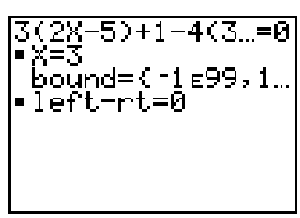


(e)



<u>**Note**</u>: You can see that Solver gives answers to an equation one at a time. We don't recommend using it if you know that the equation you are solving has more than one answer. Solver is usually used in special cases called "What if". We will demonstrate this later when we introduce the compound interest formula, continuous interest formula, and half-life formula in chapter five.

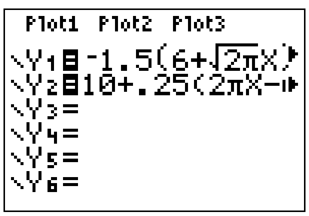
Note: The idea of using a graphing calculator in our course is to show that graphical and analytical solutions should go hand in hand. Sometimes it is a waste of time and effort to solve certain examples analytically. We will demonstrate this using the following example:

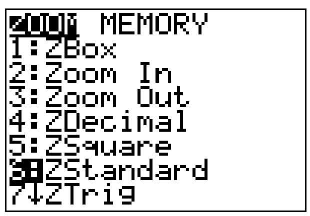
Example 6: Solve the equation: $-1.5(6+\sqrt{2\pi x})+\sqrt[7]{15}=10+.25(2\pi x-6.1)$

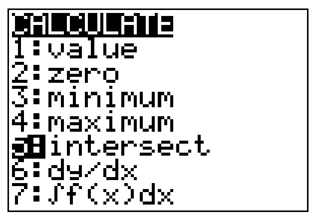
Solution: We will solve this equation by using the intersection method. Notice that the TI-84 doesn't wrap the text in the function editor. To clarify how to type each side of the equation in example 6, press the following key strokes exactly as shown next to type Y1 and Y2 in the function editor:

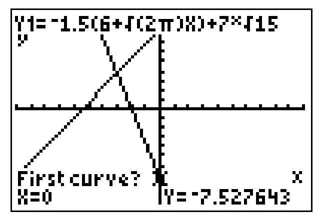
$$Y_1 = \boxed{-}\boxed{1}\boxed{.}\boxed{5}\boxed{(}\boxed{6}\boxed{+}\boxed{\text{2nd}}\boxed{x^2}\boxed{2}\boxed{\text{2nd}}\boxed{\wedge}\boxed{\triangleright}\boxed{x}\boxed{)}\boxed{+}\boxed{7}\boxed{\text{MATH}}\boxed{5:\sqrt[x]{\ }}\boxed{1}\boxed{5}$$

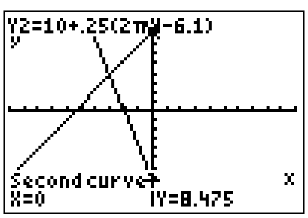
$$Y_2 = \boxed{10}\boxed{+}\boxed{.}\boxed{2}\boxed{5}\boxed{(}\boxed{2}\boxed{\text{2nd}}\boxed{\wedge}\boxed{x}\boxed{-}\boxed{6}\boxed{.}\boxed{1}\boxed{)}$$

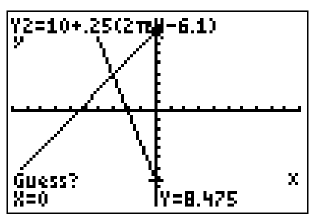


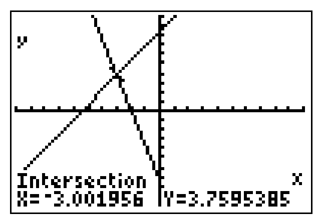












The solution is $\boxed{x \approx -3.002}$. You only take the *x*-value and ignore the *y*-value in the last screen.

<u>**Note**</u>: In typing the expression $\sqrt{2\pi}x$, we typed $\boxed{\text{2nd}}\boxed{\sqrt{\ }}\boxed{2}\boxed{\text{2nd}}\boxed{\pi}\boxed{\triangleright}\boxed{x}$. Notice that when we typed the expression, we used the right arrow $\boxed{\triangleright}$ to move the cursor out of the radical symbol before typing *x*.

<u>**Note**</u>: To find any higher root like $\sqrt[7]{2\pi}$ as an example, first press the index 7 followed by $\boxed{\text{MATH}}$ $\boxed{5: \sqrt[x]{\ }}$ then type the radicand 2π followed by $\boxed{\text{ENTER}}$. If you have a TI-83 graphing calculator, make sure to enter the radicand in parentheses especially when the radicand has more than one factor. The complete TI-83 key strokes are as follows: $\boxed{7}\boxed{\text{MATH}}$ $\boxed{5: \sqrt[x]{\ }}\boxed{(}\boxed{2}\boxed{\text{2nd}}\boxed{\pi}\boxed{)}\boxed{\text{ENTER}}$.

Exercises 1.5

In exercises 1 – 14, solve the given equation analytically.

1) $7(x-1)=4(x+2)$

2) $x-(x+4)=2x+4(x-1)$

3) $-4(2x-6)=-3(x-4)+2$

4) $4\{2-[3(x+1)-2(x+1)]\}=-2x$

5) $-2\{4(x+3)-5[3x-2(2x+7)]-8\}=-20x-12$

6) $-3(6-4x)=4-\{5x-[6x-(4x-(3x+2))]\}$

7) $\dfrac{x+3}{4}+\dfrac{2x-1}{3}=16$

8) $\dfrac{x-2}{3}-\dfrac{x-3}{4}=\dfrac{1}{6}$

9) $0.2(x-25)=1.3x+0.3(x+2)$

10) $4x+12-8x=-6(x-2)+2x$

11) $-[4-(x-2)]=2(x-1)-x$

12) $3(x+3)-4(2x-7)=-5x+2$

13) $6(x-1)=-3(2-x)+3x$

14) $4(2-3x)=-[6x-(8-6x)]$

In exercises 15 – 18, use a graphing calculator to solve each equation by using two methods:

(a) The **intersection** method

(b) The *x*-intercept (**zero or root**) method

15) $0.23\pi+1.23(3x-4)=\sqrt[5]{3\pi}+2.78\left(x-\sqrt{12.3}\right)$

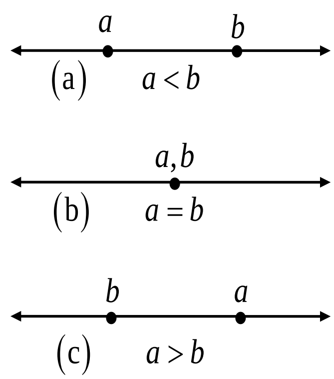
16) $5.73\pi-4.12(2x+3)=\sqrt[7]{1.24\pi}-5.12\left(3x-\sqrt{7.26}\right)$

17) $2x-4.3\left(3.12x+\sqrt[8]{3\pi}\right)=\sqrt[5]{3.2\pi}-1.73\left(2x+\sqrt{3.2}\right)$

18) $5.6x+2.54\left(3x-\sqrt[4]{9.2\pi}\right)=\sqrt[6]{4.3\pi}+5.4\left(x+\sqrt{11}\right)$

1.6 Linear Inequalities in One Variable

If we are given two numbers *a* and *b* and asked to plot the two numbers *a* and *b* on a number line, then either *a* is to the left of *b, a* and *b* are equal, or *a* is to the right of *b* as shown below.



a) If *a* is to the left of *b*, we say that "*a* is less than *b*" and write ***a* < *b***.

b) If *a* is to the right of *b*, we say that "*a* is greater than *b*" and write ***a* > *b***.

c) If *a* is at the same location as *b,* we say that "*a* is equal to *b*" and write ***a* = *b***.

