# PART I: NO CALCULATOR (144 points)

**(4.1, 4.2, 4.3, 4.4)**

For the following functions:

a) Find the amplitude, the period, any vertical translation, and any phase shift. If not applicable, write "*none*" in the blank.

b) Graph over the interval $-2\pi \le x \le 2\pi$ . Identify and label any asymptotes.

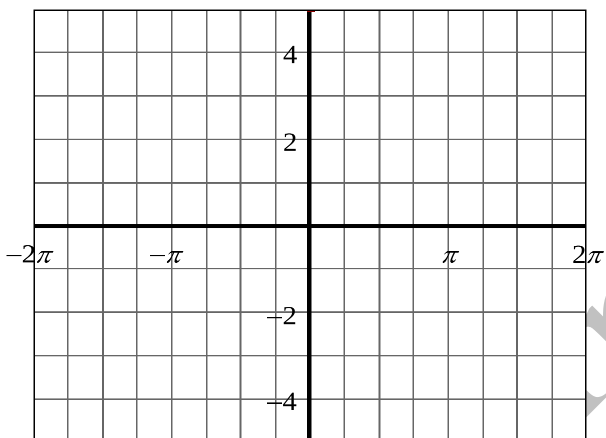
1. $y = 4\sin\frac{1}{2}x$

**amplitude:**

**period:**

**vertical translation:**

**phase shift:**



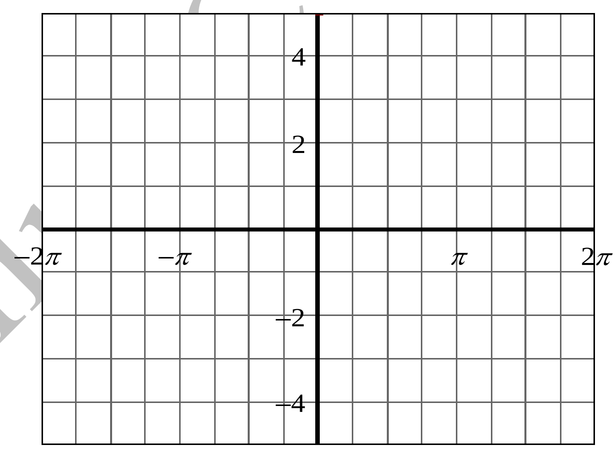

2. $y = -2\cos\left(x + \frac{3\pi}{4}\right)$

**amplitude:** :

**period:**

**vertical translation:**

**phase shift:**



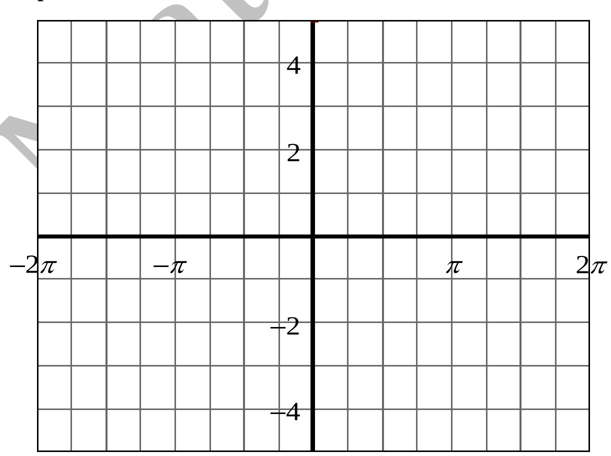

3. $y = \tan\left(x - \frac{\pi}{4}\right)$

**amplitude:**

**period:**

**vertical translation:**

**phase shift:**



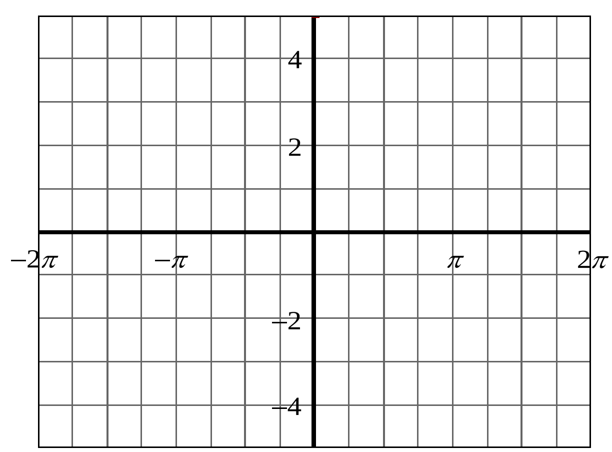

4. $y = \csc x$

**amplitude:**

**period:**

**vertical translation:**

**phase shift:**



**(4.1, 4.2, 4.3, 4.4)**
For the following functions:

a) Find the amplitude, the period, any vertical translation, and any phase shift. If not applicable, write "*none*" in the blank.

b) Graph over the interval $-2\pi \le x \le 2\pi$. Identify and label any asymptotes.

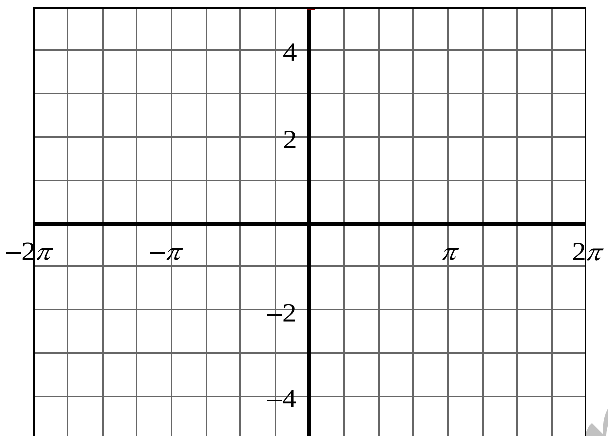
5. $y = 2 - \sec x$

**amplitude:**

**period:**

**vertical translation:**

**phase shift:**



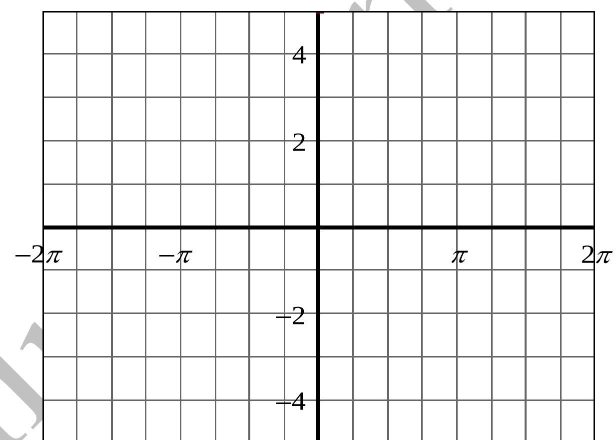

6. $y = \cot \frac{1}{2}x$

**amplitude:**

**period:**

**vertical translation:**

**phase shift:**




**(6.1)**
Give the *exact* radian measure of $y$ if it exists.

