1. **Factor completely**.

(a) $x^2 - 11x + 30 = \boxed{(x - 5)(x - 6)}$

(b) $8x^3 + 125y^3 = (2x)^3 + (5y)^3 =$

$(2x + 5y)((2x)^2 - (2x)(5y) + (5y)^2) =$

$\boxed{(2x + 5y)(4x^2 - 10xy + 25y^2)}$

(c) $3x^3 - 5x^2 - 48x + 80 = (3x^3 - 5x^2) - (48x - 80) = x^2(3x - 5) - 16(3x - 5)$

$(x^2 - 16)(3x - 5) = \boxed{(x - 4)(x + 4)(3x - 5)}$

(d) $21x^3 - 71x^2 - 22x$ Work: $21 \cdot (-22) = -462 = -77 \cdot 6$, and $-77 + 6 = -71$.

$21x^3 - 71x^2 - 22x = x(21x^2 - 71x - 22) = x(21x^2 - 77x + 6x - 22) =$

$x[(21x^2 - 77x) + (6x - 22)] = x[7x(3x - 11) + 2(3x - 11)] =$

$\boxed{x(7x + 2)(3x - 11)}$

2. **Solve** the equation $3x^2 + 13x - 10 = 0$, by factoring. Work: $3 \cdot (-10) = -30 = -2 \cdot 15$, and $-2 + 15 = 13$.

$3x^2 - 2x + 15x - 10 = 0$, $x(3x - 2) + 5(3x - 2) = 0$, $(x + 5)(3x - 2) = 0$,

$x + 5 = 0$, or $3x - 2 = 0$, so $\boxed{x = -5, \tfrac{2}{3}}$

3. **Simplify** $\dfrac{a^2 - 2a - 15}{a^2 - 7a} \cdot \dfrac{a^2}{a^2 + 2a - 35} = \dfrac{(a - 5)(a + 3)}{a(a - 7)} \cdot \dfrac{a^2}{(a - 5)(a + 7)} =$

$\boxed{\dfrac{a(a + 3)}{(a - 7)(a + 7)}}$, or $\boxed{\dfrac{a^2 + 3a}{a^2 - 49}}$

4. (a) For what value(s) of the variable a is the expression

$\dfrac{2a - 15}{a^2 - 7a} + \dfrac{3a}{a^2 - 2a - 35}$ undefined? Solve $a^2 - 7a = 0$ and $a^2 + 2a - 35 = 0$.

$a(a - 7) = 0$ or $(a + 5)(a - 7) = 0$, $\boxed{a = -5, 0, 7}$

It is **defined** for all values of a except $-5, 0, 7$.

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4. (b) **Simplify** $\frac{2a-15}{a^2-7a}+\frac{3a}{a^2-2a-35}=\frac{2a-15}{a(a-7)}+\frac{3a}{(a-7)(a+5)}=$

$$\frac{2a-15}{a(a-7)}\cdot\frac{a+5}{a+5}+\frac{3a}{(a-7)(a+5)}\cdot\frac{a}{a}=\frac{(2a-15)(a+5)+3a\cdot a}{a(a-7)(a+5)}=$$

$$\frac{2a^2-5a-75+3a^2}{a(a-7)(a+5)}=\frac{5a^2-5a-75}{a(a-7)(a+5)}=\boxed{\frac{5(a^2-a-15)}{a(a-7)(a+5)}}$$

a^2-a-15 canot be factored using rational numbers, so no cancellation occurs.