If *a* is either less than or equal to *b*, we write $a \le b$ and say that "*a* is less than or equal to *b*.

Similarly, $a \ge b$ means that *a* is either greater than or equal to *b*.

The symbols <, >, ≤, and ≥ are called inequality symbols.

Note that $a < b$ and $b > a$ mean the same thing. It does not matter whether we write $2 < 3$ or $3 > 2$.

Furthermore, if $a < b$ or if $b > a$, then the difference $(b - a)$ is positive.

Statements of the form $a < b$ or $b > a$ are called strict inequalities. Whereas statements of the form $a \le b$ or $b \ge a$ are called non-strict inequalities. An inequality is a statement in which two expressions are related by an inequality symbol.

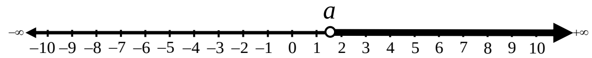
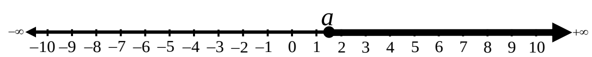
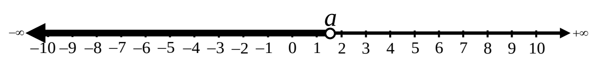
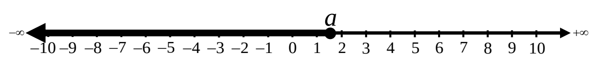
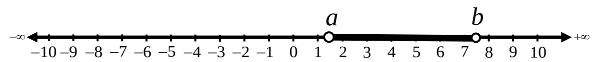
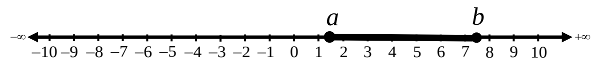
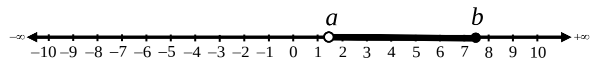
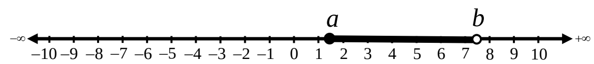
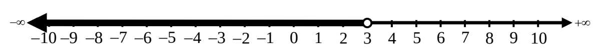
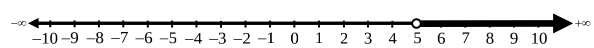
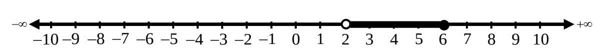
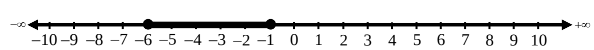
Based on the discussion so far, we conclude that:

$a > 0$ is equivalent to *a* is positive

$a < 0$ is equivalent to *a* is negative

We sometimes read $a > 0$ by saying that "*a* is positive." If $a \ge 0$, then either $a > 0$ or $a = 0$, and we can read this as "*a* is nonnegative."

Writing Inequalities in Set-Builder Notation or Interval Notation:

Solution set represented in set builder notation	Solution set indicated on a number line	Solution set represented in interval notation
$\{x \mid x > a\}$		(a, ∞)
$\{x \mid x \geq a\}$		$[a, \infty)$
$\{x \mid x < a\}$		$(-\infty, a)$
$\{x \mid x \leq a\}$		$(-\infty, a]$
$\{x \mid a < x < b\}$		(a, b)
$\{x \mid a \leq x \leq b\}$		$[a, b]$
$\{x \mid a < x \leq b\}$		$(a, b]$
$\{x \mid a \leq x < b\}$		$[a, b)$
$\{x \mid x < 3\}$		$(-\infty, 3)$
$\{x \mid x > 5\}$		$(5, \infty)$
$\{x \mid 2 < x \leq 6\}$		$(2, 6]$
$\{x \mid -6 \leq x \leq -1\}$		$[-6, -1]$

Linear (First Degree) Inequalities in One Variable:

An inequality is a statement in which two expressions are related by an inequality symbol and at least one side of the inequality contains a first degree variable. In other words, an inequality compares two first degree expressions. The solution to a linear inequality in one variable is the set of all values that makes the inequality a true statement. Inequalities have the following two properties (we will use the inequality symbol $<$ for all properties. These properties also apply for all other symbols of inequalities).

Addition Property of Inequalities:

$$\boxed{\text{If } a < b \text{ and } c \text{ is a real number, then } a + c < b + c \text{ and } a - c < b - c}$$

The above property simply says that if you add a number to one side of the inequality, you must add the same number to the other side of the inequality. Similarly, if you subtract a number from one side of the inequality, you must subtract the same number from the other side of the inequality.

Multiplication Property of Inequalities:

1) $\boxed{\text{If } a < b \text{ and } c \text{ is a positive number, then } ac < bc \text{ and } \frac{a}{c} < \frac{b}{c}}$

The above property simply says that if you multiply or divide one side of the inequality by a positive number, you must multiply or divide the other side of the inequality by the same number.

2) $\boxed{\text{If } a < b \text{ and } c \text{ is a negative number, then } ac > bc \text{ and } \frac{a}{c} > \frac{b}{c}}$

The above property simply says that if you multiply or divide one side of the inequality by a negative number, you must multiply or divide the other side of the inequality by the same number **and** you must change the direction of the inequality (sense of inequality).

To solve a linear inequality in one variable:

1. Clear all fractions from both sides of the inequality by multiplying both sides by the least common denominator.
2. Simplify each side of the inequality by using the distributive property and combining like terms.
3. Move all variable terms on one side of the inequality and all constants on the other side. This can be done by using the addition property of inequalities.
4. Simplify each side of the inequality by combining like terms.
5. Solve for the variable by multiplying each side of the inequality by the reciprocal of the variable's coefficient. Remember, if you multiply or divide by a negative number; you must change the direction of the inequality.

Note 1: If you get a true statement, the solution to the inequality will be the set of all real numbers. To graph the all real numbers solution, graph a number line and shade the whole line.

