# Simple Harmonic Motion

## Purpose:

1. To determine the spring constant k for a spring mass system using two different methods.
2. To determine the local value of gravity by using a simple pendulum

## Theory:

Periodic motion is motion of an object that regularly returns to a given position in a fixed time interval. A special kind of periodic motion occurs in mechanical systems when the force acting on an object is proportional to the position of the object relative to some equilibrium point. If this force is always directed toward the equilibrium position, the motion is called Simple Harmonic Motion. The net force acting in a Simple Harmonic System always obeys Hook's Law

The following three concepts are important in discussing any kind of periodic motion:

1. **Amplitude(A):** is the maximum distance of the object from its equilibrium position. In the absence of friction, an object in simple harmonic motion oscillates between the positions -A and +A.
2. **Period (T):** is the time it takes the object to move through one complete cycle of motion, from +A to -A and back to +A. The units of period are seconds.
3. **Frequency (f):** is the number of complete cycles or vibrations per unit time, and is the reciprocal of the period (f= 1/T). The units of frequency are Hertz.

We will study two types of harmonic motion systems in this lab: A Mass-Spring system and a pendulum system. For the Mass-Spring system we will find the constant of the spring by using hook's law and study the relation between the Period and the k. Finally, we will look at the SHM behavior of a Simple Pendulum and determine the local value of gravity.

### <u>Mass-Spring System</u>

This is called a simple harmonic oscillator and it consists of a mass couple to an ideal, mass-less spring which obeys Hook's Law. One end of the spring is attached to the mass and the other is held fixed. When such a spring is stretched a distance x (for a compressed spring, x is negative), the restoring force exerted by the spring is:

$$F = -k*\Delta x$$

The minus sign indicates that the restoring force is always opposite in direction to the distortion. The spring constant k has units of N/m and is a measure of the stiffness of the spring.

For the first phase of the experiment we will investigate an example of simple harmonic motion, or SHM: a weight on a spring. Let's hang the system vertically, so that a mass on the spring stretches it some amount. The point of rest for the system is called the equilibrium point, and we will measure all displacements relative to this point. First we need the spring constant, that quality of a spring which describes its stiffness. Since $F = -k*\Delta x$ ( see above equation) for a spring and $F = -mg$ (g down) for the mass on the spring, each added mass will produce an extension of the spring. If we assume a linear relationship (which it is to a good approximation) we can solve for k

$$k = g\, m/\Delta x.$$

Where:

**k** is the spring constant in N/m
**g** is the value of gravity of 9.81 m/$s^2$
**m** is the value hanging mass in kilograms
**Δx** is the stretched distance in meters ($x_n$ - $x_o$)

In the second part of the spring mass experiment we will study the relation between the value of k and the period of oscillation of the spring mass system.
For a spring mass system the period T is given by:

$$\mathbf{T = 2*\pi*\sqrt{(m/k)}}$$

Where:

**T** is the Mass-Spring system's period in seconds
**m** is the value hanging mass in kilograms
**k** is the spring constant in N/m

**Procedure for the Mass-Spring System:**

**Part I**

1. Record the vertical position of the spring with no load other than the 5g mass hanger.
2. Add a total of 50g to the spring in increments of 5g each time. Remember to account for the hanger mass.
3. Using Excel graph the values of the weight in the Y-axis **(<u>remember you have to multiply your mass by g</u>)** and the stretch values **(Δx)** in the X-axis. **(Use markers only. You should have 10 points)**
4. Plot the best fit line using excel's built in function(Trend line). Display the equation of the line on your graph.

**Part II**

1. Start with the 5 g mass hanger alone loaded on the spring.
2. Pull the spring down enough for you to be able to see an oscillation.
3. **Remember a complete oscillation requires the spring to go down and come back to the same initial point.**
4. Release the spring and record the time for ten oscillations
5. Divide the total time by ten to find the period of oscillation.
6. Repeat steps 1 through 4 for mass increments of 5 g until you have 50 g total (45 g plus 5 g of the hanger). You should have ten values of T at the end.
7. Graph the period square (Y axis) versus the values of the mass (x axis) using excel **(Use markers only. You should have 10 points)**.
8. Plot the best fit line using excel's built in function (Trend line). Display the equation of the line on your graph.

**Calculations:**

1. Determine the value of the spring constant **k in N/m** from the value of the slope of the graph in Part I
2. Determine the value of the spring constant **k in N/m** from the value of the slope of the graph in part II
3. Find the percent difference between the values of k in part I and part II

## <u>Simple Pendulum</u>

A simple pendulum is a device that exhibits Simple Harmonic Motion. A simple pendulum consists of a small bob suspended from a fixed point by a string or a rod. The bob is assumed to behave like a point- like particle of mass m, and the string is assumed mass less. Gravity acting on the bob provides the restoring force. When in equilibrium, the pendulum hangs vertically. When released at some angle with the vertical, the pendulum will swing back and forth along an arc of circle in a fixed vertical plane containing the equilibrium position and the initial position of the string.

The period of a simple pendulum is given by:

$$\mathbf{T = 2*\pi*\sqrt{(L/g)}}$$

Where:

**T** is the Pendulum's period in seconds
**L** is the Pendulum's length in meters
**g** is the value of gravity $m/s^2$

This is equation is only valid when the amplitude of oscillation is much less than a radian.

**Procedure for the Simple Pendulum:**

1. Setup a simple pendulum. Use the spherical steel ball as the bob.
2. Start with a pendulum length of 10 cm. The length L of the pendulum is from the point of support to the center of mass of the bob.
3. Displace the pendulum by about 10 degrees from the equilibrium position and release.
4. Measure the time of 10 oscillations; then divide by ten to obtain the period T.
5. **Remember a complete oscillation requires the pendulum to go from the initial point back to that same initial point.**
6. Repeat steps 1 to 3 for values of L in increments of 10 cm until L is equal to 100 cm.
7. Use Excel to plot the square of the Period in the vertical axis, and the length in meters in the horizontal axis. **(Use markers only. You should have 10 points).**
8. Plot the best fit line using Excel's built in function (Trend line). Display the equation of the line on the graph.

**Calculations:**

1. Determine the local value of gravity in $m/s^2$ from the slope of the graph
2. Find the percent error by using 9.81 $m/s^2$ as the theoretical value of g.