PART I: NO CALCULATOR (64 points)
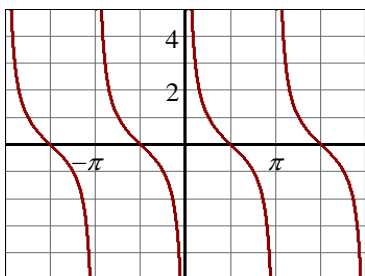
(4.1, 4.2, 4.3, 4.4)
Match each graph with one of the basic circular functions listed:
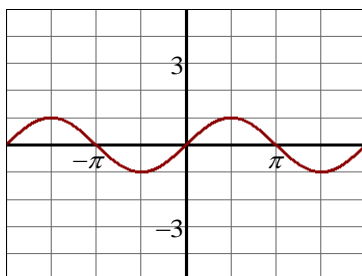
a) $y = \sin x$ b) $y = \cos x$ c) $y = \tan x$ d) $y = \csc x$ e) $y = \sec x$ f) $y = \cot x$
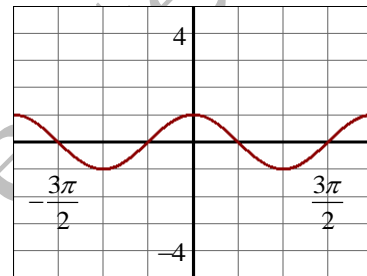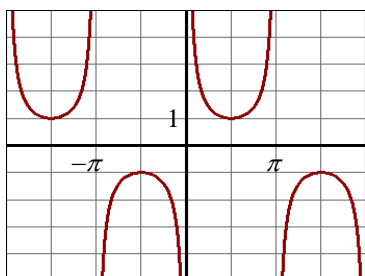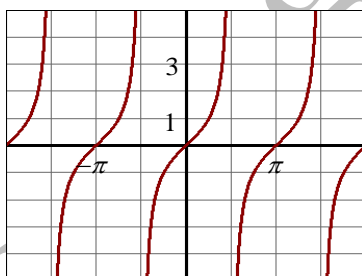
1.

2.

3.

_____

_____

_____

4.

5.

6.
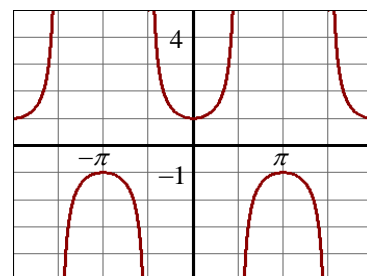
_____

_____

_____

(4.1, 4.2, 4.3, 4.4)
Find the amplitude, the period, any vertical translation, and any phase shift of the following functions.
If not applicable, write "*none*" in the blank.

7. $y = \cot 3x$

amplitude:

period:

vertical translation:

phase shift:

8. $y = 6\cos 8\left(x + \dfrac{3\pi}{4} \right)$

amplitude:

period:

vertical translation:

phase shift:

9. $y = 5 - \sin \dfrac{2}{3}x$

amplitude:

period:

vertical translation:

phase shift:

10. $y = \dfrac{1}{2}\csc\left(2x - \dfrac{\pi}{4} \right)$

amplitude:

period:

vertical translation:

phase shift:

11. $y = -9 + \tan \dfrac{1}{2}x$

amplitude:

period:

vertical translation:

phase shift:

12. $y = \sec 9\left(x - \dfrac{5\pi}{6} \right)$
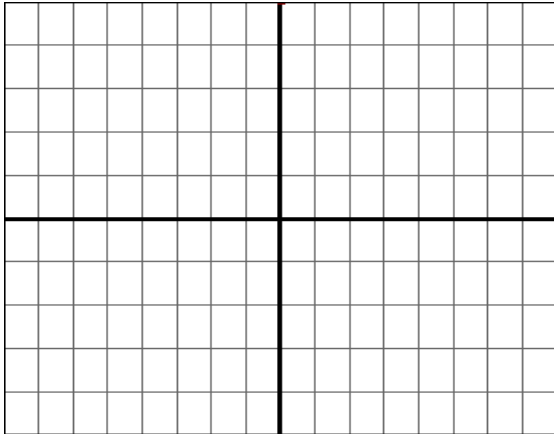
amplitude:

period:

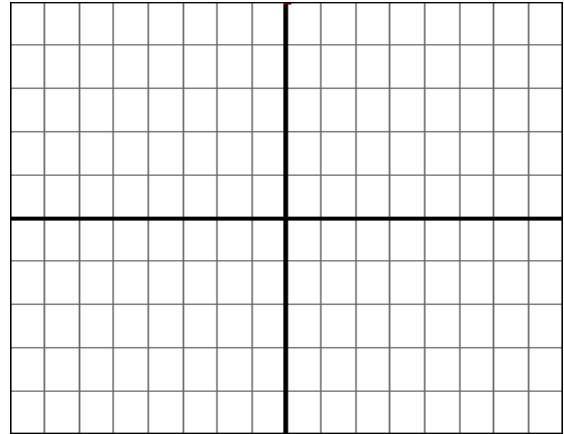**vertical translation:**

phase shift:

(4.1, 4.2, 4.3, 4.4)

Graph the following functions over a two-period interval. Identify and label any asymptotes.

13. $y = 2\csc\dfrac{1}{4}x$

14. $y = -\cos\left(x - \dfrac{\pi}{4}\right)$

15. $y = \sec 2x$
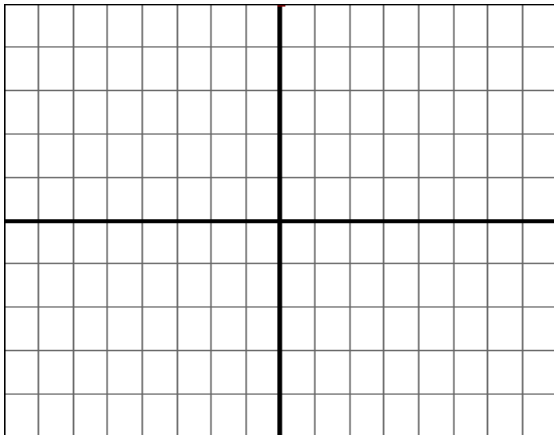
16. $y = \tan\left(x + \dfrac{\pi}{2}\right)$
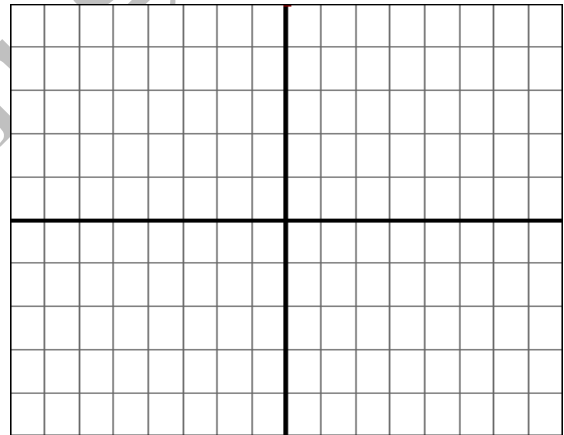
PART II: YOU MAY USE A CALCULATOR (136 points)

(1.1)

1. Convert the following angles to decimal degrees. If applicable, round to the nearest hundredth of a degree.

a) $76°48'$ b) $34°51'35''$ c) $249°15'$

2. Convert to degrees, minutes, and seconds. If applicable, round to the nearest second.

a) $310.485°$ b) $58.3°$ c) $102.9004°$

(1.2)

3.   Find the measure of θ.

a) **note:** lines *m* and *n* are parallel

b)
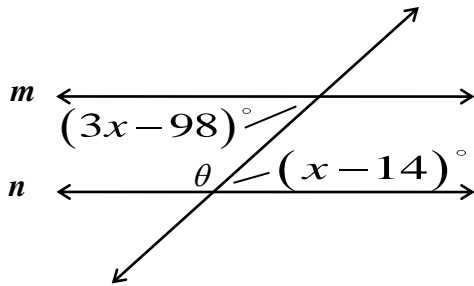
(1.3)

4.   Find the exact values of the six trigonometric functions for the angle θ in standard position having the given point on its terminal side. Rationalize denominators when applicable.