5. **Simplify** $\dfrac{\frac{2}{xy}-\frac{3}{x^2}}{\frac{3}{xy}+\frac{2}{y^2}}=\dfrac{\frac{2}{xy}\cdot\frac{x}{x}-\frac{3}{x^2}\cdot\frac{y}{y}}{\frac{3}{xy}\cdot\frac{y}{y}+\frac{2}{y^2}\cdot\frac{x}{x}}=\dfrac{\frac{2x-3y}{x^2y}}{\frac{3y+2x}{xy^2}}$

$$=\frac{2x-3y}{x^2y}\cdot\frac{xy^2}{2x+3y}=\boxed{\frac{y(2x-3y)}{x(2x+3y)}}\text{ or }\boxed{\frac{2xy-3y^2}{2x^2+3xy}}$$

6. Solve $\frac{1}{15}+\frac{1}{x}=\frac{1}{6}$ The LCD is 30x. $\frac{1}{15}\cdot 30x+\frac{1}{x}\cdot 30x=\frac{1}{6}\cdot 30x$,

$2x+30=5x$, $30=5x-2x$, $30=3x$, $\boxed{x=10.}$

Check: $\frac{1}{15}+\frac{1}{10}=\frac{2}{30}+\frac{3}{30}=\frac{5}{30}=\frac{1}{6}$

7. **Solve** the **equation** $\frac{2x}{x^2-1}+\frac{1}{x+1}=1$, and **check**.

LCD $=x^2-1=(x-1)(x+1)$

$$\frac{2x}{x^2-1}\cdot(x^2-1)+\frac{1}{x+1}\cdot(x^2-1)=1\cdot(x^2-1)$$

$2x+x-1=x^2-1$, $3x=x^2$, $x^2-3x=0$, $x(x-3)=0$, $\boxed{x=0,\ 3.}$

$$\frac{2(0)}{0^2-1}+\frac{1}{0+1}=0+1=1,\quad \frac{2(3)}{3^2-1}+\frac{1}{3+1}=\frac{6}{8}+\frac{1}{4}=1$$

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In the following word problems you must show how to use **algebra** to solve the problem. Set up an appropriate **equation**. Solve it. **Check**, and answer with a **sentence**.

8. April and May can mow the lawn in 40 minutes, working together. April can mow it alone in 90 minutes. How long would it take May, working alone? Give answer to nearest tenth of a minute. (Let May's time be x minutes.)

	Jobs	Rate	Time
April	1	1/90	90
May	1	1/x	x
Both	1	1/40	40

	Jobs	Rate	Time
April	4/9	1/90	40
May	40/x	1/x	40
Both	1	1/40	40

From the first table above, $\frac{1}{90} + \frac{1}{x} = \frac{1}{40}$. Or, from the second table

$\frac{4}{9} + \frac{40}{x} = 1$. (This equation is also obtained by multiplying by 40 in the first equation.)

Multiplying in either equation by its LCD, (360x in the first, 9x in the second) we get

$4x + 360 = 9x$, $360 = 9x - 4x$, $360 = 5x$, $x = 72$. Check in the equation used.

E.g in the first: $\frac{1}{90} + \frac{1}{72} = \frac{4}{360} + \frac{5}{360} = \frac{9}{360} = \frac{1}{40}$.

May can mow the lawn alone in 72 minutes.

9. Cody leaves home at noon, walking North at a rate of 1.5 mi/hr. At 2:00 pm that day, Mara leaves the same house, bicycling North at a rate of 3.5 mi/hr. Find **what time** Mara catches up with Cody, and **how far** from home they are then.

	Distance	Rate	Time
Cody	1.5t	1.5	t
Mara	3.5(t − 2)	3.5	t − 2

Let t be Cody's travel time in hours. Mara left later and has a shorter travel time. Cody left at noon, so his travel time is the time of day, if under 12 hours.

0.5 hour is 30 minutes. It is an accident that the two times are the same numbers as the rates.

$3.5(t - 2) = 1.5t$, $3.5t - 7 = 1.5t$, $3.5t - 1.5t = 7$,

$2t = 7$, $t = 7 \div 2 = 3.5$, $t - 2 = 3.5 - 2 = 1.5$.

Check: $3.5(t - 2) = 3.5(1.5) = 5.25$, $3.5t = 3.5(1.5) = 5.25$

Mara catches up at 3:30 pm that day. They are 5.25 miles from home.

