Note: Many other questions are possible. Study all previous tests and quizzes, plus groupwork & homework.

1. Decide if **rational or irrational**. **If rational**, write as a **simplified fraction**. **If irrational**, find a calculator **approximation** to at least 5 decimal places.

(a) $\sqrt{0.0361} = 0.19 = \boxed{\frac{19}{100}, \text{ rational}}$

(b) $\sqrt{0.361} \approx \boxed{0.6008327554, \text{ irrational}}$

2. **Solve** the following **system** of equations. **Check** any solution that you find.

$$\begin{aligned} 14x + 15y &= -5 \\ 10x + 21y &= 17 \end{aligned}$$ Since both equations are in standard form, it is convenient to use the addition method. If we decide to eliminate x, we find the LCM of its coefficients. LCM(14, 10) = 70 = 5·14 = 7·10.

$$\begin{aligned} 70x + 75y &= -25 \quad 5E_1 \\ -70x - 147y &= -119 \quad -7E_2 \end{aligned}$$ Add $5E_1 - 7E_2$. $-72y = -144$, $y = \frac{-144}{-72} = 2$

Substitute in E_1, say, to get

$14x + 15(2) = -5$, $14x + 30 = -5$, $14x = -35$, $x = \frac{-35}{14} = -2.5$.

Check: E_1 $14(-2.5) + 15(2) = -35 + 30 = -5$
E_2 $10(-2.5) + 21(2) = -25 + 42 = 17$. $\boxed{\begin{aligned} x &= -2.5 \\ y &= 2 \end{aligned}}$ or $\boxed{(-2.5, 2)}$

3. **Solve** $3(4y - 9) + 8 = 5(3y - 5) - 24$ and **check** your answer.

$12y - 27 + 8 = 15y - 25 - 24$, $12y - 19 = 15y - 49$, $49 - 19 = 15y - 12y$,

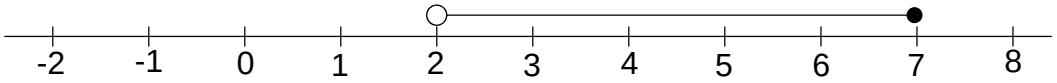
$30 = 3y$, $y = 30 \div 3$, $\boxed{y = 10.}$ Check $3(4(10) - 9) + 8 = 5(3(10) - 5) - 24$,

$3(40 - 9) + 8 = 5(30 - 5) - 24$, $3(31) + 8 = 5(25) - 24$, $93 + 8 = 125 - 24$, $101 = 101$

4. **Solve the inequality,** $-1 < 2x - 5 \leq 9$, and **graph** on the real number line.

Add 5. $-1 + 5 < 2x - 5 + 5 \leq 9 + 5$. Simplify $4 < 2x \leq 14$. Divide by 2.

$\boxed{2 < x \leq 7.}$ The inequalities do not reverse, since 2 is positive.



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5. **Factor** completely $10x^4 + 25x^3 - 165x^2 = 5x^2(2x^2 + 5x - 33) =$

$\boxed{5x^2(2x + 11)(x - 3)}$

6. **Simplify** and write with only **positive** Exponents. $\dfrac{(a^4 b^{-6})^{-4}}{(a^{-3} b^7)^3} = \dfrac{(a^4)^{-4}(b^{-6})^{-4}}{(a^{-3})^3 (b^7)^3} =$

$$\frac{a^{-16} b^{24}}{a^{-9} b^{21}} = \frac{b^{24} b^{-21}}{a^{16} a^{-9}} = \boxed{\frac{b^3}{a^7}}$$

7. **Long division**.

$$\begin{array}{r} 2x^2 + x + 4 \\ x - 8 \;\overline{)\; 2x^3 - 15x^2 - 4x - 12} \\ \underline{2x^3 - 16x^2} \\ x^2 \\ \underline{x^2 - 8x} \\ 4x \\ \underline{4x - 32} \\ 20 \end{array}$$

The quotient is $\boxed{2x^2 + x + 4 + \frac{20}{x - 8}}$

Synthetic division can also be used.

$$\begin{array}{r|rrrr} 8 & 2 & -15 & -4 & -12 \\ & & 16 & 8 & 32 \\ \hline & 2 & 1 & 4 & 20 \end{array}$$

