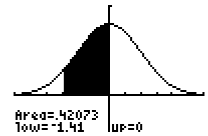
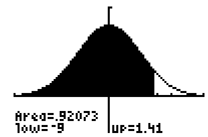
ELEMENTARY STATISTICS TEST TWO OCT. 19, 2006. SOLUTIONS.
SHOW ALL CALCULATIONS/TI COMMANDS. GIVE ANSWERS TO AT LEAST **SIX** DECIMAL PLACES.
BEST **EIGHT** PROBLEMS WILL BE COUNTED, FOR A SCORE OUT OF 100.

(12.5 points) 1. z has the standard normal distribution. Find the probabilities below, using a calculator, writing down the calculator command line. Draw a **diagram** of the normal curve with appropriate shading in each part and endpoints labeled.

(a) $P(-1.41 \le z \le 0)$ normalcdf(–1.41, 0) = **0.4207301092.**



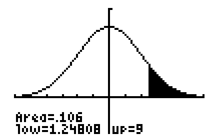
(b) $P(z \le 1.41)$ normalcdf(–9, 1.41) = **0.9207301086.**



(c) z_0, so that $P(z \ge z_0) =$ 10.6%

z(0.106) = invNorm(1 – 0.106) = invNorm(0.894)

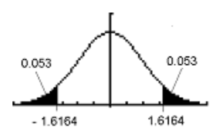
= **1.248084812**



(d) z_0, so that $P(|z| \ge z_0) =$ 10.6% $\frac{1}{2}(0.106) = 0.053$

z(0.053) = invNorm(1 – 0.053) = invNorm(0.947)

= **1.61643637**



(12.5 points) 2. Let X be the number of successes in n = 800 independent trials where the probability of success on each trial is p = 0.45. Find

(a) the **mean** value of X $\mu = np = 800(.45) =$ **360**

(b) the **standard deviation** of X $\sigma = \sqrt{npq} = \sqrt{360(.55)} \approx$ **14.07124728.**

(c) $P(350 \le X \le 367)$ by using the **normal approximation** to the binomial.

normalcdf(349.5, 367.5, 360, √(360 ∗ 0.55)) = **0.4752113799**

(d) $P(350 \le X \le 367)$ by using the **binomial functions** on the TI. .

binomcdf(800,0.45,367) – binomcdf(800,0.45,349) = **0.4753220148**

GO TO PAGE 2.

(12.5 points) 3. The heights of 240 trees in a sample are measured. If the **population mean** is 2.65 m with a **population standard deviation** of 0.60 m, find the probability that the **<u>sample</u> mean** is **less than** 2.61 m.

$\text{normalcdf}\left(0,\ 2.61,\ 2.65,\ 0.60 \div \sqrt{(240)}\right) = \boxed{0.1508498093}$

(12.5 points) 4. In a random sample of 80 cars, 12 have a satellite radio receiver. Find the 99% confidence interval for the percentage of cars with such a receiver. Show the calculation of the margin of error.

$\hat{p} = 12/80 = 0.15,\ \hat{q} = 68/80 = 0.85,\ \alpha = 0.01,\ \alpha/2 = 0.005.$

$$E = z(0.005)\sqrt{\frac{\frac{3}{20}\cdot\frac{17}{20}}{80}} = \text{invNorm}(0.995)\sqrt{51/20\text{^}2/80} = 0.1028317386.$$

Store this in variable E. Compute $\hat{p} - E$ and $\hat{p} + E$.

The 99% confidence interval is $0.0471682614 < p < 0.2528317386$.

(12.5 points) 5. If the **sample mean** is 38 for a sample of size 130, and the population standard deviation is 6.35, find the 96% **confidence interval** for the population mean. Give the endpoints accurate to six or more decimal places. Show the calculation of the margin of error.