Note 2: If you get a false statement, the inequality has no solution (or you can write "empty set" or "null set"). To graph the empty set solution, graph a number line but do not shade it.

Example 1: Solve the following inequality: $4(x-2)+5>2(x+1)-3$

Solution: $4(x-2)+5>2(x+1)-3$

$4x-3>2x-1$

$2x>2$

$x>1$

Set Notation Solution: $\{x \mid x>1\}$

Interval Notation Solution: $(1,\infty)$

Example 2: Solve the following inequality: $-11x-(6x-4)+5 \ge 3x+1$

Solution: $-11x-(6x-4)+5 \ge 3x+1$

$$-17x+9 \ge 3x+1$$

$$-20x \ge -8 \Rightarrow\ x \le \frac{8}{20} \Rightarrow\ x \le \frac{2}{5}$$

Set Notation Solution: $\left\{x \mid x \le \frac{2}{5}\right\}$, Interval Notation Solution: $\left(-\infty, \frac{2}{5}\right]$

Example 3: Solve the following equation and its related inequalities:

$a)$ $\sqrt[6]{8}+1.56(2x-1.73\pi)+2.45 = 2.3(\sqrt[3]{7}-1.15x)+3$

$b)$ $\sqrt[6]{8}+1.56(2x-1.73\pi)+2.45 \ge 2.3(\sqrt[3]{7}-1.15x)+3$

$c)$ $\sqrt[6]{8}+1.56(2x-1.73\pi)+2.45 \le 2.3(\sqrt[3]{7}-1.15x)+3$

Solution: We will solve the example by using two methods, the intersection method and *x*-intercept (zero or root) method.

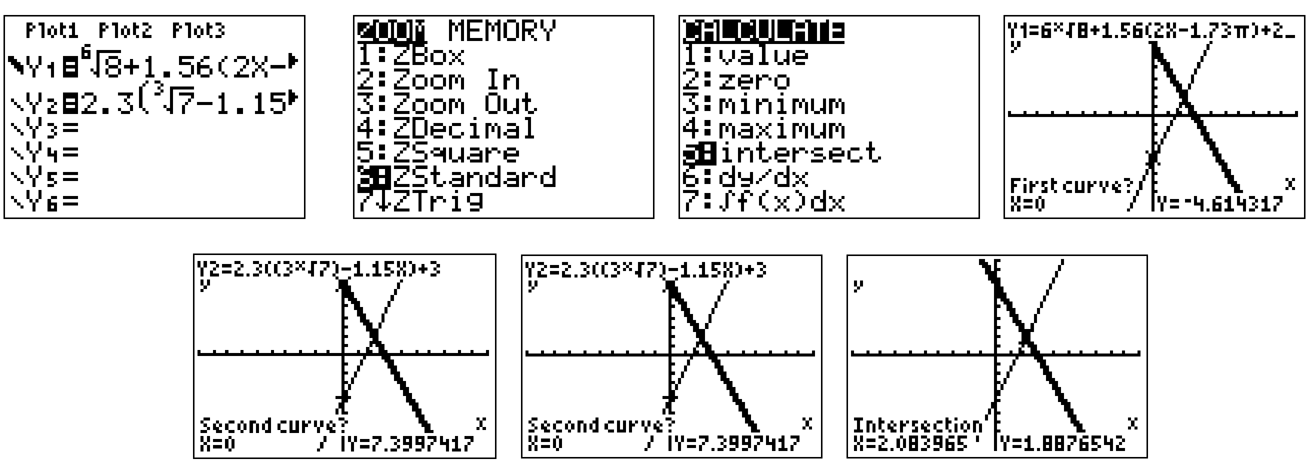
<u>First Method: Intersection method</u>:

Enter the left side of the equation in your TI as Y_1 and the right side as Y_2. For clarity, we changed the line type of Y_2 to a thick line.

Notice that the first screen doesn't show the whole algebraic expressions of Y_1 and Y_2. To type both properly, press the following key strokes exactly as shown next in the function editor of your TI.

$$Y_1 = \boxed{6}\boxed{\text{MATH}}\boxed{5:\sqrt[x]{\ }}\boxed{8}\boxed{\triangleright}\boxed{+}\boxed{1}\boxed{.}\boxed{5}\boxed{6}\boxed{(}\boxed{2}\boxed{x}\boxed{-}\boxed{1}\boxed{.}\boxed{7}\boxed{3}\boxed{\text{2nd}}\boxed{\wedge}\boxed{)}\boxed{+}\boxed{2}\boxed{.}\boxed{4}\boxed{5}$$

$$Y_2 = \boxed{2}\boxed{.}\boxed{3}\boxed{(}\boxed{\text{MATH}}\boxed{4:\sqrt[3]{\ }(}\boxed{7}\boxed{\triangleright}\boxed{-}\boxed{1}\boxed{.}\boxed{1}\boxed{5}\boxed{x}\boxed{)}\boxed{+}\boxed{3}$$



a) From the last screen, the solution to the equality is $\boxed{x \approx 2.084}$

b) The inequality asks for the interval where the thin line Y_1 is equal to or above (higher than) the thick line Y_2. From the last screen, the interval is $[2.084, \infty)$.

c) The inequality asks for the interval where the thin line Y_1 is equal to or below (lower than) the thick line Y_2. From the last screen, the interval is $(-\infty, 2.084]$.

<u>Second Method: *x*-intercept (zero or root) method</u>:

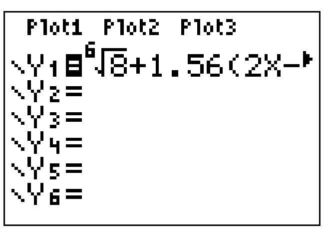
First, write the equation in standard form by moving all terms on the right side of the equation to the left hand side of the equation (remember to switch the sign of the coefficient of any term you move to the left). After writing the equation and its related inequalities in standard form we get the following:

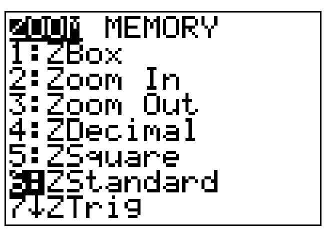
$$a)\quad \sqrt[6]{8}+1.56(2x-1.73\pi)+2.45-2.3(\sqrt[3]{7}-1.15x)-3=0$$
$$b)\quad \sqrt[6]{8}+1.56(2x-1.73\pi)+2.45-2.3(\sqrt[3]{7}-1.15x)-3\geq 0$$
$$c)\quad \sqrt[6]{8}+1.56(2x-1.73\pi)+2.45-2.3(\sqrt[3]{7}-1.15x)-3\leq 0$$

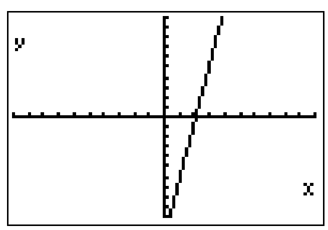
Enter the nonzero side of the equation in your TI as Y_1. Below, we are showing the key strokes needed to type the nonzero side. Due to space limitation, this time we wrote each coefficient as one string instead of showing each number as in the first method.

