(12.5 points) 1. (a) Complete the given the frequency table, with appropriate sums, and find

x	1	2	3	4	5	6	
f	10	30	20	5	10	5	$80 = \Sigma f$
xf	10	60	60	20	50	30	$230 = \Sigma xf$
x^2f	10	120	180	80	250	180	$820 = \Sigma x^2 f$
cf	10	40	60	65	75	80	

(b) the **depth** of the median $d(\tilde{x}) = \frac{n+1}{2} = \frac{80+1}{2} = \frac{81}{2} = \boxed{40.5}$

(c) the **median** $= \frac{x_{40} + x_{41}}{2} = \frac{2+3}{2} = \boxed{2.5}$

(12.5 points) 2. Using the data in problem 1., above, find (a) the **mean** $= \frac{\Sigma xf}{\Sigma f} = \frac{230}{80} = \boxed{2.875}$

(b) the **midrange** $= \frac{1+6}{2} = \boxed{3.5}$

(c) the **mode** $= \boxed{2}$

(d) the **depth** of the third quartile, Q_3 $\frac{3}{4}(80) = 60.$ $\boxed{d(Q_3) = 60.5}$

(e) the **third quartile**, Q_3. $= \frac{x_{60} + x_{61}}{2} = \frac{3+4}{2} = \boxed{3.5}$

(12.5 points) 3. Using the data in problem 1., above, find

(a) the **depth** of the 45th percentile

$.45(80) = 36$ $\boxed{d(P_{45}) = 36.5}$

(b) the **45th percentile** $= \frac{x_{36} + x_{37}}{2} = \frac{2+2}{2} = \boxed{2}$

(c) the **sample variance**

$$= s^2 = \frac{\Sigma x^2 f - \frac{(\Sigma xf)^2}{n}}{n-1} = \frac{820 - \frac{(230)^2}{80}}{79}$$

$\approx 2.0094936 7089$

(d) the **sample standard deviation**

$s \approx \sqrt{2.009493 67089}$

↑ use value from calculator

$\approx \boxed{1.417566 10812}$

GO TO PAGE 2.

(12.5 points) 4. (a) We want to include at least 72% of an unknown distribution within k (5 pts) standard deviations of its mean. Find the smallest k that will guarantee this.

$0.72 = 1 - \frac{1}{k^2}, \quad \frac{1}{k^2} = 1 - 0.72 = 0.28$

$k^2 = \frac{1}{0.28}, \quad k = \frac{1}{\sqrt{0.28}} \approx \boxed{1.8898223650S}$

(b) X has a distribution with mean 45 and standard deviation 25.
(3 pts) Find the **z-score** for X = 80.

$$z = \frac{80-45}{25} = \frac{35}{25} = \frac{7}{5} = \boxed{1.4}$$

(c) Classify the following variables as qualitative nominal, qualitative ordinal, (4½ pts) quantitative discrete, or quantitative continuous.
(a) "High", "Medium" or "Low" heat $\boxed{\text{qualitative ordinal}}$
(b) The number of calculators you have. $\boxed{\text{quantitative discrete}}$
(c) Your favorite vegetable $\boxed{\text{qualitative nominal}}$

(12.5 points) 5. (a) Complete the table, and find

	x	y	x^2	y^2	xy
1	2	1	4	1	2
2	4	3	16	9	12
3	6	4	36	16	24
4	7	4	49	16	28
	19	12	105	42	66

(b) $SS(x) = 105 - \frac{19^2}{4} = \boxed{14.75}$

(c) $SS(y) = 42 - \frac{12^2}{4} = 42 - 36 = \boxed{6}$

(d) $SS(xy) = 66 - \frac{19 \cdot 12}{4} = 66 - 57 = \boxed{9}$

(e) Find r, the **coefficient of linear correlation**.

$$r = \frac{SS(xy)}{\sqrt{SS(x) \cdot SS(y)}} = \frac{9}{\sqrt{(14.75)(6)}} \approx \boxed{0.956689206215}$$

(f) Find the **equation of line of best fit** in the form y = mx + b

$\Sigma y = m \Sigma x + bn$
$\Sigma xy = m \Sigma x^2 + b \Sigma x$

$19m + 4b = 12$
$105m + 19b = 66$

also $m = \frac{SS(xy)}{SS(x)} = \frac{9}{14.75} = \frac{36}{59}$

$4b = 12 - 19\left(\frac{36}{59}\right)$

$b = \frac{6}{59}$

$$\boxed{y = \frac{36}{59}x + \frac{6}{59}}$$

(g) Predict y if x = 8.

