ELEMENTARY STATISTICS TEST TWO OCT. 23, 2002. NAME: SOLUTIONS
SHOW ALL CALCULATIONS. GIVE ANSWERS TO AT LEAST FOUR DECIMAL PLACES.
BEST EIGHT PROBLEMS WILL BE COUNTED, FOR A SCORE OUT OF 100.

(12.5 points) 1. Given the probability distribution below, complate the table and find

x	0	1	2	3	4	5	sums
p	0.20	0.15	0.30	0.05	0.20	0.10	1.00 ✓
xp	0	0.15	0.6	0.15	0.8	0.5	2.20
x^2p	0	0.15	1.2	0.45	3.2	2.5	7.50

(a) μ, the **expected value** of x

$\mu = \Sigma xp = \boxed{2.2}$

(b) σ, the **standard deviation of** x

variance $= \sigma^2 = \Sigma x^2 p - \mu^2 = 7.5 - (2.2)^2 = 7.5 - 4.84 = 2.66$

$\sigma = \sqrt{2.66} = \boxed{1.6309506....}$

(12.5 points) 2. Let X be the number of successes in 10 independent trials, where the probability of success on each trial is p = 0.58

(a) Calculate P(X = 6), showing the formula used.

$\binom{10}{6}(.58)^6(.42)^4 = 210\,(.58)^6(.42)^4$

$= \boxed{0.2487622164 6....}$

(b) Complete the probability **distribution** table for X, using formula or a program.

(Probabilities were rounded to seven decimal places. Only four required.)

X	p
0	0.0001708
1	0.0023587
2	0.0146576
3	0.0539772
4	0.1304449
5	0.2161658
6	0.2487622
7	0.1963022
8	0.1016565
9	0.0311962
10	0.0043080

(c) Find the **mean** of the distribution.

$\mu = np = 10(0.58) = \boxed{5.8}$

(d) Find the **standard deviation** of the distribution.

$\sigma = \sqrt{npq} = \sqrt{10(0.58)(0.42)}$

$= \sqrt{(5.8)(0.42)} = \sqrt{2.436}$

$= \boxed{1.56076904...}$

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(12.5 points) 3. z has the standard normal distribution. Find, using a calculator, writing down the calculator command line. Draw a diagram of the normal curve with appropriate shading in parts (a) and (d).

(a) $P(0 \le z \le 1.35)$ (Draw a graph.)

TI83: normalcdf(0, 1.35) = **0.4114919...**

(b) $P(z \ge 1.35)$ normalcdf(1.35, 9) or normalcdf(1.35, 1E99)

= **0.0885080 5...**

(c) $P(z \ge -1.35)$ normalcdf(−1.35, 9)

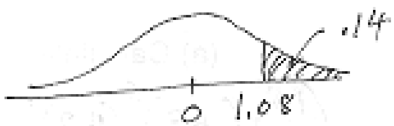
= **0.9114919...**

Note: .5 + part (a).

(d) z_0, so that $P(z \ge z_0) = 14\%$ (Draw a graph.)

$z_0 = z(0.14)$ = invNorm(1 − 0.14) = invNorm(0.86)

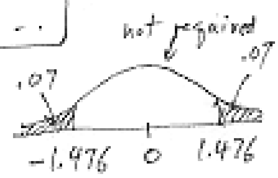
= **1.080319...**



(e) z_0, so that $P(|z| \ge z_0) = 14\%$

Two tails total 0.14. Each tail is 0.07

$z_0 = z(0.07)$ = invNorm(0.93) = **1.475791...**



(12.5 points) 4. (a) Compute $P(11, 4) = \underbrace{11 \cdot 10 \cdot 9 \cdot 8}_{4\ factors} = $ **7920**

(b) Compute $\binom{11}{4} = \frac{P(11,4)}{4!} = \frac{11 \cdot 10 \cdot 9 \cdot 8}{4 \cdot 3 \cdot 2 \cdot 1} = $ **330**

(c) When is it reasonable to use the normal approximation to the binomial?

If both np and n(1−p) equal or exceed 5.

(d) When is it reasonable to ~~use the normal approximation to the binomial?~~ assume the sampling distribution of the mean, $\bar{x}$, is normal?

1. If x is normal
2. If x is "approximately" normal and sample size is 30 or more.
3. If sample size is 50 or more.

(e) The random variable Y is normally distributed, with mean 37 and standard deviation 5.5. Find $P(29 \le Y \le 40)$

TI83: normalcdf(29, 40, 37, 5.5) = **0.63438195...**

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(12.5 points) 5. Let X be the number of successes in $n = 2400$ independent trials where the probability of success on each trial is $p = 0.4$. Find

(a) the **mean** value of X $= \mu = np = 2400(0.4) = \boxed{960}$

(b) the **standard deviation** of X $= \sqrt{npq} = \sqrt{960(0.6)}$

$= \sqrt{576} = \boxed{24}$

(c) P(X = 943) by using the **normal approximation** to the binomial.