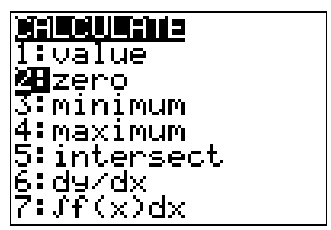
$$Y_1=\boxed{6}\boxed{\text{MATH}}\boxed{5:\sqrt[x]{\ }}\boxed{8}\boxed{\triangleright}\boxed{+}\boxed{1.56}\boxed{(}\boxed{2x}\boxed{-}\boxed{1.73\pi}\boxed{)}\boxed{+}\boxed{2.45}\boxed{-}\boxed{2.3}\boxed{(}\boxed{\text{MATH}}\boxed{4:\sqrt[3]{\ }(}\boxed{7}\boxed{\triangleright}\boxed{-}\boxed{1.15x}\boxed{)}\boxed{-}\boxed{3}$$

Make sure to use $\boxed{\text{6: ZStandards}}$ for the TI window.

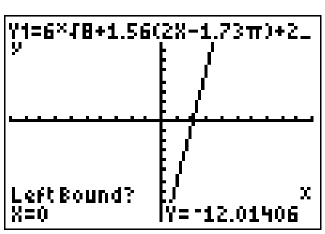


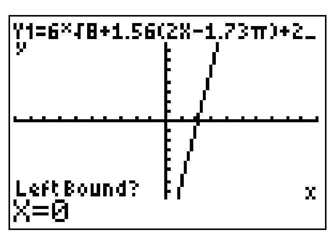




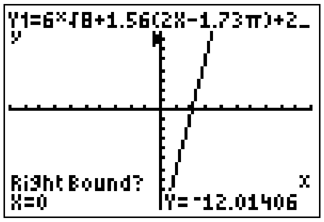


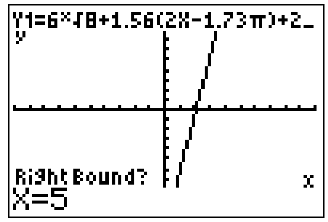
Partial TI-84+ screen *press* $\boxed{\text{Zoom}}\boxed{6}$ *press* $\boxed{\text{2nd}}\boxed{\text{TRACE}}\boxed{\text{2: zero}}$



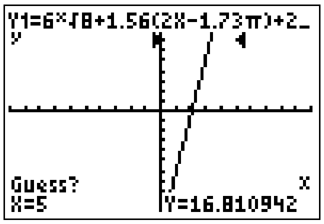


For the LB, either move cursor to the left of the x-intercept then press $\boxed{\text{ENTER}}$ *, or enter* $\boxed{0}\boxed{\text{ENTER}}$

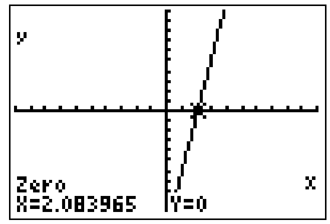




For the RB, either move cursor to the right of the x-intercept then press $\boxed{\text{ENTER}}$ *, or enter* $\boxed{5}\boxed{\text{ENTER}}$



For the Guess prompt, press $\boxed{\text{ENTER}}$



x-intercept (zero or root) is $x \approx \{2.084\}$

a) From the last screen, the solution to the equality $\sqrt[6]{8}+1.56(2x-1.73\pi)+2.45-2.3(\sqrt[3]{7}-1.15x)-3=0$ is $\boxed{x \approx 2.084}$

b) The inequality $\sqrt[6]{8}+1.56(2x-1.73\pi)+2.45-2.3(\sqrt[3]{7}-1.15x)-3 \ge 0$ asks for the interval at which the line Y_1 is equal to zero or positive (higher than the *x*-axis), from the last screen we can see that the line is equal to zero or higher than the *x*-axis when *x* is greater than or equal to 2.084.
Therefore, the interval is $[2.084, \infty)$.

c) The inequality $\sqrt[6]{8}+1.56(2x-1.73\pi)+2.45-2.3(\sqrt[3]{7}-1.15x)-3 \le 0$ asks for the interval at which the line Y_1 is equal to zero or negative (lower than the *x*-axis), from the last screen we can see that the line is equal to zero or lower than the *x*-axis when *x* is less than or equal to 2.084.
Therefore, the interval is $(-\infty, 2.084]$.

Exercises 1.6

In exercises 1 – 6, change to interval notation.

1) $\{x \mid x < 2\}$

2) $\{x \mid x > -3\}$

3) $\{x \mid x \geq -5\}$

4) $\{x \mid x \leq 4\}$

5) $\{x \mid -6 < x \leq 1\}$

6) $\{x \mid 2 \leq x < 9\}$

In exercises 7 – 14, change to set notation.

7) $(-\infty, 2)$

8) $(-2, 5)$

9) $[-5, 4]$

10) $(-2, 3]$

11) $[5, \infty)$

12) $(-\infty, 0]$

13) $(3, \infty)$

14) $[0, 7)$

In exercises 15 – 28, solve the given inequality analytically.

15) $7(x-1) \leq 4(x+2)$

16) $x-(x+4) \leq 2x+4(x-1)$

17) $-4(2x-6) \geq -3(x-4)$

18) $4\{2-[3(x+1)-2(x+1)]\} \geq -2x$

19) $-2\{4(x+3)-5[3x-2(2x+7)]-8\} < -20x-12$

20) $-3(6-4x) < 4-\{5x-[6x-(4x-(3x+2))]\}$

21) $\dfrac{x+3}{4}+\dfrac{2x-1}{3} > 16$

22) $\dfrac{x-2}{3}-\dfrac{x-3}{4} \leq \dfrac{1}{6}$

23) $0.2(x-25) > 1.3x+0.3(x+2)$

24) $-4x+12-8x > -6(x-2)+2x$

25) $-[4-(x-2)] < 2(x-1)-x$

26) $3(x+3)-4(2x-7) \leq -5x+2$

27) $6(x-1) \geq -3(2-x)+3x$

28) $4(2-3x) \geq -[6x-(8-6x)]$

In exercises 29 – 31, use your graphing calculator to solve the given equation and its related inequalities.

