

CALCULUS TWO TEST THREE SAMPLE.

Show all calculations and simplify answers. Page 1 of 2.

1. Compute  $\sum_{n=1}^{\infty} 3^{(-2n)}$

2.  $\sum_{n=1}^{\infty} \frac{n}{\sqrt{n^3 + n^2 + 1}}$  (Apply **limit comparison test** or **comparison test**.)

3.  $\sum_{n=1}^{\infty} n^2 \cdot e^{-n}$  Apply **integral test**. (Integration by parts, or tables.)

4.  $\sum_{n=1}^{\infty} n! \cdot e^{-n}$  Apply **ratio test**.

5. Classify as **divergent**, **conditionally convergent** or **absolutely convergent**.  
**Explanation required.**

(a)  $\frac{1}{2\sqrt{2}} - \frac{1}{3\sqrt{3}} + \frac{1}{4\sqrt{4}} - \frac{1}{5\sqrt{5}} + \dots$

(b)  $\sum_{n=1}^{\infty} \frac{(-1)^n n \sqrt{n}}{n^2 + n}$

6. Compute by using power series.  $\lim_{x \rightarrow 0} \frac{1 - \exp(x^2)}{x^2}$

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7. Find the first four non-zero terms of the Maclaurin series for the following functions.  
(Assume removable discontinuities have been removed.)

(a) 
$$\frac{x^2 - \tan^{-1}(x^2)}{x^4}$$

(b) 
$$(1 - 2x)^{\frac{5}{2}}$$

(c) 
$$\sin(e^x - 1)$$

8. Suppose a function  $f$  has derivatives of all orders, and that  
 $f(0) = 2$ ,  $f'(0) = 3$ ,  $f''(0) = -4$ ,  $f'''(0) = 0$ ,  $f^{(4)}(0) = 6$ , and

$$|f^{(5)}(x)| \leq 15 \text{ if } |x| < \frac{1}{2}.$$

- (a) Find the **4th Maclaurin polynomial**.

- (b) Find an **upper bound** on the absolute value of the error in using the 4th Maclaurin polynomial for  $f(x)$ ,  $|x| < \frac{1}{2}$ .

9. Classify as **divergent**, **conditionally convergent** or **absolutely convergent**.  
**Explanation required.**

(a) 
$$\frac{1}{\sqrt{2}} - \frac{1}{4} + \frac{1}{\sqrt{3}} - \frac{1}{8} + \frac{1}{\sqrt{4}} - \frac{1}{16} + \frac{1}{\sqrt{5}} - \frac{1}{32} + \dots$$

(b) 
$$\sum_{n=1}^{\infty} \frac{(-1)^n \sqrt{n}}{n^2 + 1}$$

10. **Estimate**  $\int_0^{0.1} \ln(1 + x) dx$  accurate to **three decimal places** by using Maclaurin series. What test justifies this accuracy?

( 100 points total. )