

1. (a) State if **rational** or **irrational**. If irrational, give your calculator **approximation**.
If rational write as a **reduced fraction**.

(i) $\sqrt{0.64} = 0.8 = \frac{4}{5}$, **rational**

(ii) $\sqrt{0.46} \approx 0.678232998$, **irrational**

- (b) Name the property illustrated

(i) $3 + (w + x) = (3 + w) + x$ **Associative** property of addition.

(ii) $3(w + x) = 3w + 3x$ **Distributive** property of multiplication over addition.

2. (a) **Evaluate** $4x^2 - 3y$ if $x = -3$ and $y = -4$. Show the substitution correctly.

$$4(-3)^2 - 3(-4) = 4(9) + 12 = 36 + 12 = 48$$

- (b) Find the **sum** of the polynomials: $3y^2 - 4y + 7 + y^2 + 5y - 11$.

$$3y^2 + y^2 - 4y + 5y + 7 - 11 = 4y^2 + y - 4$$

- (c) **Simplify** the expression $2(9x - 7) - 3(4x + 3) = 2(9x) - 2(7) - 3(4x) - 3(3) =$

$$18x - 14 - 12x - 9 = 6x - 23.$$

- (d) **Simplify** the expression $5x - 6(5 - 7x) = 5x - 6(5) - 6(-7x) =$

$$5x - 30 + 42x = 47x - 30$$

3. Simplify, and write with positive exponents only.

(a) $(2x^{-2}y^3)^{-2} = 2^{-2}(x^{-2})^{-2}(y^3)^{-2} = \frac{1}{2^2} x^{(-2)(-2)} y^{3(-2)} = \frac{1}{4} x^4 y^{-6} = \frac{x^4}{4y^6}$

(b) $\frac{(x^{-3}y^{-1})^{-3}}{(x^4y^{-3})^{-2}} = \frac{x^9y^3}{x^{-8}y^6} = \frac{x^9x^8}{y^{-3}y^6} = \frac{x^{17}}{y^3}$

4. Solve the equations and check your answers.

(a) $2x - 8 = 9(x + 3)$, distribute: $2x - 8 = 9x + 27$,
 addition property: $2x - 9x = 27 + 8$, collect like terms: $-7x = 35$,
 multiplication property : $x = 35/(-7)$, $x = -5$.

Check in original equation: $2(-5) - 8 = 9((-5) + 3)$, $-10 - 8 = 9(-2)$, $-18 = -18$.

(b) $16x - 7 = 11x + 18$, $16x - 11x = 18 + 7$, $5x = 25$, $x = 5$.

Check in original equation: $16(5) - 7 = 11(5) + 18$, $80 - 7 = 55 + 18$, $73 = 73$.

(c) $\frac{3}{2}x + \frac{5}{6} = \frac{2}{3}x + 1$ Multiply by 6, the LCD of the fractional coefficients.

$6(\frac{3}{2}x + \frac{5}{6}) = 6(\frac{2}{3}x + 1)$, $6(\frac{3}{2}x) + 6(\frac{5}{6}) = 6(\frac{2}{3}x) + 6(1)$,

$9x + 5 = 4x + 6$, $9x - 4x = 6 - 5$, $5x = 1$, $x = \frac{1}{5}$

Check in original equation: $\frac{3}{2}(\frac{1}{5}) + \frac{5}{6} = \frac{2}{3}(\frac{1}{5}) + 1$, $\frac{3}{10} + \frac{5}{6} = \frac{2}{15} + 1$,

$\frac{9}{30} + \frac{25}{30} = \frac{2}{15} + \frac{15}{15}$, $\frac{34}{30} = \frac{17}{15}$, $\frac{17}{15} = \frac{17}{15}$

5. (a) **Solve** for y in the equation $7x + 5y = 4$. $5y = 4 - 7x$,

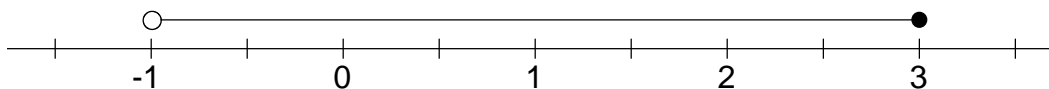
$y = \frac{4 - 7x}{5}$

(b) **Solve the inequality** $3x - 4 \geq 5x + 6$, $3x - 5x \geq 6 + 4$,

$-2x \geq 10$, $\frac{-2x}{-2} \leq \frac{10}{-2}$, $x \leq -5$

↑ Note: inequality is reversed here.

(c) **Graph the inequality** $-1 < x \leq 3$



(d) Write an **algebraic expression** for "three times the difference of a number and Five".

$3(x - 5)$

6. Consider the straight line through the points $(1, 3)$ and $(4, 1)$.

(a) Find the **slope** of the line. $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 3}{4 - 1} = \boxed{\frac{-2}{3}}$

(b) Find the **slope-intercept equation** of the line. We want the form $y = mx + b$.