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10. Juanita has $6,000 to invest and she wants to make 4% in interest. How much should she place into a 3% investment and a 6% investment, respectively, to get exactly 4% of $6,000 in interest in one year?

	Principal	Rate	Interest
First part	x	0.06	0.06x
2nd part	6000 – x	0.03	0.03(6000 − x)
Total	6000	0.04	240

I = Prt. Since t = 1, the interest amount is the product of the principal and the rate. Let $x be invested at the higher rate. 4% of $6000 is $240.

The interest amounts have to add up, so

$0.06x + 0.03(6000 - x) = 0.04(6000)$, $0.06x + 180 - 0.03x = 240$,

$0.03x = 240 - 180 = 60$, $x = 60 \div 0.03 = 2000$,

$6000 - x = 6000 - 2000 = 4000$

Check: 6% of 2000 is 120, 3% of 4000 is 120, 120 + 120 = 240 = 4% of 6000

Juanita invested $2,000 at 6% and $4,000 at 3%.

It is an accident that the interest amounts are the same.

11. A fish swims at 2 miles per hour in still water. It swims 24 miles upstream, then returns the same 24 miles. The round trip takes 25 hours. Find the speed of the current.

	Distance	Rate	Time
Upstream	24	2 − x	$\frac{24}{2-x}$
Downstream	24	2 + x	$\frac{24}{2+x}$

Let the current be x miles per hour. Speed upstream is then $2 - x$ miles per hour, speed downstream is $2 + x$ miles per hour. We must have $0 \le x < 2$. The times upstream and downstream add to the total time.

$\frac{24}{2-x} + \frac{24}{2+x} = 25$. The LCD is $(2-x)(2+x) = 4 - x^2$. Multiply by LCD.

$24(2+x) + 24(2-x) = 25(4-x^2)$, $48 + 24x + 48 - 24x = 100 - 25x^2$,

$25x^2 = 100 - 96 = 4$, $x^2 = 0.16$, $x = +\sqrt{0.16} = 0.4$. Check: $0 \le 0.4 < 2$,

$$\frac{24}{2-0.4} + \frac{24}{2+0.4} = \frac{24}{1.6} + \frac{24}{2.4} = 15 + 10 = 25.$$

The current is 0.4 miles per hour.

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12. Find the **slope** of the straight line through the points $(-5,\ 7)$ and $(3,\ -3)$.
Note: Either point can be taken as the first.

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(-3) - 7}{3 - (-5)} = \frac{-10}{8} = \boxed{\frac{-5}{4}}$$

13. A straight line has slope $m = \frac{3}{5}$ and passes through the point $(10,\ -5)$.

(a) Write down the **point-slope equation**. $y - y_1 = m(x - x_1)$,

$\boxed{y - (-5) = \frac{3}{5}(x - 10)}$ or, partially simplify to $\boxed{y + 5 = \frac{3}{5}(x - 10)}$

(b) Find the **slope-intercept equation**. We want the form $y = mx + b$.

Distribute the $\frac{3}{5}$ in the point-slope equation above.

$y + 5 = \frac{3}{5}x - 6$. Then subtract 5. $\boxed{y = \frac{3}{5}x - 11.}$

Check: If $x = 10$, $y = \frac{3}{5}(10) - 11 = 6 - 11 = -5$. Coefficient of x is $\frac{3}{5}$.

Technically, the slope-intercept form is $\boxed{y = \frac{3}{5}x + (-11)}$

14. Consider the line with equation $5x - 4y = 20$.

(a) Find the (x, y)-coordinates of the **x-intercept**. Let $y = 0$ in the equation.

$5x - 4(0) = 20$, $5x = 20$, $x = 4$. The x-intercept is $\boxed{(4,\ 0).}$.

(b) Find (x, y)-coordinates of the **y-intercept**. Let $x = 0$ in the equation.

$5(0) - 4y = 20$, $-4y = 20$, $y = -5$. The y-intercept is $\boxed{(0,\ -5).}$.

