SOLUTIONS

1. An urn contains 5 red marbles, 6 white marbles, and 7 blue marbles. Six marbles are drawn at random from the urn, one at a time, without replacement.

$5+6+7=18$

(5 points) (a) Find the **probability** of getting the sequence "red, white, white, blue, blue, blue".

$$\frac{5}{18}\cdot\frac{6}{17}\cdot\frac{5}{16}\cdot\frac{7}{15}\cdot\frac{6}{14}\cdot\frac{5}{13}=\frac{5\cdot\cancel{6}\cdot5\cdot\cancel{7}\cdot\cancel{6}\cdot5}{\cancel{18}\cdot17\cdot16\cdot\cancel{15}\cdot\cancel{14}\cdot13}=\boxed{\frac{25}{10608}}\approx\boxed{0.0023567119}$$

(5 points) (b) Find the **probability** of getting the combination "1 red, 2 white, 3 blue".

$$\frac{\binom{5}{1}\cdot\binom{6}{2}\cdot\binom{7}{3}}{\binom{18}{6}}=\frac{5\cdot6\cdot5\cdot7\cdot6\cdot5\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{1\cdot2\cdot1\cdot3\cdot2\cdot1\cdot18\cdot17\cdot16\cdot15\cdot14\cdot13}\approx\boxed{0.1414027149}$$

$$=\boxed{\frac{125}{884}}$$

(8 points) 2. Given the bivariate data

x	1	2	3	4	sums $\Sigma x=10$
y	9	4	4	2	$\Sigma y=19$
x^2	1	4	9	16	$\Sigma x^2=30$
xy	9	8	12	8	$\Sigma xy=37$
y^2	81	16	16	4	$\Sigma y^2=117$

$SS(x)=30-\frac{10^2}{4}=5$

$SS(y)=117-\frac{19^2}{4}=26.75$

$SS(xy)=37-\frac{10\cdot19}{4}=-10.5$

$y=mx+b$

$\Sigma y=m\Sigma x+bn$

$\Sigma xy=m\Sigma x^2+b\Sigma x$

$10m+4b=19 \quad \times(-3)$

$30m+10b=37$

$-30m-12b=-57$

add $-2b=-20$

$b=10$

$m=-2.1$

(a) find the **correlation coefficient, r** $=\dfrac{SS(xy)}{\sqrt{SS(x)\cdot SS(y)}}$

$$=\frac{-10.5}{\sqrt{5(26.75)}}=\boxed{-0.9079091725}$$

(b) find the slope-intercept **equation** of the line of best fit

$$\boxed{y=-2.1x+10}$$

(9 points) 3. We get half our light bulbs from store A, and one fourth from each of stores B and C. Only 1% of bulbs from A are defective, but 1.5% of those from B are defective, and 2.5% of those from C are defective.

(a) Make a complete labeled table to apply Bayes' Theorem

(b) Find the overall rate of defectives in our light bulbs. 1.5% (Coincidence that this is B's defective rate.)

(c) What is the probability that a defective bulb came from store B? 25%

Store	P(store)	P(D|store)	P(D∩store)	P(Store|D)
A	.50	.010	.00500	1/3 = .333...
B	.25	.015	.00375	1/4 = .25
C	.25	.025	.00625	5/12 = .4166...
			.01500	12/12 = 1

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(6 points) 4. Let x be the number of successes in n = 13 independent binomial trials, where the probability of success in one trial is p = 0.6. Find P(x > 11) using appropriate formulas. (Do not use normal approximation.)

$$P(X=12,13) = \binom{13}{12}(.6)^{12}(.4)^{1} + \binom{13}{13}(.6)^{13}(.4)^{0}$$

$$= \binom{13}{1}(.6)^{12}(.4) + 1\,(.6)^{13} \doteq .0113192681 + .001306069$$

$$= \boxed{0.0126253375}$$

$\boxed{\text{Program TI gives } 0.012625}$

(10 points) 5. Let x be the number of successes in n = 1300 independent binomial trials, where the probability of success in one trial is p = 0.6.
Estimate P(x ≥ 800) using the normal approximation to the binomial.

$\mu = 1300(.6) = 780, \quad \sigma = \sqrt{780(.4)} = \sqrt{312} = 17.6635\ldots$

Let Y be normal with same mean and S.D.

$$P(X \geq 800) \approx P(799.5 < Y < 1300.5)$$

$$\approx P\left(\frac{799.5-780}{\sqrt{312}} < \frac{Y-\mu}{\sigma} < \frac{1300.5-780}{\sqrt{312}}\right)$$

$$\approx P(1.10397 < Z < 29.4675) \doteq \boxed{0.1348}$$

(6 points) 6. A machine makes parts whose weights are normally distributed with a mean of 30 grams and a standard deviation of 0.32 gram. Find the percentage of parts that weigh between 29.5 and 30.5 grams.

$$P(29.5 < X < 30.5) = P\left(\frac{29.5-30}{.32} < Z < \frac{30.5-30}{.32}\right)$$

$$= P(-1.5625 < Z < 1.5625)$$

$$\approx 0.881829736$$

$\boxed{\text{About } 88.18\% \text{ lie between } 29.5\text{ g and } 30.5\text{ g.}}$