Check: $(x - 8)(2x^2 + x + 4) + 20 = \ldots\ldots = 2x^3 - 15x^2 - 4x - 12.$
A quick, but incomplete, check, is to substitute 8 into the dividend. The result must equal the remainder.
Let $x = 8$. Then $2x^3 - 15x^2 - 4x - 12 = 2(8)^3 - 15(8)^2 - 4(8) - 12 = 20$

8. **Add** $\dfrac{x+3}{x^2 - 4x - 77} + \dfrac{x-3}{x^2 - 13x + 22} = \dfrac{x+3}{(x-11)(x+7)} + \dfrac{x-3}{(x-11)(x-2)} =$

$$\frac{x+3}{(x-11)(x+7)} \cdot \frac{x-2}{x-2} + \frac{x-3}{(x-11)(x-2)} \cdot \frac{x+7}{x+7} =$$

$$\frac{(x+3)(x-2) + (x-3)(x+7)}{(x-11)(x+7)(x-2)} = \frac{x^2 + x - 6 + x^2 + 4x - 21}{(x-11)(x+7)(x-2)} =$$

$$\frac{x^2 + x - 6 + x^2 + 4x - 21}{(x-11)(x+7)(x-2)} = \boxed{\frac{2x^2 + 5x - 27}{(x-11)(x+7)(x-2)}}$$

(The numerator does not factor using integers, so there is no cancellation.)

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9. **Multiply** $\dfrac{x^2-8x+16}{x^2-4x-77} \cdot \dfrac{x^2-121}{x^2-16} = \dfrac{(x-4)(x-4)}{(x+7)(x-11)} \cdot \dfrac{(x+11)(x-11)}{(x+4)(x-4)}$

Cancelling $\dfrac{(x-4)(x-11)}{(x-11)(x-4)}$ leaves $\boxed{\dfrac{(x-4)(x+11)}{(x+4)(x+7)}}$ or $\boxed{\dfrac{x^2+7x-44}{x^2+11x+28}}$

10. **Solve** $12x^2 = 11x + 15$. $12x^2 - 11x - 15 = 0$ $12(-15) = -180 = -20(9)$,

with $-20 + 9 = -11$. $12x^2 - 11x - 15 = (12x^2 - 20x) + (9x - 15) =$

$4x(3x - 5) + 3(3x - 5) = (4x + 3)(3x - 5) = 0$. $4x + 3 = 0$, or $3x - 5 = 0$.

$4x = -3$, or $3x = 5$. $\boxed{x = \frac{-3}{4},\ \frac{5}{3}}$

11. **Solve** $9x^2 - 30x + 22 = 0$ using the **quadratic formula**. $a = 9$, $b = -30$, $c = 22$,

Compute the **Discriminant.** $b^2 - 4ac = (-30)^2 - 4(9)(22) = 900 - 792 = 108$. $x =$

$$\frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-(-30) \pm \sqrt{108}}{2(9)} = \frac{30 \pm 6\sqrt{3}}{18} = \frac{6(5 \pm \sqrt{3})}{18} = \boxed{\frac{5 \pm \sqrt{3}}{3}}$$

12. **Rationalize the denominator** $\dfrac{5-3\sqrt{7}}{3+\sqrt{7}} = \dfrac{5-3\sqrt{7}}{3+\sqrt{7}} \cdot \dfrac{3-\sqrt{7}}{3-\sqrt{7}} =$

$$= \frac{(5-3\sqrt{7})(3-\sqrt{7})}{(3+\sqrt{7})(3-\sqrt{7})} = \frac{(5)(3)-5\sqrt{7}-9\sqrt{7}+3(\sqrt{7})^2}{(3)^2-(\sqrt{7})^2} =$$

$$\frac{15-14\sqrt{7}+3(7)}{9-7} = \frac{36-14\sqrt{7}}{2} = \frac{36}{2} - \frac{14\sqrt{7}}{2} = \boxed{18-7\sqrt{7}.}$$

13. (a) Find the **slope-intercept equation** of the line through the point $(4, 3)$ with slope, $m = \frac{7}{4}$. We want the form $y = mx + b$.

Write down the **point-slope equation**. $y - y_1 = m(x - x_1)$, $y - 3 = \frac{7}{4}(x - 4)$.