(c) Find the **slope** of the line. Solve for y to get the slope-intercept equation and read off the slope, or use the two intercepts. $-4y = -5x + 20$, $y = \frac{5}{4}x - 5$, and The slope is the coefficient of x, namely $\boxed{5/4.}$ Or,

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - (-5)}{4 - 0} = \boxed{\frac{5}{4}}$$

(d) Find the **slope-intercept equation** of the Line. $\boxed{y = \frac{5}{4}x - 5}$
(Using above slope and y-intercept values.)

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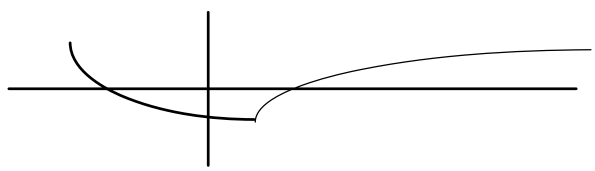
15. Determine if each of the relations below is a **function or not**, giving a detailed, **valid reason** in each case.

(a) $\{(2, 8), (3, 9), (6, 2), (4, 9)\}$ **Function.** No number is repeated as a first component. (So no element of the domain can correspond to more than one element of the range. Note that the domain and range are sets.)

(b) $x^2 + y = 20$ **Function.** We can solve for y uniquely in terms of x. $y = -x^2 + 20$

(c) $x + |y| = 20$ **Not a function.** If $x = 0$, $y = \pm 20$.

(d) **Function.**



The graph passes the vertical line test.

16. If $f(x) = 7x - 3$, compute $f(5)$. $f(5) = 7(5) - 3 = 35 - 3 =$ **32.**

17. **Solve** the system of equations, using **substitution** or **addition** method, and **check** your answer.

$$\begin{aligned} 12x + 5y &= 19 \\ 8x - 7y &= -8 \end{aligned}$$

Since both equations are in standard form, it is convenient to use the addition method. Since the coefficients of y are of opposite sign with an easy to find LCD, it is convenient to eliminate y.

$$\begin{aligned} 84x + 35y &= 133 \quad 7E_1 \\ 40x - 35y &= -40 \quad 5E_2 \end{aligned}$$

Add $7E_1 + 5E_2$ $124x = 93$, $x = \frac{93}{124} = \frac{3}{4}$.

Substitute in E_1, say. $12(\frac{3}{4}) + 5y = 19$, $9 + 5y = 19$, $5y = 10$, $y = 2$.

Check: E_1 $12(\frac{3}{4}) + 5(2) = 9 + 10 = 19$
E_2 $8(\frac{3}{4}) - 7(2) = 6 - 14 = -8$.

$x = 3/4$, $y = 2$ or **$(\frac{3}{4}, 2)$**

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18. (a) **<u>Set up</u> a system of two equations in x and y for the following problem:**

Al wants to make 14 liters of a 36% acid solution. Al has some 30% acid solution and some 50% acid solution on hand. How much of each should Al mix?

Mix x liters of 30% solution and y liters of 50% solution.

$$\begin{aligned} x + y &= 14 \\ 0.3x + 0.5y &= 0.36(14) \end{aligned} \quad \text{or} \quad \begin{aligned} x + y &= 14 \\ 0.3x + 0.5y &= 5.04 \end{aligned}$$

(b) Solve the problem.

$$\begin{aligned} -0.3x - 0.3y &= -4.2 \quad && -0.3E_1 \\ 0.3x + 0.5y &= 5.04 && E_2 \end{aligned}$$

Add $E_2 - 0.3E_1$. $0.2y = .84$, $y = 4.2$.

Substitute in the first Equation. $x + 4.2 = 14$, $x = 14 - 4.2 = 9.8$.

Check:
E_1 $4.2 + 9.8 = 14$
E_2 $0.3(9.8) + 0.5(4.2) = 2.94 + 2.1 = 5.04$

Mix 9.8 liters of 30% solution and 4.2 liters of 50% solution.

(100 points total.)