29. (a) $0.25\pi + 5.23(2x-5) = \sqrt[5]{2\pi} + 2.23\left(x - \sqrt{2.3}\right)$

    (b) $0.25\pi + 5.23(2x-5) \le \sqrt[5]{2\pi} + 2.23\left(x - \sqrt{2.3}\right)$

    (c) $0.25\pi + 5.23(2x-5) \ge \sqrt[5]{2\pi} + 2.23\left(x - \sqrt{2.3}\right)$

30. (a) $2.53\pi - 3.13(3x-7) = \sqrt[7]{2.34\pi} - 2.13\left(2x - \sqrt{5.37}\right)$

    (b) $2.53\pi - 3.13(3x-7) > \sqrt[7]{2.34\pi} - 2.13\left(2x - \sqrt{5.37}\right)$

    (c) $2.53\pi - 3.13(3x-7) < \sqrt[7]{2.34\pi} - 2.13\left(2x - \sqrt{5.37}\right)$

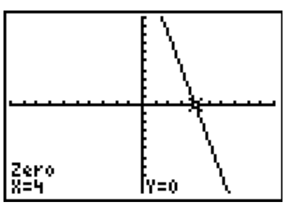
31. (a) $2.45x - 5.24\left(1.54x + \sqrt[8]{2.3\pi}\right) = \sqrt[5]{4.5\pi} - 3.63\left(3x + \sqrt{4.57}\right)$

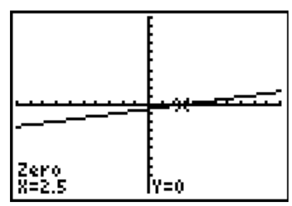
    (b) $2.45x - 5.24\left(1.54x + \sqrt[8]{2.3\pi}\right) < \sqrt[5]{4.5\pi} - 3.63\left(3x + \sqrt{4.57}\right)$

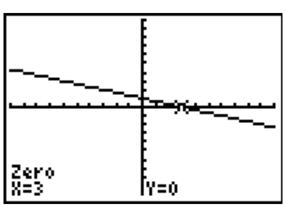
    (c) $2.45x - 5.24\left(1.54x + \sqrt[8]{2.3\pi}\right) > \sqrt[5]{4.5\pi} - 3.63\left(3x + \sqrt{4.57}\right)$

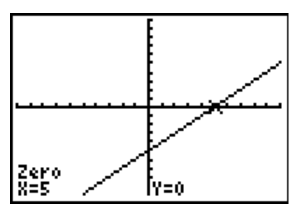
In exercises 32 – 35, refer to the graph of the linear function $y = f(x)$ and answer the following:

a. $f(x) = 0$, b. $f(x) > 0$, c. $f(x) \ge 0$, d. $f(x) < 0$, e. $f(x) \le 0$; Write your answers in set notation.

32) 

33) 

34) 

35) 

1.7 Applications

Formulas: A formula is an equation that contains two or more letters. To solve for a letter in a formula simply consider all other letters as constants and follow the steps for solving equations.

Example 1: In the formula $\frac{1}{z} = \frac{1}{x} + \frac{1}{y}$, solve for x.

Solution: $\frac{1}{z} = \frac{1}{x} + \frac{1}{y}$, multiply every term by the LCD xyz.

$xy = yz + xz$, isolate the terms that contain x.

$xy - xz = yz$, factor out x.

$x(y - z) = yz$, divide by the coefficient of x.

$$\boxed{x = \frac{yz}{y - z}}$$

Direct Variation: A variable y varies directly as another variable x

$\boxed{y = kx}$, where k is called the constant of variation

(i.e. As x increases, y increases. As x decreases, y decreases.)

Example 2: Write a formula for the following: y varies directly as the square root of x.

Solution: $y = k\sqrt{x}$

Example 3: y varies directly as the square of x. If y is 400 when x is 4, find the value of y when x is 3.

Solution: 1. First write the formula: $y = kx^2$

2. Use the first set of given values to solve for $k \Rightarrow 400 = k(4)^2 = 16k, \ \therefore k = \frac{400}{16} = 25$

3. Substitute the second set of given values and the value of k you already found in step 2 in the formula you got in step 1 $\Rightarrow \ y = kx = (25)(3)^2 = (25)(9) = 225$

Note: Most of the applications in this section follow the model $\boxed{z = x \cdot y}$

Simple Interest Formula:

$I = P \cdot r \cdot t$, where I is the interest, P is the principal (include money invested or borrowed), r is the interest rate, and t is the time in years.

$I = P \cdot r$, when the time is just one year.

Example 4: You invested \$10,000 in two accounts. The first account pays an 8% annual interest rate and the second account pays a 6% annual interest rate. If the total interest after one year is \$720; how much did you invest in each account?

Solution: If we assume that the amount of money invested in the first account is *x*, then the amount of money invested in the second account must be (10,000 – *x*). The interest from the first account only should be the product of the principal and the interest rate, that is 0.08*x.* Similarly, the interest from the second account should be 0.06(10000 - *x*). If we add both interests, we should get the total interest which is \$720. To organize our work, we can arrange all the given values in a table as follows:

	P •	r =	I
Account 1	x	0.08	$0.08x$
Account 2	$10000-x$	0.06	$0.06(10000-x)$
Total	10000		720

The equation can be written from the last column: $0.08x+0.06(10000-x)=720$

We solve the equation using the following steps:

$$0.08x+0.06(10000-x)=720$$
$$0.08x+600-0.06x=720$$
$$0.02x=120$$
$$x=6000$$

$\therefore$ \$6000 was invested in the 8% account and \$4000 was invested in the 6% account.

<u>**Note**</u>: If you prefer, you can eliminate all decimals by multiplying the equation by 100, but the equation is simple enough to solve without doing that.

<u>**Mixing Solutions Problems**</u>: We can use any three letters to introduce this type of problem. For clarity, we will use three letters that will relate to our problem.

$\boxed{Q=A\cdot r}$, where Q is the quantity (or volume), A is the amount, and r is the concentration (percentage)

Example 5: A 20% alcohol solution is mixed with an 8% alcohol solution to obtain 30 liters of a 12% alcohol solution. How many liters of the 20% solution and the 8% solution are in the 12% solution?

Solution: If we assume that the amount of the 20% alcohol solution is *x*, then the amount of the 8% alcohol solution must be (30 – *x*). Therefore the amount of pure alcohol in the 20% alcohol solution must be the product of the amount and the percentage, that is 0.20*x*. Similarly, the amount of pure alcohol in the 8% alcohol solution is 0.08(30 – *x*). Also, the amount of pure alcohol in the mix should be 0.12(30). This amount must be equal to the sum of the amounts of the pure alcohol in the 20% and the 8% alcohol solutions. If we arrange these values in a table as before, we get the following table:

	A •	r =	Q
20% solution	x	0.20	$0.20x$
8% solution	$30-x$	0.08	$0.08(30-x)$
12% solution	30	0.12	$(0.12)(30)$

We see that our equation is written from last column: $0.20x+0.08(30-x)=0.12(30)$

We solve the equation using the following steps:

$$0.20x+0.08(30-x)=0.12(30)$$
$$0.20x+2.4-0.08x=3.60$$
$$0.12x=1.20$$
$$x=\frac{1.20}{0.12}=10$$

$\therefore$ 10 liters of the 20% solution and 20 liters of the 8% solution are in the 12% solution.