7. $y = \arctan\left(-\sqrt{3}\right)$

8. $y = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

9. $y = \sec^{-1}(2)$

10. $y = \arcsin\left(-\frac{\sqrt{2}}{2}\right)$

11. $y = \csc^{-1}\left(\frac{\sqrt{2}}{2}\right)$

12. $y = \cot^{-1}(-1)$

Write the following trigonometric expression as an algebraic expression in *u*, for $u > 0$.

13. $\cot\left(\sec^{-1} u\right)$

14. $\cos\left(\arcsin u\right)$

## PART II: YOU MAY USE A CALCULATOR (256 points)

### DOUBLE-ANGLE IDENTITIES

$\sin 2A = 2\sin A\cos A$

$\cos 2A = \cos^2 A - \sin^2 A$

$\cos 2A = 2\cos^2 A - 1$

$\cos 2A = 1 - 2\sin^2 A$

$\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$

### SUM AND DIFFERENCE IDENTITIES

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$$

### HALF-ANGLE IDENTITIES

$$\sin\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}}$$

$$\cos\frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}}$$

$$\tan\frac{A}{2} = \frac{\sin A}{1+\cos A}$$

$$\tan\frac{A}{2} = \frac{1-\cos A}{\sin A}$$

$$\tan\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{1+\cos A}}$$

### LAW OF COSINES

$a^2 = b^2 + c^2 - 2bc\cos A$

$b^2 = a^2 + c^2 - 2ac\cos B$

$c^2 = a^2 + b^2 - 2ab\cos C$

### LAW OF SINES

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

### DE MOIVRE'S THEOREM

$$\left[r\left(\cos\theta + i\sin\theta\right)\right]^n = r^n\left(\cos n\theta + i\sin n\theta\right)$$

