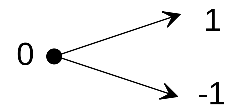
ELEMENTARY ALGEBRA SOLUTIONS TO TEST 3 SAMPLE PROBLEMS.

SHOW ALL CALCULATIONS. SIMPLIFY ANSWERS. .

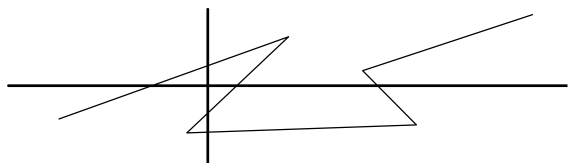
1. Determine if each of the relations below is a **function or not**, giving a detailed, **valid reason** in each case.

   (a) $\{(5, 8), (3, 8), (6, 2), (1, 9)\}$ **[Function.]** No number is repeated as a **first** component.

   (b) $x + y^2 = 1$ **[Not a function.]** If $x = 0$, $y = \pm 1$.



   (c)



   [Not a function.]

   The graph fails the **vertical line test**.

2. (a) If $f(x) = 3x^2 - 5x + 4$, compute $f(-5)$, showing correct substitution.

   $f(-5) = 3(-5)^2 - 5(-5) + 4 = 3(25) + 25 + 4 = 75 + 25 + 4 = \boxed{104.}$

   (b) State the **domain** of the relation $\{(5, 8), (3, 8), (6, 2), (1, 9)\}$

   The domain is $\boxed{\{1, 3, 5, 6\}.}$

   (c) State the **range** of the relation $\{(5, 8), (3, 8), (6, 2), (1, 9)\}$

   The range is $\boxed{\{2, 8, 9\}.}$

3. Use the **addition** method to solve the system of equations, if possible.
   Check the solution, if there is one.

   (a) $\begin{aligned} 8x + 5y &= 7 \\ 2x - 3y &= 6 \end{aligned}$ Since one coefficient of x is a multiple of the other,

   it is convenient to eliminate x.

   $\begin{aligned} 8x + 5y &= 7 & E_1 \\ -8x + 12y &= -24 & -4E_2 \end{aligned}$ Add $E_1 - 4E_2$. $17y = -17$, $y = \dfrac{-17}{17} = -1$.

   Substitute in E_2, say. $2x - 3(-1) = 6$, $2x + 3 = 6$, $2x = 3$, $x = \dfrac{3}{2}$

   Check: $\begin{aligned} E_1 \quad 8(\tfrac{3}{2}) + 5(-1) &= 12 - 5 = 7 \\ E_2 \quad 2(\tfrac{3}{2}) - 3(-1) &= 3 + 3 = 6 \end{aligned}$. $\boxed{\begin{aligned} x &= 3/2 \\ y &= -1 \end{aligned}}$ or $\boxed{(\tfrac{3}{2}, -1).}$

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3. (b) $-4x + 6y = 5$, $2x - 3y = 6$. Since one coefficient of x is a multiple of the other, it is convenient to eliminate x. (We could also eliminate y.)

$-4x + 6y = 5 \quad E_1$
$4x - 6y = 12 \quad 2E_2$

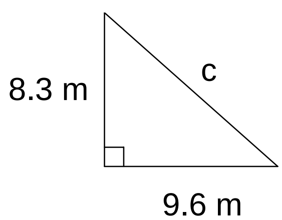
Add $E_1 + 2E_2$. $0 = 17$. **No solution.**

4. (a) Find a **calculator approximation** for $\sqrt{450}$ On the TI-84, press [2nd] [x^2] ($\sqrt{\ }$) [4] [5] [0] [)] [ENTER] to get **21.21320344**

(b) **Simplify** $\sqrt{450} = \sqrt{9 \cdot 25 \cdot 2} = \sqrt{9} \cdot \sqrt{25} \cdot \sqrt{2} = 3 \cdot 5 \cdot \sqrt{2} = \boxed{15\sqrt{2}}$.