$y = \frac{36}{59}(8) + \frac{6}{59} = \boxed{4\frac{58}{59}}$

GO TO PAGE 3.

(12.5 points) 6. One card is drawn at random from a standard deck of 52 cards. Events are D = diamond, R = red card, L = card lower than a six, F = face card.

(a) From these events list a **pair of events** that are

(i) **mutually exclusive** $\boxed{L, F}$

(ii) **independent** $\boxed{L, R}$ (Also L, D; F, D; F, R.)

(iii) **neither** independent not mutually exclusive. $\boxed{D, R}$

(b) Find the **probabilities**, as simplified fractions.

(i) $P(F) = \frac{12}{52} = \boxed{\frac{3}{13}}$

(ii) $P(D \cap F) = \boxed{\frac{3}{52}}$

(iii) $P(D \mid F) = \frac{P(D \cap F)}{P(F)} = \frac{n(D \cap F)}{n(F)} = \frac{3}{12} = \boxed{\frac{1}{4}}$

(12.5 points) 7. A natural number from 1 to 60, inclusive, is drawn at random. (Each of the 60 numbers has probability 1/60 of being drawn.) Let E be the event "the number is even", and G be the event "the number is a multiple of 11". Find

$G = \{11, 22, 33, 44, 55\}$

$E \cap G = \{22, 44\}$

(a) $P(E) = \frac{30}{60} = \boxed{\frac{1}{2}}$

(b) $P(G) = \frac{5}{60} = \boxed{\frac{1}{12}}$

(c) $P(E \cap G) = \frac{2}{60} = \boxed{\frac{1}{30}}$

(d) $P(E \cup G) = \frac{30 + 5 - 2}{60} = \frac{33}{60} = \boxed{\frac{11}{20}}$

(e) $P(G \mid E) = \frac{P(G \cap E)}{P(E)} = \frac{n(G \cap E)}{n(E)} = \frac{2}{30} = \boxed{\frac{1}{15}}$

(f) if E and G are independent or not. **(explain)**

$P(G \mid E) = \frac{1}{15} \neq \frac{1}{12} = P(G)$

G and E are $\boxed{\text{not independent}}$

GO TO PAGE 4.

ELEMENTARY STATISTICS TEST ONE NAME: SOLUTIONS
SHOW ALL CALCULATIONS. GIVE ANSWERS AS REDUCED FRACTIONS, OR TO 6 DECIMALS.
BEST EIGHT PROBLEMS COUNT, FOR A SCORE OUT OF 100. Sept. 25, 2002.

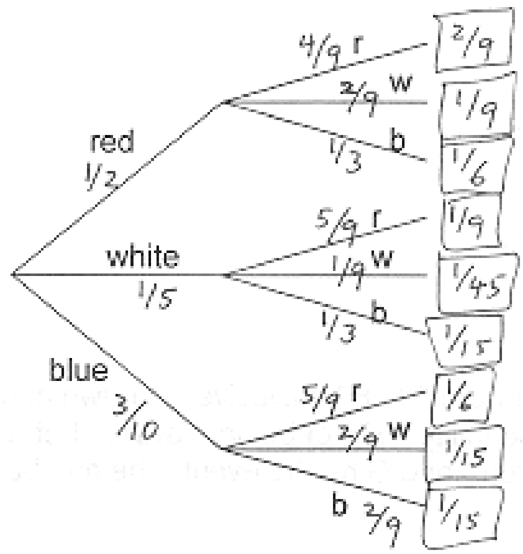
(12.5 points) 8. An urn contains five red marbles, two white marbles and three blue marbles. We draw two marbles, one at a time, without replacement.