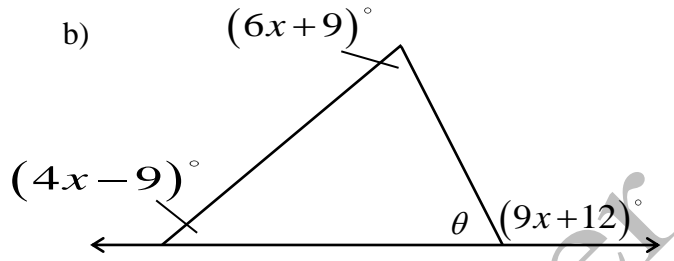a) $(-8, 15)$ b) $(9, -2)$ c) $(0, -2)$

(1.4)

5.   Find the exact values of the five remaining trigonometric functions for each angle θ. Rationalize denominators when applicable.

a) $\sin \theta = \dfrac{\sqrt{3}}{5}$, and $\cos \theta < 0$

b) $\sec \theta = -\dfrac{5}{4}$ and θ is in quadrant III.

(2.1, 2.2)

Find the exact value of each expression.

6. $\cos 30°$ _____

7. $\sin 270°$ _____

8. $\cot 315°$ _____

9. $\tan 90°$ _____

10. $\sin 240°$ _____

11. $\csc 210°$ _____

12. $\sec(-45°)$ _____

13. $\tan(-300°)$ _____

(2.3)

14. Find a value of θ in the interval $\left[0°, 90°\right)$ that satisfies the given statement.

    Write your answer in decimal degrees to four decimal places.
   a) $\csc\theta = 2.3861147$ b) $\tan\theta = 2.674321$

15. Find *all* values of θ in the interval $\left[0°, 360°\right)$ that satisfies the given statement.

    Write your answer in decimal degrees to two decimal places.
    a) $\sec\theta = -9.56677$ b) $\sin\theta = -0.53$

(2.4)

16. Solve the following right triangles where $C = 90°$.
    Make sure to use the correct number of significant digits in your final answer.
    a) $B = 47°53', b = 298.6\,m$ b) $A = 58°30', c = 748\,in.$ c) $a = 129.7\,ft, b = 368.1\,ft$

(2.5)

17. Find h as indicated in the figure.
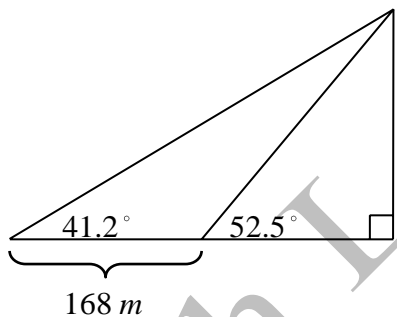
18. Suppose that an observer for a radar station is located at the origin of a coordinate system.
    Find the bearing of an airplane located at the following points.
    Express the bearing giving the direction from a north-south line.
    a) $(3, -3)$ b) $(-5, 5)$

19. Solve the following problem. Include a labeled sketch in your work.
    A ship leaves a pier on a bearing of $118°$ and travels for 75 km. It then turns and continues on a bearing of $28°$ for 53 km. How far is the ship from the pier, to the nearest km?

(3.1)

20. Convert the following angles to radians. Leave answers as multiples of π.

    a) $110°$ b) $216°$

(3.2)

21. A central angle of a circle with radius 8.973 cm intercepts an arc of 7.683 cm.
    a) Find the radian measure of the angle.   b) Find the measure of the angle in degrees.
    Make sure to use the correct number of significant digits in your final answer.

(3.3)

Find each *exact* function value. Rationalize denominators when applicable.

22. $\tan \dfrac{3\pi}{4}$ _____   23. $\cos \dfrac{\pi}{6}$ _____

24. $\cot \dfrac{11\pi}{6}$ _____   25. $\sin \dfrac{5\pi}{3}$ _____

26. $\cos \dfrac{\pi}{2}$ _____   27. $\csc \dfrac{5\pi}{4}$ _____

28. $\sec \dfrac{2\pi}{3}$ _____   29. $\csc(-\pi)$ _____

30. Find the *exact* value(s) of β in the interval $[0, 2\pi]$ that makes the given statement true.

    a) $\sin \beta = -\dfrac{\sqrt{3}}{2}$ b) $\cos \beta = \dfrac{\sqrt{2}}{2}$ c) $\tan \beta = -\sqrt{3}$

Part I Answers:

1)   *f*) $y = \cot x$

2)   *a*) $y = \sin x$

3)   *b*) $y = \cos x$

4)   *d*) $y = \csc x$

5)   *c*) $y = \tan x$

6)   *e*) $y = \sec x$

7) **amplitude:** *not applicable (or none)*   **period:** $\dfrac{\pi}{3}$

    vertical translation: *none*   **phase shift:** *none*

8) **amplitude:** 6   **period:** $\dfrac{\pi}{4}$

    vertical translation: *none*   **phase shift:** $\dfrac{3\pi}{4}$ *to the left*

9) **amplitude:** 1   **period:** 3π

    vertical translation: 5 *up*   **phase shift:** *none*

Part I Answers:

10) **amplitude:** *not applicable (or none)*   **period:** π

   vertical translation: *none*   **phase shift:** $\frac{\pi}{8}$ *to the right*

11) **amplitude:** *not applicable (or none)*   **period:** 2π

   vertical translation: 9 *down*   **phase shift:** *none*

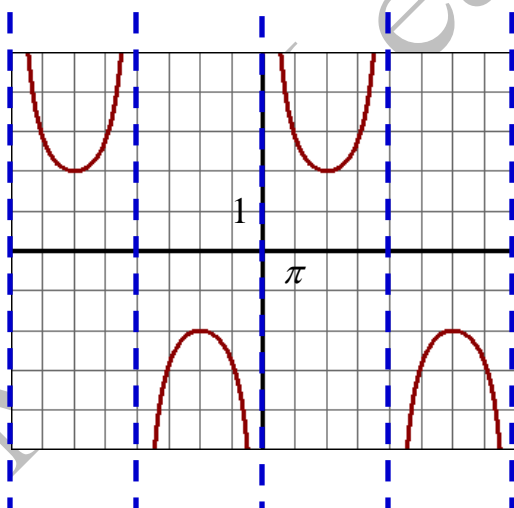**12)** **amplitude:** *not applicable (or none)*   **period:** $\frac{2\pi}{9}$

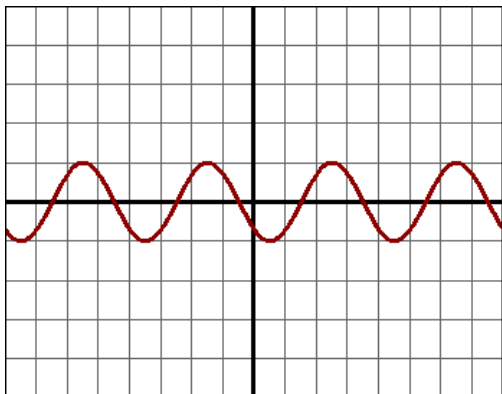   **vertical translation:** *none*   **phase shift:** *none*
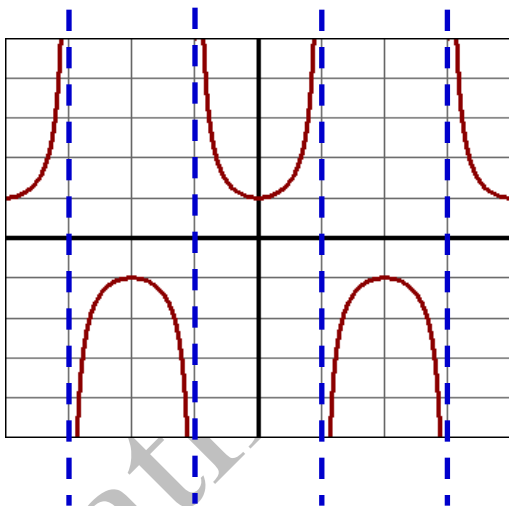
13)

   asymptotes:
   $x = -8\pi$ $x = -4\pi$ $x = 0$ $x = 4\pi$ $x = 8\pi$
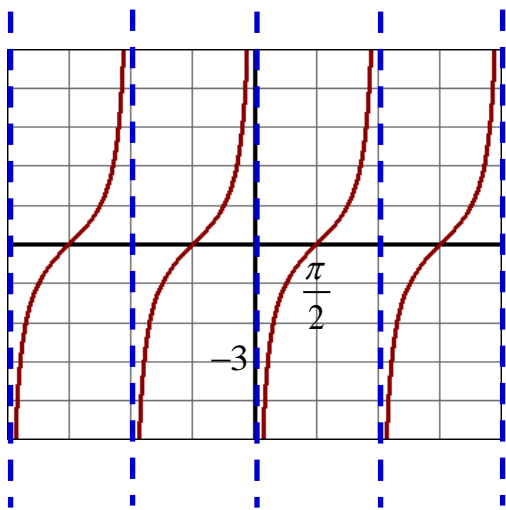
Part I Answers:

14)

15)

asymptotes:

$$x = -\frac{3\pi}{4} \quad x = -\frac{\pi}{4} \quad x = \frac{\pi}{4} \quad x = \frac{3\pi}{4}$$

Part I Answers:

16)

asymptotes:

$x = -2\pi \quad x = -\pi \quad x = 0 \quad x = \pi \quad x = 2\pi$

Part II Answers:

1)   **a)** $76.8°$ **b)** $34.86°$ **c)** $249.25°$

2)   **a)** $310°29'6"$ **b)** $58°18'$ **c)** $102°54'1"$

3)   **a)** $\theta = 152°$ **b)** $\theta = 60°$

4a)

$$\sin\theta = \frac{15}{17} \qquad \cos\theta = -\frac{8}{17} \qquad \tan\theta = -\frac{15}{8}$$

$$\csc\theta = \frac{17}{15} \qquad \sec\theta = -\frac{17}{8} \qquad \cot\theta = -\frac{8}{15}$$

4b)

$$\sin\theta = -\frac{2\sqrt{85}}{85} \qquad \cos\theta = \frac{9\sqrt{85}}{85} \qquad \tan\theta = -\frac{2}{9}$$

$$\csc\theta = -\frac{\sqrt{85}}{2} \qquad \sec\theta = \frac{\sqrt{85}}{9} \qquad \cot\theta = -\frac{9}{2}$$

4c)

$$\sin\theta = -1 \qquad \cos\theta = 0 \qquad \tan\theta = undefined$$

$$\csc\theta = -1 \qquad \sec\theta = undefined \qquad \cot\theta = 0$$

Part II Answers:

5a)

$$\cos\theta = -\frac{\sqrt{22}}{5} \qquad \tan\theta = -\frac{\sqrt{66}}{22} \qquad \csc\theta = \frac{5\sqrt{3}}{3}$$

$$\sec\theta = -\frac{5\sqrt{22}}{22} \qquad \cot\theta = -\frac{\sqrt{66}}{3}$$

5b)

$$\sin\theta = -\frac{3}{5} \qquad \cos\theta = -\frac{4}{5} \qquad \tan\theta = \frac{3}{4}$$

$$\csc\theta = -\frac{5}{3} \qquad \cot\theta = \frac{4}{3}$$

6) $\dfrac{\sqrt{3}}{2}$

7) -1

8) -1

9) *undefined*

10) $-\dfrac{\sqrt{3}}{2}$

11) -2

12) $\sqrt{2}$

13) $\sqrt{3}$

Part II Answers:

14)   **a)** $24.7772°$ **b)** $69.4979°$

15)   **a)** $96.00°, 264.00°$ **b)** $212.01°, 327.99°$

16a) $a = 270.0\,m$ $c = 402.5\,m$ $A = 42°07'$

16b) $a = 638\,in.$ $b = 391\,in.$ $B = 31°30'$

16c) $c = 390.3\,ft$ $A = 19°25'$ $B = 70°35'$

17) $h = 448\,m$

18)   **a)** $S45°E$ **b)** $N45°W$

19) $92\,km$

20)   **a)** $\dfrac{11\pi}{18}$ **b)** $\dfrac{6\pi}{5}$

21)   **a)** 0.8562 **b)** $49.06°$

22) -1

23) $\dfrac{\sqrt{3}}{2}$

24) $-\sqrt{3}$

25) $-\dfrac{\sqrt{3}}{2}$

26) 0

27) $-\sqrt{2}$

28) -2

29)   *undefined*

30)   **a)** $\dfrac{4\pi}{3}, \dfrac{5\pi}{3}$ **b)** $\dfrac{\pi}{4}, \dfrac{7\pi}{4}$ **c)** $\dfrac{2\pi}{3}, \dfrac{5\pi}{3}$