Distribute the slope to get $y - 3 = \frac{7}{4}x - 7$, then add 3. $\boxed{y = \frac{7}{4}x - 4.}$

Check: If $x = 4$, $y = \frac{7}{4}(4) - 4 = 7 - 4 = 3$. Coefficient of x is $\frac{7}{4}$.

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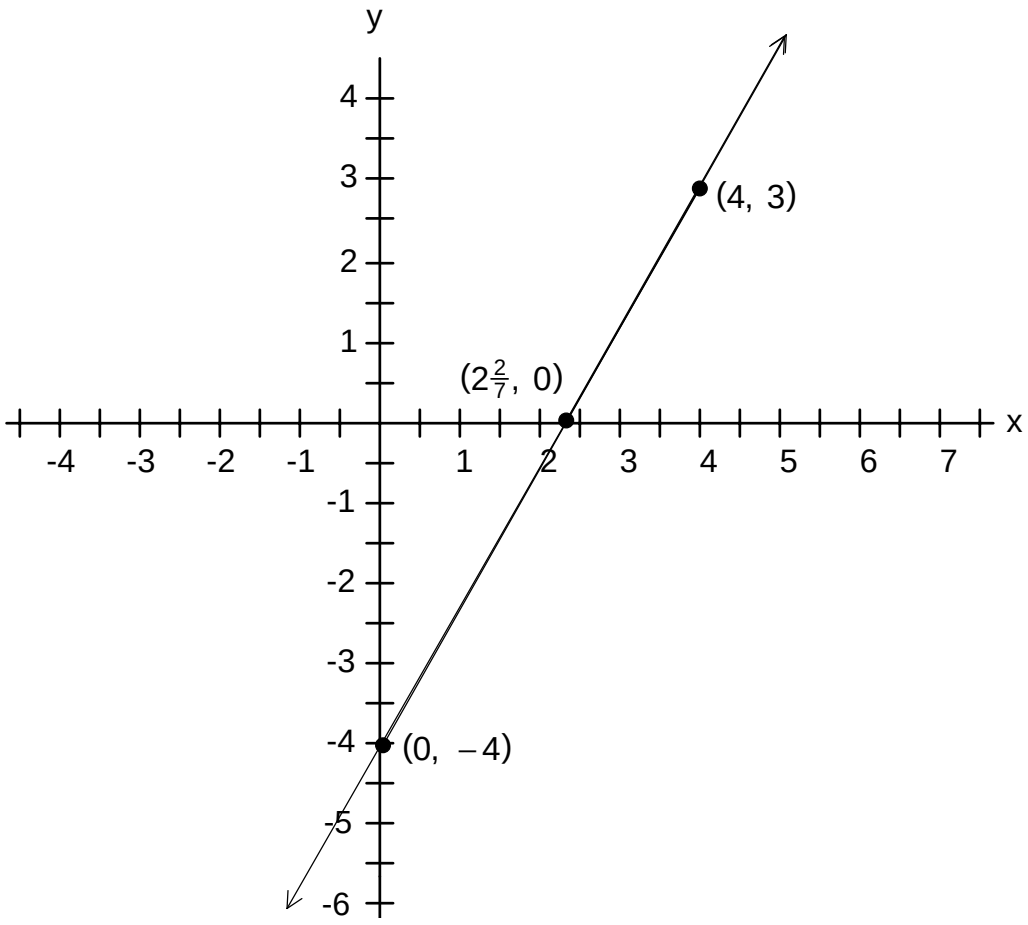
13. (b) On graph paper, draw a large, accurate **graph** of the line in part (a).



14. Given the line with equation $-7x + 6y = 21$, find

(a) the **x-intercept**. Let $y = 0$ in the equation. $-7x + 6(0) = 21$,

$-7x = 21$, $x = -3$. The x-intercept is $(-3, 0)$. .

(b) the **y-intercept** Let $x = 0$ in the equation. $-7(0) + 6y = 21$,

$6y = 21$, $y = \frac{21}{6} = \frac{7}{2} = 3.5$. The x-intercept is $(0, 3.5)$. .

(c) find the **slope** of the line. We can solve for Y. $6y = 7x + 21$, $y = \frac{7}{6}x + \frac{7}{2}$

The slope is $\frac{7}{6}$. ($\frac{7}{6}$ is the coefficient of x in the slope-intercept equation.