We can start with the point-slope form with either point. Using the first point,

$$y - y_1 = m(x - x_1), \quad y - 3 = -\frac{2}{3}(x - 1), \quad y - 3 = -\frac{2}{3}x + \frac{2}{3},$$

$$y = -\frac{2}{3}x + \frac{2}{3} + 3, \quad \boxed{y = -\frac{2}{3}x + \frac{11}{3}}$$

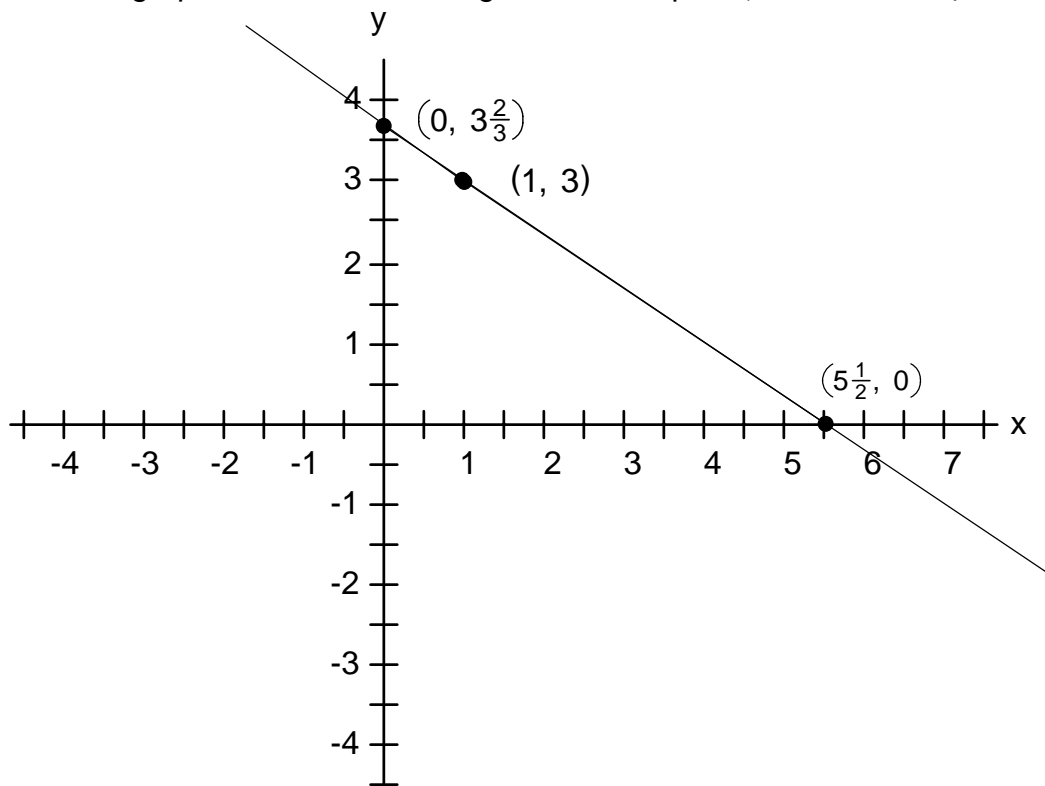
Check: If $x = 1$, $y = -\frac{2}{3}(1) + \frac{11}{3} = \frac{-2+11}{3} = \frac{9}{3} = 3$.
 If $x = 4$, $y = -\frac{2}{3}(4) + \frac{11}{3} = \frac{-8+11}{3} = \frac{3}{3} = 1$.

(c) Find the (x, y) -coordinates of the **x-intercept**. Let $y = 0$ in the equation.

$$0 = -\frac{2}{3}x + \frac{11}{3}, \quad \frac{2}{3}x = \frac{11}{3}, \quad x = \frac{11}{3} \cdot \frac{3}{2} = \frac{11}{2}$$

The x-intercept is $\boxed{(5\frac{1}{2}, 0)}$.

(d) Draw a graph of the line, showing both intercepts. (Can check a 3rd point, e.g. $(1, 3)$.)



7. (a) Find the **difference** of the polynomials $5x^2 - 7x + 2$ and $2x^2 - 4x + 7$.

$$5x^2 - 7x + 2 - (2x^2 - 4x + 7) = 5x^2 - 7x + 2 - 2x^2 + 4x - 7 = \boxed{3x^2 - 3x - 5}$$

(b) Find the product $(2x + 7)(5x - 2) = (2x)(5x) + (2x)(-2) + 7(5x) + 7(-2) =$

$$10x^2 - 4x + 35x - 14 = \boxed{10x^2 + 31x - 14}$$

(c) Expand $(5x + 7)^2 = (5x)^2 + 2(5x)(7) + 7^2 =$

$$\boxed{25x^2 + 20x + 49}$$

8. (a). Write the **quotient in scientific notation** $(2 \times 10^{97}) \div (5 \times 10^{22})$

$$= 2 \div 5 \times 10^{97} \div 10^{22} = 0.4 \times 10^{97-22} = 4 \times 10^{-1} \times 10^{75} = \boxed{4 \times 10^{74}}$$