Mixing Problems: Again, we can use any three letters to introduce this type of problem. For clarity, we will use three letters that will relate to our problem.

$\boxed{V=A\cdot C}$, where V is the value, A is the amount, and C is the cost per unit

Example 6: A pound of cashews costs \$9 and a pound of almonds costs \$6. A mix of forty-five pounds of cashews and almonds costs \$8 per pound. How many pounds of cashews and how many pounds of almonds are in the mix?

Solution: If we assume that the amount of the cashews is x, then the amount of almonds must be $(45-x)$. Therefore the total value of cashews is $9x$. Similarly, the total value of almonds is $6(45-x)$. Also, the total value of the mix must be (45)(8). This value must be equal to the sum of the values of almonds and cashews. If we arrange these values in a table as before, we get the following table:

	A •	C =	V
Cashews	x	9	$9x$
Almonds	$45-x$	6	$6(45-x)$
Mix	45	8	$(8)(45)$

We see that our equation is written from the last column: $9x+6(45-x)=8(45)$

We solve the equation using the following steps:

$$9x+6(45-x)=8(45)$$
$$9x+270-6x=360$$
$$3x=90\to x=\frac{90}{3}=30$$

$\therefore$ 30 pounds of cashews and 15 pounds of almonds are in the mix.

Economics Problems: We will introduce a very basic economics problem. In this example we assume that whatever is produced is sold.

• The Cost Function = Total Cost = Fixed Costs + Variable Costs, where variable costs are the number of units produced multiplied by the cost per unit, and the Fixed Costs are the overhead expenses.

• The Revenue Function = Total Revenue = number of units sold multiplied by the selling price per unit.

If we assume that we produce x units and sell all of them then:

$$C(x) = \text{fixed costs } + \text{variable costs}$$
$$R(x) = p(x)\cdot x, \text{ where } p(x) \text{ is the selling price.}$$

• Therefore the profit should be the difference between the total revenue and the total cost.

$$\text{Total profit} = P(x) = R(x) - C(x)$$

The Break-Even Point is the number of units x that are produced and sold such that the profit is equal to zero. This will occur when the total revenue is equal to the total cost.

Break-Even Point: $R(x) = C(x)$ or $R(x) - C(x) = 0,$ solve for x

Example 7: Tom bakes cakes and sells them at county fairs. His initial cost for the Orange county fair was \$50.00. He figures that each cake costs \$2.50 to make, and he charges \$7.50 per cake. Find the cost function, the revenue function, the profit function, and the break-even point.

Solution: Let x be the number of cakes baked and sold.

Cost Function $= C(x) = 50 + 2.50x$

Revenue Function $= R(x) = 7.50x$

Profit Function $= P(x) = 7.50x - (50 + 2.50x) = 5x - 50$

Break-Even Point: $P(x) = 0 = 5x - 50 \rightarrow \boxed{x = 10}$

$\therefore$ Tom must bake and sell 10 cakes to break even.

Example 8: Mary runs a copying service and she charges 3 cents per copy. The cost of the copy machine is \$4500, the cost of a life time maintenance service is \$2500, and the cost of making a single copy is 1 cent. Find the cost function, the revenue function, the profit function, and the break-even point.

Solution: let x be the number of copies made.

Cost function $= C(x) = 4500 + 2500 + 0.01x = 7000 + 0.01x$

Revenue Function $= R(x) = 0.03x$

Profit Function $= P(x) = 0.03x - (7000 + 0.01x) = 0.02x - 7000$

Break Even Point: $P(x) = 0 = 0.02x - 7000$

$\boxed{x = 350000}$

$\therefore$ Mary must make 350,000 copies to break even.

Exercises 1.7

In exercises 1 – 4, solve the formula for the given variable.

1) $\frac{1}{w}=\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$; solve for y

2) $\frac{1}{w}=\frac{1}{x}+\frac{1}{y}+\frac{1}{z}$; solve for x

3) $\frac{2x-5}{y}=\frac{3x+4}{2}$; solve for x

4) $\frac{3x+1}{2y}=\frac{2x-3}{4}$; solve for x

5) L varies directly as M. If L is 30 when M is 10, find L when M is 5.

6) Y varies directly as X. If Y is 25 when X is 0.5, find Y when X is 10.

7) You invested \$5,000 in two accounts. The first account pays a 4% annual interest rate and the second account pays a 3% annual interest rate. If the total interest after one year is \$180, how much did you invest in each account?

8) You invested \$7,000 in two accounts. The first account pays a 5% annual interest rate and the second account pays a 4% annual interest rate. If the total interest after one year is \$330, how much did you invest in each account?

9) An 18% alcohol solution is mixed with a 12% alcohol solution to obtain 24 liters of a 14% alcohol solution. How many liters of the 18% solution and the 12% solution are in the 14% solution?

10) A 30% alcohol solution is mixed with an 18% alcohol solution to obtain 12 liters of a 22% alcohol solution. How many liters of the 30% solution and the 18% solution are in the 22% solution?

11) How much pure acid should be added to 10 liters of 40% acid solution to increase the concentration to 50% acid?

12) How much pure acid should be added to 10 liters of 44% acid solution to increase the concentration to 65% acid?

13) A pound of cashews costs $6 and a pound of almonds costs $2. A mix of sixty pounds of cashews and almonds costs $4 per pound. How many pounds of cashews and how many pounds of almonds are in the mix?

14) A pound of cashews costs $8 and a pound of almonds costs $2. A mix of forty pounds of cashews and almonds costs $6 per pound. How many pounds of cashews and how many pounds of almonds are in the mix?

15) A coin bank has 20 coins in quarters and dimes. If the total value of the coins is $3.65, how many quarters and how many dimes are in the coin bank?

16) A coin bank has 27 coins in quarters and dimes. If the total value of the coins is $5.70, how many quarters and how many dimes are in the coin bank?

17) Gabriel bakes cakes and sells them at county fairs. His initial cost for the Orange County Fair was $200. He figures that each cake costs $4.50 to make, and he charges $7.00 per cake. Find the cost function, the revenue function, the profit function, and the break-even point.

18) Dara bakes cakes and sells them at county fairs. Her initial cost for the Orange County Fair was $280. She figures that each cake costs $4.00 to make, and she charges $7.50 per cake. Find the cost function, the revenue function, the profit function, and the break-even point.

19) Dominic runs a copying service, and he charges 5 cents per copy. The cost of the copy machine is $3600, the cost of a life time maintenance service is $1500, and the cost of making a single copy is 2 cents. Find the cost function, the revenue function, the profit function, and the break-even point.