We can also use the two intercepts and the slope formula.)

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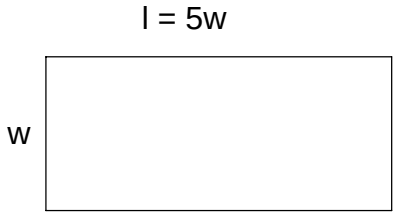
In the word problems, set up an appropriate equation to solve the problem. Remember to <u>check</u>, and answer with a <u>sentence</u>.

15. The **perimeter** of a rectangle is 78 meters. The length is five times the width. Find the dimensions of the rectangle.



$P = 2(l + w)$, $78 = 2(5w + w)$,

$2(6w) = 78$, $12w = 78$,

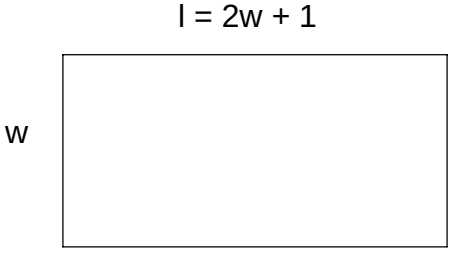
$w = 78 \div 12 = 6.5$,

$l = 5w = 5(6.5) = 32.5$.

Check: $2(32.5 + 6.5) = 2(39) = 78$.

The rectangle is 6.5 meters by 32.5 meters.

16. The **area** of a rectangle is 78 square meters. The length is one meter more than twice the width. Find the dimensions of the rectangle.



$l = 2w + 1$, $A = lw$,
$w(2w + 1) = 78$, $2w^2 + w = 78$,
$2w^2 + w - 78 = 0$. If necessary, we could use the quadratic formula here to solve.
$(w - 6)(2w + 13) = 0$, so $w = 6, -6.5$.
We reject $-6.5 < 0$.
Using $w = 6$, $l = 2w + 1 = 12 + 1 = 13$.
Check: $lw = 13(6) = 78$.

The rectangle is 6 meters by 13 meters.

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17. Al starts walking north at noon, at 3 miles per hour. Sal leaves the same spot one hour later, running after Al at 5 miles per hour. What time is it when Sal catches up with Al, and how far have they gone?

	Distance	Rate	Time
Al	$3t$	3	t
Sal	$5(t-1)$	5	$t-1$

Let t be the Al's time in hours. Sal left later and has a shorter travel time.

$5(t-1) = 3t$, $5t - 5 = 3t$, $5t - 3t = 5$, $2t = 5$, $t = 2.5$,

$t - 1 = 2.5 - 1 = 1.5$. Check: $5(t-1) = 5(1.5) = 7.5$, $3t = 3(2.5) = 7.5$

2.5 hours after noon is 2:30 pm.

Sal catches up at 2:30 pm. They each travel 7.5 miles.

18. Fred's motorboat can travel at 5 miles per hour in still water. He motors **downstream** for 13 miles, then cuts the motor and drifts at the speed of the current for 3 miles. The total time is 4 hours. Find the speed of the current.

	Distance	Rate	Time
Motoring	13	$x + 5$	$\frac{13}{x+5}$
Drifting	3	x	$\frac{3}{x}$

Let the current be x miles per hour. Speed motoring downstram is $x + 5$ miles per hour. We must have $0 \le x$. The times motoring and drifting add to the total time.

$\frac{13}{x+5} + \frac{3}{x} = 4$. The LCD is $x(x+5) = x^2 + 5x$. Multiply by LCD.

$13x + 3(x+5) = 4(x^2 + 5x)$, $13x + 3x + 15 = 4x^2 + 20x$,

$0 = 4x^2 + 4x - 15$, $0 = (2x-3)(2x+5)$, $2x - 3 = 0$, or $2x + 5 = 0$.

$x = 1.5, -2.5$ Reject $-2.5 < 0$. Check 1.5.

$\frac{13}{1.5+5} + \frac{3}{1.5} = \frac{13}{6.5} + \frac{3}{1.5} = 2 + 2 = 4$. (Note: It is an accident that the times are equal.)

The current is 1.5 miles per hour. .

(100 points total.)