$P(X=943) \approx P(942.5 \le Y \le 943.5)$

where Y is normal, with mean 960 and standard deviation 24.

normal cdf (942.5, 943.5, 960, 24)

$= \boxed{0.01293399....}$

(Compare binompdf (2400, 0.4, 943) = 0.01296193...)

(d) $P(951 \le X \le 980)$ by using the **normal approximation** to the binomial.

$\approx P(950.5 \le Y \le 980.5)$ Y as above

normal cdf (950.5, 980.5, 960, 24)

$= \boxed{0.45737968.....}$

(Compare binomcdf (2400, .4, 980)

− binomcdf (2400, .4, 950)

= 0.45704520...)

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(10 points) 6. The heights of 196 trees in a sample are measured. If the **population mean** is 3.88 m with a **population standard deviation** of 0.7 m, find the probability that the **sample mean** is **less than** 3.81 m.

$P(\overline{x} < 3.81) = \text{normalcdf}(-1E99, 3.81, 3.88, .7/\sqrt{(196)})$

$= \text{normalcdf}(-1E99, 3.81, 3.88, .05)$

$\left(\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.7}{\sqrt{196}} = \frac{0.7}{14} = 0.05 \right)$

0.0807567...

(12.5 points) 7. If the **sample mean** is 29 for a sample of size 160, and the population standard deviation is 8.50, find the 92% **confidence interval** for the population mean. Give the endpoints accurate to three or more decimal places.

$C = 0.92, \quad \alpha = 1 - C = 1 - 0.92 = 0.08, \quad \frac{\alpha}{2} = 0.04$

$z\left(\frac{\alpha}{2}\right) = z(.04) = \text{invNorm}(.96) = 1.750686...$

$\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}} = \frac{8.50}{\sqrt{160}}, \quad E = z\left(\frac{\alpha}{2}\right)\sigma_{\overline{x}} = (1.750686...)\frac{8.5}{\sqrt{160}}$

$= 1.1764330...$ (store this value.)

$\overline{x} \pm E = 29 \pm 1.1764... = 27.82356697, 30.17643303$

The 92% confidence interval is 27.824 to 30.176.

(12.5 points) 8. Using a sample of size n, the endpoints of a confidence interval for the population mean are given by $\overline{x} \pm 1.514(\sigma/\sqrt{n})$.

(a) What is the value of α? (Can round to three digits.) $1.514 = z\left(\frac{\alpha}{2}\right)$

$\frac{\alpha}{2} = P(z \geq 1.514) = \text{normalcdf}(1.514, 9) = 0.0650129432$

$\alpha = 2(0.0650129432) = 0.1300258864$

α is about 13%

(b) What is the **confidence level**?

$C = 1 - \alpha =$ **.87** or **87%**

(c) If σ is 5.0, find a **minimum sample size** so that the maximum error, E, is under 0.04 in size.

$E = z\left(\frac{\alpha}{2}\right)\frac{\sigma}{\sqrt{n}} = (1.514)\frac{5}{\sqrt{n}}, \quad \sqrt{n} = \frac{(1.514)(5)}{E}$

Want $\sqrt{n} \geq \frac{(1.514)(5)}{0.04} = 189.25, \quad n \geq (189.25)^2 = 35815.5625$

The sample size should be at least 35816.

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(12.5 points) 9. 55% of our gaskets come from company A, 25% from company B, and and 20% from company C. 4% of A gaskets are defective, 8% of B gaskets are defective, and 9% of C gaskets are defective..

(a) Make a complete table with **labeled** headings to apply Bayes' Theorem to find the probabilities that a defective gasket came from each company.

(b) What is the probability that a randomly chosen gasket is defective? 6% or 0.06

(b) What is the probability that a defective gasket came from company B? 1/3 or $33\frac{1}{3}\%$

X = company, D = defective

X	P(X)	P(D|X)	P(D∩X)	P(X|D)	
A	0.55	0.04	0.022	$\frac{11}{30}$, or $0.3\bar{6}$	$\frac{.022}{.060}$
B	0.25	0.08	0.020	$\frac{1}{3}$, or $0.\bar{3}$	$\frac{.020}{.060}$
C	0.20	0.09	0.018	$\frac{3}{10}$, or 0.3	$\frac{.018}{.060}$
	1.00 ✓		0.060	1 ✓	

$$P(D \cap X) = P(X) \cdot P(D \mid X)$$

$$P(X \mid D) = \frac{P(X \cap D)}{P(D)}$$

(100 points, total.)