where $r\left(\cos\theta + i\sin\theta\right)$ is a complex number and $n$ is any real number.

**(1.1)**

1. Convert the following angles to decimal degrees. If applicable, round to the nearest hundredth of a degree.

   a) $76^\circ 48'$ b) $34^\circ 51' 35''$ c) $249^\circ 15'$

**(1.4)**

2. Identify the quadrant satisfying the given conditions:

   a) $\cos\theta < 0$ *and* $\cot\theta > 0$ b) $\tan\theta < 0$ *and* $\csc\theta > 0$

**(2.1, 2.2)**

3. Find the *exact* values of the six trigonometric functions for the given angles. Rationalize denominators when applicable.

   a) $135^\circ$ b) $210^\circ$ c) $270^\circ$ d) $300^\circ$

**(5.5)**

4. Given $\cos 2x = -\frac{5}{12}$ and $90° < x < 180°$, find the exact values of the following.

   $\sin x =$ ____________________

   $\cos x =$ ____________________

   $\tan x =$ ____________________

5. Given $\csc x = -\frac{7\sqrt{5}}{5}$ and $\cos x > 0$, find the exact values of the following.

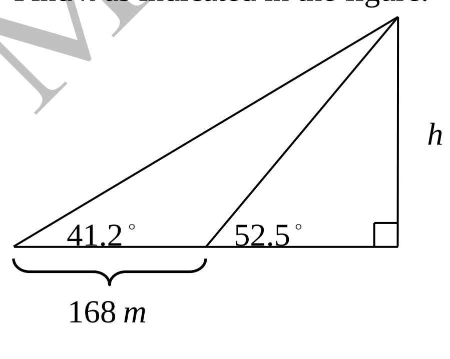
   $\sin 2x =$ ____________________

   $\cos 2x =$ ____________________

   $\tan 2x =$ ____________________

**(2.5)**

6. Find *h* as indicated in the figure.



**(7.3)**

Solve the following problem. Include a labeled sketch in your work.

7. Starting at point *A*, a ship sails 18.2 km on a bearing of $190^\circ$, then turns and sails 47.4 km on a bearing of $319^\circ$. Find the distance of the ship from point *A to the nearest tenth of a kilometer*.

**(7.1)**

Solve the following problem. Include a labeled sketch in your work.

8. Francesco is flying in a hot air balloon directly above a straight road 3.4 mi long that joins two towns. He finds that the town closer to him is at an angle of depression of $52.9^\circ$, and the farther town is at an angle of depression of $28.5^\circ$. How high above the ground is the balloon? *Round your answer to the nearest hundredth of a mile.*

**(5.6)**

9. Use the appropriate half-angle identity to find the exact value of the following. **Work demonstrating use of appropriate identity must be shown.**

   a) $\sin 202.5°$ b) $\tan 75°$

**(5.3, 5.4)**

10. Use the appropriate sum or difference identity to find the exact value of the following. **Work demonstrating use of appropriate identity must be shown.**

    a) $\cos\frac{5\pi}{12}$ b) $\tan\frac{13\pi}{12}$ c) $\sin\frac{\pi}{12}$

**(3.1)**

11. Convert the following angles to radians. Leave answers as multiples of $\pi$.

    a) $110^\circ$ b) $216^\circ$

**(3.1)**

12. Convert the following angles to degrees.

    a) $-\frac{4\pi}{15}$ b) $\frac{8\pi}{5}$

**(5.1, 5.2, 5.5)**

Verify that each equation is an identity.