20) Lora runs a copying service, and she charges 6 cents per copy. The cost of the copy machine is $5000, the cost of a life time maintenance service is $2000, and the cost of making a single copy is 2 cents. Find the cost function, the revenue function, the profit function, and the break-even point.

Answers – Exercises 1.1

1. $\mathbb{N}=\{7\}$, $\mathbb{W}=\{0,7\}$, $\mathbb{Z}=\left\{-10,-\frac{8}{4},0,7\right\}$, $\mathbb{Q}=\left\{-10,-\frac{8}{4},0,7\right\}$, $\mathbb{H}=\left\{\sqrt{2},e\right\}$,

   $\mathbb{R}=\left\{-10,-\frac{8}{4},0,\sqrt{2},e,7\right\}$

2. $\mathbb{N}=\left\{\sqrt{4}\right\}$, $\mathbb{W}=\left\{\sqrt{4}\right\}$, $\mathbb{Z}=\left\{\sqrt{4}\right\}$, $\mathbb{Q}=\left\{\frac{1}{3},\frac{10}{6},\sqrt{4}\right\}$, $\mathbb{H}=\left\{-\pi,\sqrt{17}\right\}$, $\mathbb{R}=\left\{-\pi,\frac{1}{3},\frac{10}{6},\sqrt{4},\sqrt{17}\right\}$

3. $\mathbb{N}=\phi$, $\mathbb{W}=\phi$, $\mathbb{Z}=\phi$, $\mathbb{Q}=\{-5.6,-2.5,0.1,1.25\}$, $\mathbb{H}=\left\{\sqrt{3}\right\}$, $\mathbb{R}=\left\{-5.6,-2.5,0.1,1.25,\sqrt{3}\right\}$

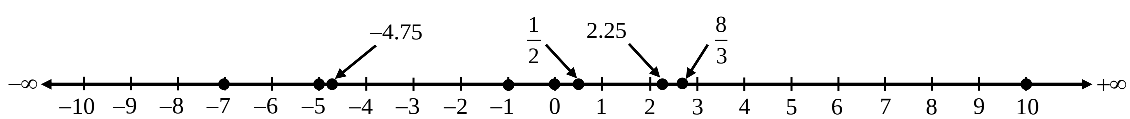
4. $\mathbb{N}=\left\{\frac{14}{2}\right\}$, $\mathbb{W}=\left\{\frac{14}{2}\right\}$, $\mathbb{Z}=\left\{\frac{14}{2}\right\}$, $\mathbb{Q}=\left\{-0.\bar{3},\frac{1}{7},0.7,1.\overline{234},\frac{14}{2}\right\}$, $\mathbb{H}=\phi$, $\mathbb{R}=\left\{-0.\bar{3},\frac{1}{7},0.7,1.\overline{234},\frac{14}{2}\right\}$

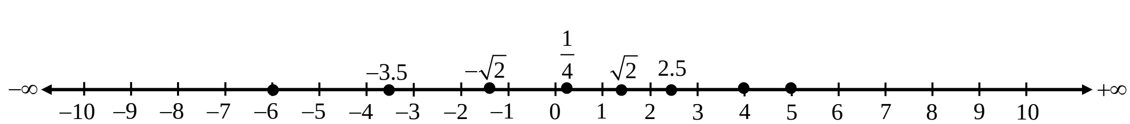
5. $\mathbb{N}=\phi$, $\mathbb{W}=\phi$, $\mathbb{Z}=\left\{-\sqrt{25},-3\right\}$, $\mathbb{Q}=\left\{-\sqrt{25},-3,-1.25,\frac{5}{4}\right\}$, $\mathbb{H}=\sqrt{7}$, $\mathbb{R}=\left\{-\sqrt{25},-3,-1.25,\frac{5}{4},\sqrt{7}\right\}$

6. $\mathbb{N}=\phi$, $\mathbb{W}=\phi$, $\mathbb{Z}=\left\{-\frac{6}{2}\right\}$, $\mathbb{Q}=\left\{-4.25,-\frac{6}{2},0.125,0.\bar{4},0.\overline{56}\right\}$, $\mathbb{H}=\phi$, $\mathbb{R}$

   $=\left\{-4.25,-\frac{6}{2},0.125,0.\bar{4},0.\overline{56}\right\}$

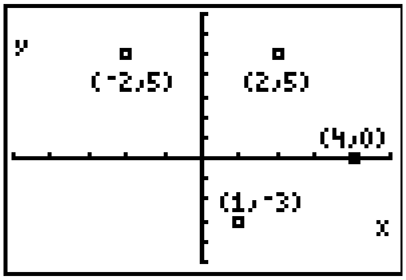
7) 0   8) -7   9) $\frac{8}{3}$ 10) -1   11) 10   12) -4.75   13) $\frac{1}{2}$ 14) -5   15) 2.25

$7-15 \Rightarrow$ 

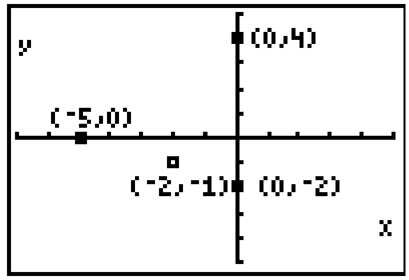
$16-23 \Rightarrow$ 

Answers – Exercises 1.2

1 – 4



5 - 8



9. 10
10. 13
11. (-5, 5), (11, 5)
12. (-4, -4), (20, -4)
13. (-2, 1), (-2, -7)
14. (3, -4), (3, 8)
15. (4, 1)
16. (-1, 4)
17. a = 11, b = -13
18. a = 8, b = -1

Answers - Exercises 1.3

1. Function, Domain = {-1, 0, 3, 7, 10}, Range = {-4, 2, 3, 5, 10}

2. Function, Domain = {-2, -1, 3, 5, 7}, Range = {1, 2, 4, 5, 9}

3. Function, Domain = {-5, 0, 1, 3, 6}, Range = {2}

4. Function, Domain = {-3, -1, 3, 5, 7}, Range = {1}

5. Not a function, Domain = {-6, -3, 0, 5}, Range = {-4, -1, 1, 4, 6}

6. Not a function, Domain = {-4, 1, 3, 6}, Range = {-1, 2, 4, 5, 8}

7. Function, Domain = {-5, -2, 0, 1, 4, 5}, Range = {-3, 2, 3, 4}

8. Function, Domain = {-5, -2, 0, 2, 4, 5}, Range = {-5, -3, 2, 3, 4}

9. Not a function, Domain = {-3, 1, 2, 3, 4}, Range = {-4, 2, 3, 4, 5}

10. Not a function, Domain = {-4, -3, -1, 2, 3}, Range = {-4, -3, 2, 3, 4, 9}

11. Function, Domain $= [-3, 4]$, Range $= [0, 3]$

12. Function, Domain $= [-2, 2]$, Range $= [0, 2]$

13. Not a function, Domain $= [-5, 5]$, Range $= [-5, 5]$

14. Not a function, Domain $= [-3, 3]$, Range $= [-3, 3]$

15. Not a function, Domain $= [-4, 4]$, Range $= [-3, 3]$

16. Not a function, Domain $=[-5,5]$, Range $=[-2,2]$

17. Not a function, Domain $=[-3,3]$, Range $=[-5,5]$

18. Not a function, Domain $=[-1,1]$, Range $=[-4,4]$

19. 10	20. -13	21. 2	22. -11	23. 4	24. 3
25. 8	26. 17	27. 8	28. 5/7	29. -7/6	30. zero
31. Undefined	32. 1	33. 2/3	34. 3/2	35. -2	36. -3/4