Check by evaluating $15\sqrt{2}$ on your calculator, and comparing the approximations.

(c) A right triangle has sides of lengths 9.60 m and 8.30 m forming the right angle. Find the length of the **hypotenuse**, rounded to two decimal places.



$c^2 = (8.3)^2 + (9.6)^2 = 68.89 + 92.16 = 161.05.$

$c = \sqrt{161.05} \approx 12.69054766$

The hypotenuse has length 12.69 m, approximately.

5. **Solve** the equation $\sqrt{6x + 1} = 9 - x$. Be sure to **check.**

$\left(\sqrt{6x + 1}\right)^2 = (9 - x)^2$, $6x + 1 = 81 - 18x + x^2$, $0 = x^2 - 24x + 80$,

$0 = (x - 4)(x - 20)$, $x = 4, 20$. Check $x = 4$. $\sqrt{6(4) + 1} = \sqrt{25} = 5 = 9 - (4)$.

Check $x = 20$. $\sqrt{6(20) + 1} = \sqrt{121} = 11 \neq -11 = 9 - (20)$. Reject 20. $\boxed{x = 4.}$

6. **Rationalize the denominator** and simplify.

(a) $$\frac{2\sqrt{10}}{\sqrt{14}} = \frac{2\sqrt{2}\sqrt{5}}{\sqrt{2}\sqrt{7}} = \frac{2\sqrt{5}}{\sqrt{7}} \cdot \frac{\sqrt{7}}{\sqrt{7}} = \boxed{\frac{2\sqrt{35}}{7}}$$

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6. (b) $\dfrac{\sqrt{11}+3\sqrt{5}}{\sqrt{11}-\sqrt{5}} = \dfrac{\sqrt{11}+3\sqrt{5}}{\sqrt{11}-\sqrt{5}} \cdot \dfrac{\sqrt{11}+\sqrt{5}}{\sqrt{11}+\sqrt{5}} = \dfrac{(\sqrt{11}+3\sqrt{5})(\sqrt{11}+\sqrt{5})}{(\sqrt{11}-\sqrt{5})(\sqrt{11}+\sqrt{5})} =$

$\dfrac{(\sqrt{11})^2+\sqrt{11}\sqrt{5}+3\sqrt{5}\sqrt{11}+3(\sqrt{5})^2}{(\sqrt{11})^2-(\sqrt{5})^2} = \dfrac{11+4\sqrt{55}+15}{11-5} = \dfrac{4\sqrt{55}+26}{6} =$

$\boxed{\dfrac{2\sqrt{55}+13}{3}}$

7. (a) Compute $\sqrt[4]{81}$. $\sqrt[4]{81} = \boxed{3,}$ since $3^4 = 81$.

(b) Give your calculator display for $\sqrt[4]{243}$. On a TI-84 press [4] [MATH] [5]

then [2] [4] [3] [ENTER] to get [3.948222039] Also

[2] [4] [3] [^] [(] [1] [÷] [4] [)] [ENTER]

(b) Simplify $\sqrt[4]{243} = \sqrt[4]{3^5} = \sqrt[4]{3^4 \cdot 3} = \sqrt[4]{3^4} \cdot \sqrt[4]{3} = \boxed{3\sqrt[4]{3}}$

(Be careful not to write $3^4\sqrt{3}$)

(c) Simplify $\sqrt[3]{80x^4y^3z^8} = \sqrt[3]{8x^3y^3z^6 \cdot 10xz^2} = \sqrt[3]{8x^3y^3z^6}\sqrt[3]{10xz^2} =$

$\boxed{2xyz^2 \cdot \sqrt[3]{10xz^2}}$

8. (a) Solve the system of equations $\begin{aligned} 8x - 5y &= 2 \\ y &= 3x + 2 \end{aligned}$ by the **substitution method**,

and **check** your answer. The second equation is a formula for y in terms of x. Substitute the formula for y in the first equation. $8x - 5(3x + 2) = 2$, $8x - 15x - 10 = 2$, $-7x = 12$, $x = 12/(-7) = -12/7$. Use the formula to find y. $y = 3x + 2 = 3(\frac{-12}{7}) + 2 = \frac{-36}{7} + \frac{14}{7} = \frac{-36+14}{7} = -22/7$. Since we just did the computation that checks the second equation, we only have to check the first. $8(-12/7) - 5(-22/7) = -96/7 + 110/7 = 14/7 = 2$.