13. $\dfrac{\sin\theta+\tan\theta}{1+\cos\theta}=\tan\theta$

14. $\sec^2\theta\csc^2\theta=\sec^2\theta+\csc^2\theta$

15. $\cot\theta-\tan\theta=\dfrac{\cos 2\theta}{\sin\theta\cos\theta}$

16. $\tan 8\theta-\tan 8\theta\tan^2 4\theta=2\tan 4\theta$

**(6.2)**

17. Solve the following equations for <u>all</u> exact solutions in radians.
*Write answers using the least possible nonnegative angle measures.*
a) $2\sin x-\sqrt{3}=0$
b) $\cos x+1=2\sin^2 x$

**(6.3)**

18. Solve the following for <u>all</u> solutions in degrees. Use exact values for $x$ whenever possible. If necessary, approximate answers to the nearest tenth of a degree.
*Write answers using the least possible nonnegative angle measures.*
a) $3\cot 3x=\sqrt{3}$
b) $2-\sin 2x=4\sin 2x$

**(6.2, 6.3)**

19. Solve the following equation over the interval $\left[0^\circ, 360^\circ\right)$ Use exact values for $x$ whenever possible. If necessary, approximate answers to the nearest tenth of a degree.
a) $5\tan^2 x+16\tan x=40$
b) $5\sec^2 x=3+3\sec x$
c) $2\sin^2 x=1$

**(8.4)**

20. Use DeMoivre's Theorem to find $\left(2-2i\sqrt{3}\right)^6$. Write your answer in rectangular form.

**(8.5)**

21. Convert the following to polar coordinates with $0^\circ\le\theta<360^\circ$ and $r>0$.
a) $\left(-1,\sqrt{3}\right)$
b) $\left(\sqrt{2},-\sqrt{2}\right)$

**(8.6)**

22. A golf ball is hit from the ground with initial velocity of 150 feet per second at an angle of $60°$ with the ground.

    The parametric equations that model the path of the rocket are given by $x = 75\,t$, $y = -16t^2 + 75\sqrt{3}\,t$

    Determine a rectangular equation that models the path of the projectile.
    *Use exact values for any numbers in the equation.*

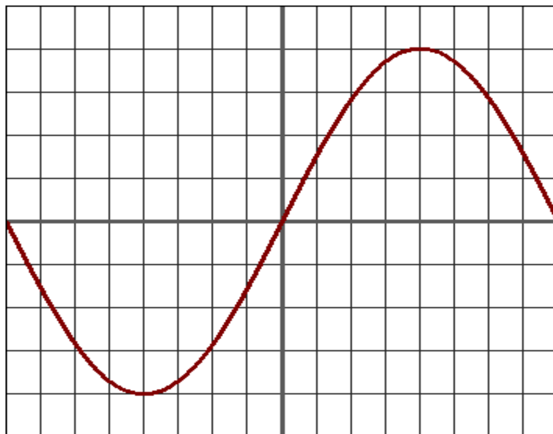
## Part I Answers:

**1)**

**a) amplitude:** $4$ **vertical translation:** *none*

**period:** $4\pi$ **phase shift:** *none*

**b)**

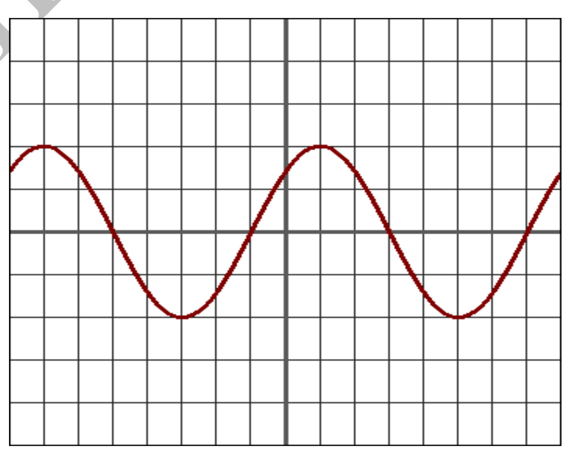


**2)**

**a) amplitude:** $2$ **vertical translation:** *none*

**period:** $2\pi$ **phase shift:** $\frac{3\pi}{4}$ *left*

**b)**



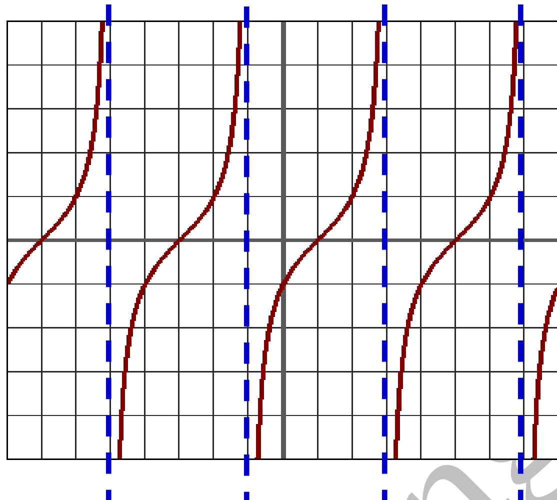
## Part I Answers:

**3)**

**a) amplitude:** *none* **vertical translation:** *none*

**period:** $\pi$ **phase shift:** $\frac{\pi}{4}$ *right*

**b)**



**asymptotes:**

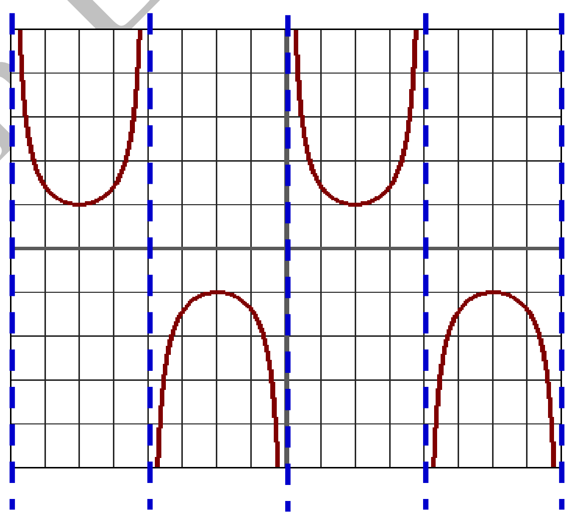
$x = -\frac{5\pi}{4}$ $x = -\frac{\pi}{4}$ $x = \frac{3\pi}{4}$ $x = \frac{7\pi}{4}$

**4)**

**a) amplitude:** *none* **vertical translation:** *none*

**period:** $2\pi$ **phase shift:** *none*

**b)**



**asymptotes:**

$x = -2\pi$ $x = -\pi$ $x = 0$ $x = \pi$ $x = 2\pi$

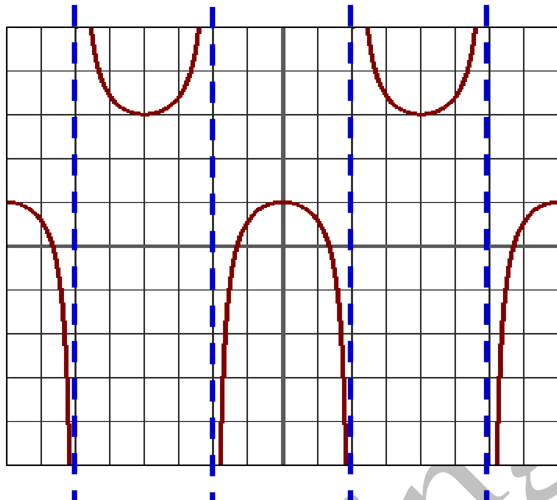
## Part I Answers:

**5)**

**a) amplitude:** *none* **vertical translation:** $2\ up$

**period:** $2\pi$ **phase shift:** *none*

**b)**



**asymptotes:**

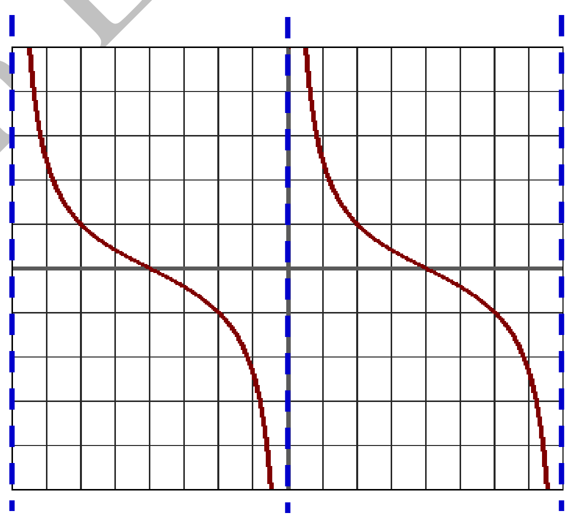
$x=-\frac{3\pi}{2}$ $x=-\frac{\pi}{2}$ $x=\frac{\pi}{2}$ $x=\frac{3\pi}{2}$

**6)**

**a) amplitude:** *none* **vertical translation:** *none*

**period:** $2\pi$ **phase shift:** *none*

**b)**



**asymptotes:**

$x=-2\pi$ $x=0$ $x=2\pi$

## Part I Answers:

**7)** $-\frac{\pi}{3}$

**8)** $\frac{5\pi}{6}$

**9)** *does not exist*

**10)** $-\frac{\pi}{4}$

**11)** *does not exist*

**12)** $-\frac{\pi}{4}$

**13)** $\frac{\sqrt{u^2-1}}{u^2-1}$

**14)** $\sqrt{1-u^2}$

## Part II Answers:

**1)** **a)** $78.8^\circ$ **b)** $34.86^\circ$ **c)** $249.25^\circ$

**2)** **a)** III **b)** II

**3a)** $\sin 135^\circ = \dfrac{\sqrt{2}}{2}$ $\cos 135^\circ = -\dfrac{\sqrt{2}}{2}$ $\tan 135^\circ = -1$

$\csc 135^\circ = \sqrt{2}$ $\sec 135^\circ = -\sqrt{2}$ $\cot 135^\circ = 1$

**3b)** $\sin 210^\circ = -\dfrac{1}{2}$ $\cos 210^\circ = -\dfrac{\sqrt{3}}{2}$ $\tan 210^\circ = \dfrac{\sqrt{3}}{3}$

$\csc 210^\circ = -2$ $\sec 210^\circ = -\dfrac{2\sqrt{3}}{3}$ $\cot 210^\circ = \sqrt{3}$

**3c)** $\sin 270^\circ = -1$ $\cos 270^\circ = 0$

**3d)** $\sin 300^\circ = -\dfrac{\sqrt{3}}{2}$ $\cos 300^\circ = \dfrac{1}{2}$ $\tan 300^\circ = -\sqrt{3}$

$\csc 300^\circ = -\dfrac{2\sqrt{3}}{3}$ $\sec 300^\circ = 2$ $\cot 300^\circ = -\dfrac{\sqrt{3}}{3}$