37. zero	38. zero	39. undefined	40. undefined

Answers - Exercises 1.4

1. $y=2x+5$	2. $y=-5x+13$	3. $y=\frac{2}{3}x+\frac{19}{3}$	4. $y=-\frac{2}{5}x+\frac{26}{5}$
5. $y=-2x+1$	6. $y=x-5$	7. $y=-\frac{5}{3}x+\frac{5}{3}$	8. $y=\frac{1}{2}x+\frac{5}{2}$
9. $y=5$	10. $y=3$	11. $x=5$	12. $x=3$

13. $m=-\frac{2}{3}, b=2$	14. $m=-\frac{5}{4}, b=2$	15. $m=\frac{3}{7}, b=-1$
16. $m=\frac{5}{2}, b=-2$	17. Slope is undefined, no *y*-intercept	
18. $m=0, b=-5$	19. Perpendicular	20. Neither
21. Parallel	22. Neither	23. Neither
24. $y=-\frac{3}{4}x+3$	25. $y=-\frac{3}{5}x+3$	26. $y=\frac{1}{5}x+2$
27. $y=\frac{3}{4}x+3$	28. $y=-\frac{4}{5}x+\frac{7}{5}$	29. $y=-\frac{2}{7}x-\frac{33}{7}$
30. $y=-\frac{1}{3}x+\frac{11}{3}$	31. $y=-\frac{3}{4}x-\frac{13}{4}$	32. $y=x-4$
33. $y=-2x$	34. $y=-\frac{2}{7}x-\frac{19}{7}$	35. $y=\frac{4}{5}x-\frac{41}{5}$
36. $y=\frac{3}{4}x+3$	37. $y=\frac{3}{2}x+3$	38. $y=-x+5$
39. $y=-\frac{3}{5}x+3$	40. $y=3$	41. $y=-2$
42. $x=4$		43. $x=-3$

Answers - Exercises 1.5

1. 5
2. 0
3. 2
4. 2
5. 68
6. 2
7. 17
8. 1
9. -4
10. Identity, all real numbers
11. Contradiction, no solution
12. Contradiction, no solution
13. Identity, all real numbers
14. Identity, all real numbers
15. -4.3804
16. 1.3158
17. -0.52585
18. 3.2407

Answers - Exercises 1.6

1. $(-\infty, 2)$
2. $(-3, \infty)$
3. $[-5, \infty)$
4. $(-\infty, 4]$
5. $(-6, 1]$
6. $[2, 9)$
7. $\{x \mid x < 2\}$
8. $\{x \mid -2 < x < 5\}$
9. $\{x \mid -5 \le x \le 4\}$
10. $\{x \mid -2 < x \le 3\}$
11. $\{x \mid x \ge 5\}$
12. $\{x \mid x \le 0\}$
13. $\{x \mid x > 3\}$
14. $\{x \mid 0 \le x < 7\}$
15. $x \le 5$
16. $x \ge 0$
17. $x \le 12/5$
18. $x \le 2$
19. $x < 68$
20. $x < 2$
21. $x > 17$
22. $x \le 1$
23. $x < -4$
24. $x < 0$
25. $\mathbb{R}$
26. ϕ
27. $\mathbb{R}$
28. $\mathbb{R}$
29. a) $x = 2.8465$ b) $x \le 2.8465$ c) $x \ge 2.8465$
30. a) $x = 4.5989$, b) $x < 4.5989$, c) $x > 4.5989$
31. a) $x = 0.1229$, b) $x < 0.1229$, c) $x > 0.1229$
32. a) $x = \{4\}$, b) $\{x \mid x < 4\}$, c) $\{x \mid x \le 4\}$, d) $\{x \mid x > 4\}$, e) $\{x \mid x \ge 4\}$
33. a) $x = \{2.5\}$, b) $\{x \mid x > 2.5\}$, c) $\{x \mid x \ge 2.5\}$, d) $\{x \mid x < 2.5\}$, e) $\{x \mid x \le 2.5\}$
34. a) $x = \{3\}$, b) $\{x \mid x < 3\}$, c) $\{x \mid x \le 3\}$, d) $\{x \mid x > 3\}$, e) $\{x \mid x \ge 3\}$
35. a) $x = \{5\}$, b) $\{x \mid x > 5\}$, c) $\{x \mid x \ge 5\}$, d) $\{x \mid x < 5\}$, e) $\{x \mid x \le 5\}$

Answers - Exercises 1.7

1. $y = \dfrac{wxz}{xz - wz - wx}$

2. $x = \dfrac{wyz}{yz - wz - wy}$

3. $x = \dfrac{10 + 4y}{4 - 3y}$

4. $x = \dfrac{3y + 2}{2y - 6}$

5. $L = 15$

6. $y = 500$

7. \$3000 @ 4%, \$2000 @ 3%

8. \$5000 @ 5%, \$2000 @ 4%

9. $8Lt$ @ 18%, $16Lt$ @ 12%

10. $4Lt$ @ 30%, $8Lt$ @ 18%

11. 2 liters

12. 6 liters

13. 30 Ib of cashews, 30 Ib of almonds

14. $26\frac{2}{3}$ Ib of cashews, $13\frac{1}{3}$ Ib of almonds

15. 11 quarters, 9 dimes

16. 20 quarters, 7 dimes

17. $C(x) = 200 + 4.50x$, $R(x) = 7x$, $p(x) = 2.50x - 200$, break even point: $x = 80$ cakes

18. $C(x) = 280 + 4x$, $R(x) = 7.50x$, $p(x) = 3.50x - 280$, break even point: $x = 80$ cakes

19. $C(x) = 5100 + 0.02x$, $R(x) = 0.05x$, $p(x) = 0.03x - 5100$, break even point: $x = 170,000$ copies

20. $C(x) = 7000 + 0.02x$, $R(x) = 0.06x$, $p(x) = 0.04x - 7000$, break even point: $x = 175,000$ copies