$5+2+3=10$

(a) What is the probability of getting a red marble on the second draw, **given that** we got a blue one on the first draw?

$$P(R_2 \mid B_1) = \boxed{\frac{5}{9}}$$

(b) In the **tree diagram** below, label each branch with its **conditional** probability, and each branch tip with its probability.



$\frac{4}{9}\cdot\frac{1}{2}=\frac{2}{9}$

$\frac{1}{2}\cdot\frac{2}{9}=\frac{1}{9}$

$\frac{1}{2}\cdot\frac{1}{3}=\frac{1}{6}$

$\frac{1}{5}\cdot\frac{5}{9}=\frac{1}{9}$

$\frac{1}{5}\cdot\frac{1}{9}=\frac{1}{45}$

$\frac{1}{5}\cdot\frac{1}{3}=\frac{1}{15}$

$\frac{3}{10}\cdot\frac{5}{9}=\frac{1}{6}$

$\frac{3}{10}\cdot\frac{2}{9}=\frac{1}{15}$

" " "

(c) What is the **sum** of the branch tip probabilities? What should it be?

$$\frac{2}{9}+\frac{1}{9}+\frac{1}{6}+\frac{1}{9}+\frac{1}{45}+\frac{1}{15}+\frac{1}{6}+\frac{1}{15}+\frac{1}{15}$$

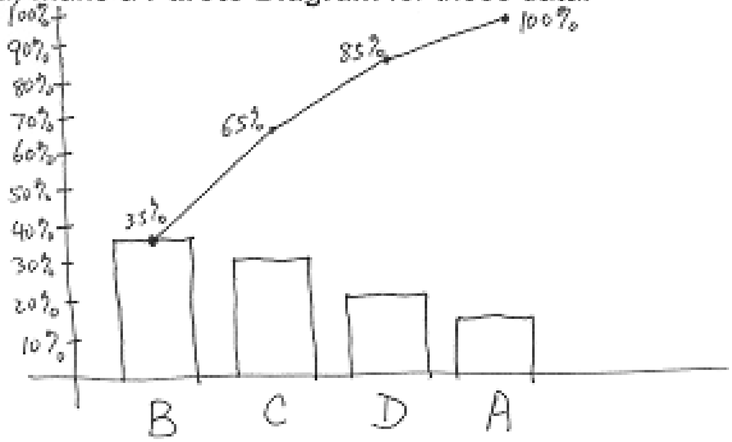
$$= \frac{20+10+15+10+2+6+15+6+6}{90} = \frac{90}{90} = \boxed{1}$$ ✓

It should be 1.

GO TO PAGE 5.

(12.5 points) 9. We get 15% of our parts from supplier A, 35% from supplier B, 30% from supplier C, and 20% from supplier D.

(a) Make a **Pareto Diagram** for these data.



$35+30=65$

$65+20=85$

(b) Defective rates are 8% of A parts, 2% of B parts, 4% of C parts, and 6% of D parts. (These are conditional probabilities.)

(i) Find the probability that a randomly chosen part is both defective **and** from supplier A. (Hint: multiplication rule for conditional probability.)

$P(A \cap D_e) = P(A) \cdot P(D_e | A) = 0.15\,(0.08) = \boxed{0.012}$

(Let D_e = defective) or $\boxed{1.2\%}$

(ii) By finding the probabilities for 'defective and from B', 'defective and from C', and 'defective and from D', and then adding these to the answer for (i), obtain the probability that a randomly chosen part is defective.

$P(B \cap D_e) = P(B) \cdot P(D_e | B) = 0.35\,(0.02) = 0.007$

$P(C \cap D_e) = P(C) \cdot P(D_e | C) = 0.30\,(0.04) = 0.012$

$P(D \cap D_e) = P(D) \cdot P(D_e | D) = 0.20\,(0.06) = 0.012$

$P(D_e) = 0.012 + 0.007 + 0.012 + 0.012$

$= \boxed{0.043}$ or $\boxed{4.3\%}$

(100 points, total.)