**4)** $\sin x = \dfrac{\sqrt{102}}{12}$ $\cos x = -\dfrac{\sqrt{42}}{12}$ $\tan x = \dfrac{\sqrt{119}}{7}$

## Part II Answers:

**5)** $\sin 2x = -\dfrac{4\sqrt{55}}{49}$ $\cos 2x = \dfrac{39}{49}$ $\tan 2x = \dfrac{-4\sqrt{55}}{39}$

**6)** $448\ m$

**7)** $38.6\ km$

**8)** $1.31\ miles$

**9)** **a)** $-\dfrac{\sqrt{2-\sqrt{2}}}{2}$ **b)** $-2-\sqrt{3}$

**10)** **a)** $\dfrac{\sqrt{6}-\sqrt{2}}{4}$ **b)** $2-\sqrt{3}$ **c)** $\dfrac{\sqrt{6}-\sqrt{2}}{4}$

**11)** **a)** $\dfrac{11\pi}{18}$ **b)** $\dfrac{6\pi}{5}$

**12)** **a)** $-48^\circ$ **b)** $288^\circ$

## Part II Answers:

**13) One possible way of verifying**

$$\frac{\sin\theta+\tan\theta}{1+\cos\theta}=\tan\theta\textbf{:}$$

$$\begin{aligned}
&\frac{\sin\theta+\tan\theta}{1+\cos\theta}\\
&=\frac{\sin\theta+\dfrac{\sin\theta}{\cos\theta}}{1+\cos\theta}\\
&=\frac{\sin\theta\left(1+\dfrac{1}{\cos\theta}\right)}{1+\cos\theta}\\
&=\frac{\sin\theta\left(\dfrac{\cos\theta}{\cos\theta}+\dfrac{1}{\cos\theta}\right)}{1+\cos\theta}\\
&=\frac{\sin\theta\left(\dfrac{\cos\theta+1}{\cos\theta}\right)}{1+\cos\theta}\\
&=\frac{\dfrac{\sin\theta}{\cos\theta}\cancel{(\cos\theta+1)}}{\cancel{1+\cos\theta}}\\
&=\tan\theta
\end{aligned}$$

**14) One possible way of verifying**

$$\sec^2\theta\csc^2\theta=\sec^2\theta+\csc^2\theta\textbf{:}$$

$$\begin{aligned}
&\sec^2\theta+\csc^2\theta\\
&=\frac{1}{\cos^2\theta}+\frac{1}{\sin^2\theta}\\
&=\frac{\sin^2\theta+\cos^2\theta}{\sin^2\theta\cos^2\theta}\\
&=\frac{1}{\sin^2\theta\cos^2\theta}\\
&=\sec^2\theta\csc^2\theta
\end{aligned}$$

## Part II Answers:

**15) One possible way of verifying**

$$\cot\theta - \tan\theta = \frac{\cos 2\theta}{\sin\theta\cos\theta}:$$

$$\begin{aligned}
&\frac{\cos 2\theta}{\sin\theta\cos\theta} \\
&= \frac{\cos^2\theta - \sin^2\theta}{\sin\theta\cos\theta} \\
&= \frac{\cos^2\theta}{\sin\theta\cos\theta} - \frac{\sin^2\theta}{\sin\theta\cos\theta} \\
&= \frac{\cos\theta\cdot\cancel{\cos\theta}}{\sin\theta\cancel{\cos\theta}} - \frac{\cancel{\sin\theta}\cdot\sin\theta}{\cancel{\sin\theta}\cos\theta} \\
&= \cot\theta - \tan\theta
\end{aligned}$$

**16) One possible way of verifying**

$$\tan 8\theta - \tan 8\theta\tan^2 4\theta = 2\tan 4\theta:$$

$$\begin{aligned}
&\tan 8\theta - \tan 8\theta\tan^2 4\theta \\
&= \tan 8\theta\left(1-\tan^2 4\theta\right) \\
&= \tan 2(4\theta)\left(1-\tan^2 4\theta\right) \\
&= \frac{2\tan 4\theta}{\cancel{1-\tan^2 4\theta}}\cdot\frac{\cancel{\left(1-\tan^2 4\theta\right)}}{1} \\
&= 2\tan 4\theta
\end{aligned}$$

## Part II Answers:

**17)** **a)** $\frac{\pi}{3}+2k\pi$, $\frac{2\pi}{3}+2k\pi$, where $k$ is an integer

**b)** $\pi+2k\pi$, $\frac{\pi}{3}+2k\pi$, $\frac{5\pi}{3}+2k\pi$, where $k$ is an integer

**18)** **a)** $20^\circ+60^\circ k$, where $k$ is an integer

**b)** $11.8^\circ+180^\circ k$, $78.2^\circ+180^\circ k$, where $k$ is an integer

**19)** **a)** $58.8^\circ, 101.7^\circ, 238.8^\circ, 281.7^\circ$

**b)** $27.8^\circ, 332.2^\circ$

**c)** $45^\circ, 135^\circ, 225^\circ, 315^\circ$

**20)** $4096$ *or* $4096+0i$

**21)** **a)** $\left(2, 120^\circ\right)$

**b)** $\left(2, 315^\circ\right)$

**22)** $y=-\frac{16}{5625}x^2+\sqrt{3}x$