(5 points) 7. Let x be normal with S.D. = 9. We wish to test the null hypothesis $H_0 : \mu = 60$, (versus $H_a : \mu \neq 60$) with $\alpha = 0.05$. We want the standard error of the sample mean to be 4, or less. Find the minimum sample size, n, needed to achieve this.

$$E = z\left(\frac{\alpha}{2}\right)\cdot\frac{\sigma}{\sqrt{n}}, \quad \sqrt{n} = z\left(\frac{\alpha}{2}\right)\cdot\frac{\sigma}{E}, \quad n = \left[z\left(\frac{\alpha}{2}\right)\frac{\sigma}{E}\right]^2$$

$$\left[z(.025)\,\frac{9}{4}\right]^2 = (1.95996\ldots)^2(2.25)^2 = (4.4099\ldots)^2$$

$$= 19.447\ldots$$

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$\boxed{\text{The sample size should be at least } 20.}$

8. A sample from a normal population has mean equal to 16 and S.D. equal to 2. Find the 90% confidence interval for the population mean, if

(6 points) (a) the sample size is 400.

$t(0.05, df=399) = 1.64868....$
$z(0.05) = 1.649853626$ close - can use z.

Using t, $\bar{x} \pm t(0.05, df=399)\frac{2}{\sqrt{400}} = 16 \pm 0.164868$

$\boxed{\text{The 90\% confidence interval is 15.835 to 16.165.}}$

(7 points) (b) the sample size is 10. $\bar{x} \pm t(\frac{\alpha}{2}, df=9)\frac{s}{\sqrt{10}}$

$= 16 \pm t(0.05, df=9)\frac{2}{\sqrt{10}} = 16 \pm (1.833...)(\frac{2}{\sqrt{10}})$

$= 16 \pm 1.159362409$

$\boxed{\text{The 90\% confidence interval is 14.84 to 17.16.}}$

9. Independent samples are taken from two normal populations.
$n_1 = 15, n_2 = 18, s_1 = 4, s_2 = 8, \overline{x_1} = 80, \overline{x_2} = 95$

(6 points) (a) Test the hypothesis that $H_0: \sigma_1 = \sigma_2$, at the 10% level. versus $H_a: \sigma_1 \neq \sigma_2$

$F^* = \frac{s_2^2}{s_1^2} = \frac{8^2}{4^2} = 4$ $2P(F > 4, dfn = 17, dfd = 14)$

$= 2(0.0060235...) = 0.012047... < 0.10$

Reject H_0 at the 10% level.

$\boxed{\text{The sample variances are significantly different at the 10\% level.}}$

(7 points) (b) Test the hypothesis that $\mu_1 = \mu_2$ (versus $\mu_1 \neq \mu_2$), at the 5% level.

$$t^* = \frac{(\bar{x}_2 - \bar{x}_1) - 0}{\sqrt{\frac{8^2}{18} + \frac{4^2}{15}}} = \frac{95-80}{\sqrt{\frac{64}{18} + \frac{16}{15}}} = \frac{15}{\sqrt{\frac{208}{45}}}$$

$= 6.97695...$

Program gives $t^* = -6.97695..$
$p \doteq 2.127 \times 10^{-7}$
$df = 25.8995...$

$2P(t > 6.97695... | df = 14) = 2(0.0000324....)$
$= 0.0000648...$
$< .05$

$\boxed{\text{the means are significantly different, at the 5\% level.}}$

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(10 points) 10. In a test of drugs A and B, the following results were obtained. Test the hypothesis that there is no difference between the two drugs, at the 10% level. O/E

$df = (2-1)(3-1) = 1(2) = 2$

	Worse	Same	Improved	
Drug A	25 / 23.75	20 / 23.75	50 / 47.5	95
Drug B	15 / 16.25	20 / 16.25	30 / 32.5	65
	40	40	80	160

$\boxed{\text{No significant difference at the 10\% level.}}$

$$\chi^{2*} = \sum \frac{(O-E)^2}{E} = 1.943319838$$

$$p(\chi^2 > 1.9433\ldots \mid df = 2) = 0.37845 > 0.10$$

(10 points) 11. A researcher tests drugs A, B and C, and uses a quantitative measure of improvement. Three groups of people tried the drugs with results below. Use ANOVA and test the hypothesis that the mean improvement is the same for all three drugs, at the 5% level.

$k_1 = 6$ $k_2 = 4$ $k_3 = 5$

	Drug A	Drug B	Drug C
	10	6	8
	9	8	8
	8	7	6
	9	11	7
	11		7
	11		
Totals	$C_1 = 58$	$C_2 = 32$	$C_3 = 36$

	df	SS	MS
factor	2	$17.4\overline{6}$	$8.7\overline{3}$
error	12	$24.1\overline{3}$	$2.0\overline{1}$
total	14	41.6	

$$F^* = \frac{8.7\overline{3}}{2.0\overline{1}} = 4.34254\ldots$$

$$p(F > 4.34254, dfn = 2, dfd = 12) = 0.0381192094 < 0.05$$

Reject H_0.

$\boxed{\text{There is a significant difference among the drugs, at the 5\% level.}}$

(100 points, total.)