$n = 130,\ \bar{x} = 38,\ \sigma = 6.35,\ \alpha = 0.04,\ \alpha/2 = 0.02.$

$$E = z(0.02)\cdot\frac{6.35}{\sqrt{130}} = \text{invNorm}(0.98) * 6.35 \div \sqrt{130} = 1.143798164.$$

Store this in variable E. Compute $\bar{x} - E$ and $\bar{x} + E$.

The 96% confidence interval is $36.85620184 < \mu < 39.143798164$.

GO TO PAGE 3.

(12.5 points) 6. Using a sample of size n, the endpoints of a confidence interval for the population mean are given by $\bar{x} \pm 1.5141(\sigma/\sqrt{n})$.

(a) What is the value of α? (Can round to three decimal places.)

$z(\alpha/2) = 1.5141$. $\alpha/2 = P(z \geq 1.5141) \approx$ normalcdf(1.5141, 9) $\approx$

0.0650002515. Multiply by 2. $\alpha \approx 0.1300005255$. Or, about 13.0%.

(b) What is the **confidence level**? $c = 1 - \alpha$. About 87.0%.

(c) If σ is 8.0, find the **minimum sample size** so that the margin of error, E, is under 0.55 in size.

$$n \geqq \left(\frac{z(\alpha/2)\cdot\sigma}{E}\right)^2 = \left(\frac{1.5141(8.0)}{0.55}\right)^2 = 485.0245416$$

The minimum sample size is 486.

(12.5 points) 7. If the **sample mean** is 45, for a sample of size 15, and the **sample** standard deviation is 3.9, find the 93% **confidence interval** for the population mean. (The population is normally distributed. Show calculation of maximum error.)

$n = 15$, $df = n - 1 = 14$, $\bar{x} = 45$, $s = 3.9$, $\alpha = 0.07$, $\alpha/2 = 0.035$.

$t(0.035, 14) =$ invT(0.965, 14) $= 1.961655649$.

Or, solve $0 = 0.035 -$ tcdf(X, 10^99, 14)

$$E = t(0.035, 14)\cdot\frac{3.9}{\sqrt{15}} = (1.961655649) * 3.9 \div \sqrt{15} = 1.975339512.$$

Store this in variable E. Compute $\bar{x} - E$ and $\bar{x} + E$.

The 93% confidence interval is $43.02466049 < \mu < 46.975339512$.

GO TO PAGE 4.

(12.5 points) 8. The maximum error of the estimate is to be 0.07 in a 90% confidence interval for the proportion. What should the **minimum sample size** be? (Assume we cannot estimate p and q.)

$E = 0.07, \ \alpha = 0.10, \ \alpha/2 = 0.05.$

For the product, pq, use the maximum value of 0.25.

$$n \geqq \left(\frac{z(\alpha/2)}{E}\right)^2 p^* q^* = \left(\frac{\text{invNorm}(0.95)}{0.07}\right)^2 \cdot (0.25) = 138.0379312.$$

The minimum sample size is 139.

(12.5 points) 9. The weights of 33 loaves of bread are measured.
The **sample standard deviation** is 0.85 ounces.
Find the 94% confidence interval for the population variance, including the calculation of the critical values.

$n = 33, \ df = n - 1 = 32, \ s = 0.85, \ \alpha = 0.06, \ \alpha/2 = 0.03.$

For χ^2_L, solve $0 = 0.03 - \chi^2\text{cdf}(0, X, 32)$, or $0 = 0.97 - \chi^2\text{cdf}(X, 10\text{^}99, 32)$.

For χ^2_R, solve $0 = 0.97 - \chi^2\text{cdf}(0, X, 32)$, or $0 = 0.03 - \chi^2\text{cdf}(X, 10\text{^}99, 32)$.

$$\frac{(n-1)s^2}{\chi^2_R} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_L}$$

$$\frac{32(0.85)^2}{48.64108801} < \sigma^2 < \frac{32(0.85)^2}{18.72746818}$$

The 94% confidence interval is $0.4753183152 < \sigma^2 < 1.234550222.$

(100 points, total.)