(b) **Long division**

$$\begin{array}{r}
 3x^2 + 2x + 2 \\
 x - 5 \overline{) 3x^3 - 13x^2 - 8x + 2} \\
 \underline{3x^3 - 15x^2} \\
 2x^2 \\
 \underline{2x^2 - 10x} \\
 2x \\
 \underline{2x - 10} \\
 12
 \end{array}$$

The quotient is $\boxed{3x^2 + 2x + 2 + \frac{12}{x-5}}$

Synthetic division can also be used.

$$\begin{array}{r|rrrrr}
 5 & 3 & -13 & -8 & 2 & \\
 & & 15 & 10 & 10 & \\
 \hline
 & 3 & 2 & 2 & 12 &
 \end{array}$$

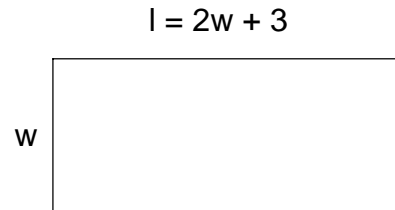
Check: $(x - 5)(3x^2 + 2x + 2) + 12 = \dots = 3x^3 - 13x^2 - 8x + 2$.

A quick, but incomplete, check, is to substitute 5 into the dividend. The result must equal the remainder.

Let $x = 5$. Then $3x^3 - 13x^2 - 8x + 2 = 3(5)^3 - 13(5)^2 - 8(5) + 2 = 12$

**IN THE FOLLOWING WORD PROBLEMS, SET UP AN APPROPRIATE EQUATION,
 SOLVE, CHECK, AND ANSWER WITH A SENTENCE.**

9. The length of a rectangle is 3 meters more than **twice** the width. The perimeter of the rectangle is 54 meters. Find the dimensions of the rectangle, using algebra.



$$P = 2(l + w), \quad 54 = 2(2w + 3 + w),$$

$$2(3w + 3) = 54, \quad 6w + 6 = 54, \quad 6w = 54 - 6 = 48,$$

$$w = 48 \div 6 = 8, \quad l = 2(8) + 3 = 19.$$

Check: $2(19 + 8) = 2(27) = 54.$

The rectangle is 8 meters by 19 meters.

10. One cyclist rides away at 9 miles per hour. One-half hour later another cyclist follows the first at 12 miles per hour. How long will it take the second cyclist to catch up with the first? How many miles has each cycled?

	Distance	Rate	Time
First cyclist	$9(t + \frac{1}{2})$	9	$t + \frac{1}{2}$
2nd cyclist	$12t$	12	t

Let t be the 2nd cyclist's time in hours. The first left earlier and has a longer travel time.

$$12t = 9(t + \frac{1}{2}), \quad 12t = 9t + \frac{9}{2}, \quad 12t - 9t = 4.5,$$

$$3t = 4.5, \quad t = 4.5 \div 3 = 1.5, \text{ or } 1\frac{1}{2},$$

$$t + \frac{1}{2} = 1\frac{1}{2} + \frac{1}{2} = 2.$$

Check: $9(t + \frac{1}{2}) = 9(2) = 18, \quad 12t = 12(1.5) = 18$

The second cyclist takes 1.5 hours to catch up. They each travel 18 miles.

11. Maria splits her \$15,000 lottery prize into two parts, one part invested at 4%, the other part at 7%. How much is invested at each rate, if the total interest after one year is 5% of \$15,000 ?

	Principal	Rate	Interest
First part	x	0.04	$0.04x$
2nd part	$15000 - x$	0.07	$0.07(15000 - x)$
Total	15000	0.05	750

$I = Prt$. Since $t = 1$, the interest amount is the product of the principal and the rate.

The interest amounts have to add up, so

$$0.04x + 0.07(15000 - x) = 0.05(15000), \quad 0.04x + 1050 - 0.07x = 750,$$

$$-0.03x = 750 - 1050 = -300, \quad x = -300 \div (-0.03) = 10000,$$

$$15000 - x = 15000 - 10000 = 5000$$

Check: 4% of 10000 is 400, 7% of 5000 is 350, $400 + 350 = 750 = 5\%$ of 15000

Maria invested \$10,000 at 4% and \$5,000 at 7%.

12. Find how many liters of a 2% solution and a 7% solution must be mixed to make 60 liters of a 4% solution.

	Solution	Rate	Solute
First part	x	0.02	$0.02x$
2nd part	$60 - x$	0.07	$0.07(60 - x)$
Total	60	0.04	2.4

The rate here is usually called **strength**, or **concentration**.

$$0.02x + 0.07(60 - x) = 0.04(60), \quad 0.02x + 4.2 - 0.07x = 2.4,$$

$$-0.05x = 2.4 - 4.2 = -1.8, \quad x = -1.8 \div (-0.05) = 36,$$

$$60 - x = 60 - 36 = 24,$$

Check: 2% of 36 is 0.72, 7% of 24 is 1.68, $0.72 + 1.68 = 2.4 = 4\%$ of 60.

Mix 36 liters of 2% solution and 24 liters of 7% solution.

(100 points, total.)