$\boxed{(x, y) = (-12/7, -22/7)}$

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8. (b) **<u>Set up</u> a system of two equations in x and y for the following problem:**
"Maria wants to make 5 liters of a 28% alcohol solution. 20% alcohol solution and 40% acid solution are available. How much of each should she mix?" (Set up only, do not solve.)

Mix x liters of 20% solution and y liters of 40% solution.

$$\boxed{\begin{aligned} x + y &= 5 \\ 0.2x + 0.4y &= 0.28(5) \end{aligned}} \quad \text{or} \quad \boxed{\begin{aligned} x + y &= 5 \\ 0.2x + 0.4y &= 1.4 \end{aligned}}$$

9. **Solve** the equation $x^2 + 16x - 5 = 0$ by **completing the square**.

$x^2 + 16x = 5$, $\frac{1}{2}(16) = 8$, $8^2 = 64$, $x^2 + 16x + 64 = 5 + 64$, $(x + 8)^2 = 69$,

$x + 8 = \pm\sqrt{69}$, $\boxed{x = -8 \pm \sqrt{69}}$

10. Given the quadratic equation $3x^2 - 10x + 5 = 0$,

(a) compute the **discriminant** $a = 3$, $b = -10$, $c = 5$,

$b^2 - 4ac = (-10)^2 - 4(3)(5) = 100 - 60 = \boxed{40.}$

(b) **solve** the equation by using the **quadratic formula**. $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} =$

$$\frac{-(-10) \pm \sqrt{40}}{2(3)} = \frac{10 \pm 2\sqrt{10}}{6} = \frac{2(5 \pm \sqrt{10})}{6} = \boxed{\frac{5 \pm \sqrt{10}}{3}}$$

12. Find the **distance** between the two points: $(-1, 7)$, $(5, -1)$

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{(-1 - 5)^2 + (7 - (-1))^2} =$$

$$\sqrt{(-6)^2 + (8)^2} = \sqrt{36 + 64} = \sqrt{100} = \boxed{10.}$$

13. Simplify $(81)^{-3/4}$ (Write answer in **fraction** form.) $(81)^{-3/4} = \left[(81)^{1/4}\right]^{-3} = 3^{-3} = \boxed{\frac{1}{27}}$

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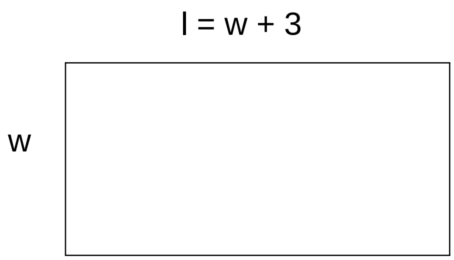
14. A rectangle has a length 3 meters longer than its width.
Its area is 154 square meters. Find the dimensions of the rectangle.



Let the width be w meters, and the length l Meters. $l = w + 3$, $A = lw$,
$w(w + 3) = 154$, $w^2 + 3w = 154$,
$w^2 + 3w - 154 = 0$. If necessary, we could use the quadratic formula here to solve.
$(w - 11)(w + 14) = 0$, so $w = 11, -14$.
We reject $-14 < 0$.

Using $w = 11$, $l = w + 3 = 11 + 3 = 14$. Check: $lw = 11(14) = 154$.

The rectangle is 11 meters by 14 meters.

Note that in factoring above, we had to find two integers differing by 3, whose product is 154. This is actually the original problem. However, we could have used the quadratic formula if the factoring was difficult, or if the dimensions were irrational numbers. Also there might be two answers to a problem, and solving the quadratic equation would find both.

(100 points total.)