1. Given the probability distribution

x	1	2	3	4	5	sum
p	0.15	0.25	0.20	0.10	0.30	1.00 ✓
xp	0.15	0.50	0.60	0.40	1.50	3.15
x^2p	0.15	1.00	1.80	1.60	7.50	12.05

(3 points) find (a) μ, the **expected value** of x

$\mu = \Sigma xp = \boxed{3.15}$

(5 points) (b) σ, the **standard deviation of** x

$\sigma^2 = E(x^2) - \mu^2 = 12.05 - (3.15)^2$
$= 12.05 - 9.9225 = 2.1275$

$\sigma = \sqrt{2.1275} \cong \boxed{1.458595215}$

2. Let X be the number of successes in 7 independent trials, where the probability of success on each trial is p = 0.65 .

$q = 1 - p = 1 - 0.65 = 0.35$
$7 - 3 = 4$

(6 points) (a) Calculate P(X = 3), showing the formula used.

$\binom{7}{3}(0.65)^3(0.35)^4 \cong \boxed{0.1442382}$
$= 35(0.274625)(0.01500625)$

(6 points) (b) Complete the probability **distribution** table for X, using formula or a program. →

X	p (rounded to 6 places)
0	0.000643
1	0.008364
2	0.046600
3	0.144238
4	0.267871
5	0.298485
6	0.184776
7	0.049022

(3 points) (b) Find the **mean** of the distribution.

$\mu = np = 7(0.65) = \boxed{4.55}$

(4 points) (c) Find the **standard deviation** of the distribution.

$\sigma = \sqrt{npq} = \sqrt{7(0.65)(0.35)}$
$= \sqrt{1.5925} \cong \boxed{1.261942946}$

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(10 points) 3. Z has the standard normal distribution. Find, from tables or program,

(a) $P(0 \le z \le 1.45) \cong \boxed{0.4265}$ or

TI-83 or TI-86 DISTR normalcdf $(0, 1.45) \cong \boxed{0.4264706991}$

(b) $P(z \ge 1.45) = P(1.45 \le z < \infty)$ (Can use 9 for infinity.)

$\cong \boxed{0.0735}$ or $\boxed{0.0735293004}$

(c) $P(z \ge -1.45) = 1 - P(z \le -1.45)$

$= 1 - P(z \ge 1.45) \cong 1 - 0.0735$

$= \boxed{0.9265}$

(d) z_0, so that $P(z \ge z_0) = 6\%$ $\boxed{z_0 \approx 1.55}$ using program

or invNorm $(0.06) = -1.554773593$, $\boxed{z_0 \cong 1.554773593}$

(e) z_0, so that $P(|z| \ge z_0) = 6\%$

$1 - .03 = .97$

invNorm $(0.97) = \boxed{1.88079361}$ or $\boxed{1.88}$ with program.

two tails, each 0.03

(6 points) 4. The random variable Y is normally distributed, with mean 25 and standard deviation 8.

Find $P(Y \le 31) = P(Y - 25 \le 31 - 25) = P(Y - 25 \le 6)$

$= P\left(\frac{Y-25}{8} \le \frac{6}{8}\right) = P(z \le 0.75)$

(use -9 for $-\infty$)

$\approx \boxed{0.7733727206}$

or $\cong \boxed{0.7734}$ with program

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(3 points.) 5. When is it reasonable to use the normal approximation to the binomial?

np and $n(1-p)$ should be 5 or larger.

6. Let X be the number of successes in $n = 2700$ independent trials where the probability of success on each trial is $p = 0.25$. Find

(3 points) (a) the mean value of X $= \mu = np = 2700(0.25) = \boxed{675}$

(4 points) (b) the standard deviation of X $= \sigma = \sqrt{npq} = \sqrt{675(0.75)}$

$= \sqrt{506.25} = \boxed{22.5}$

(10 points) (c) P(X = 680) by using the **normal approximation** to the binomial.

$\doteq P(679.5 \le Y \le 680.5) = P(679.5-675 \le Y-\mu \le 680.5-675)$

$= P(4.5 \le Y-\mu \le 5.5) = P\left(\frac{4.5}{22.5} \le \frac{Y-\mu}{\sigma} \le \frac{5.5}{22.5}\right)$

$= P(0.2 \le Z \le 0.24444\ldots) \cong \boxed{0.0173}$

(Using binomial formula on a TI-86 gives 0.0172543...)

(10 points) (d) P($653 \le X \le 762$) by using the **normal approximation** to the binomial.

$\doteq P(652.5 \le Y \le 762.5) = P(-22.5 \le Y-\mu \le 87.5)$

$= P(-1 \le Z \le 3.\overline{8}) \cong \boxed{0.8413}$

(3 points) 7. When is it reasonable to assume the sampling distribution of the mean is normal?

1. If the random variable X is normal
2. If $n \ge 50$
3. If $n \ge 30$ and X is "close" to normal

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(10 points) 8. The diameters of 256 bolts in a sample are measured. If the **population** mean is 0.600 cm with a **population standard deviation** of 0.020 cm, find the probability that the **sample mean** is greater than 0.602 cm.

$\mu_{\bar{x}} = \mu = 0.600,\quad \sigma = 0.020,\quad \sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.020}{\sqrt{256}} = \frac{0.020}{16}$

$= 0.00125$

$P(\bar{x} > 0.602) = P\left(z > \frac{0.602 - 0.600}{0.00125}\right)$

$= P(z > 1.6) \cong \boxed{0.0548}$

(7 points) 9. If the **sample mean** is 30 for a sample of size 320, and the population standard deviation is 5.00, find the 90% **confidence interval** for the population mean. Give the endpoints accurate to three or more decimal places.

$\bar{x} = 30,\quad n = 320,\quad \sigma = 5.00,\quad \sigma_{\bar{x}} = \frac{5}{\sqrt{320}} = \frac{5}{8\sqrt{5}} = \frac{\sqrt{5}}{8}$

$\alpha = 100\% - 90\% = 10\%,\quad \frac{\alpha}{2} = \frac{10\%}{2} = 5\%,\quad z(0.05) \cong 1.645$

$\bar{x} \pm 1.645\left(\frac{\sqrt{5}}{8}\right) \cong 30 \pm 0.45979\ldots$

$\boxed{\text{The 90\% confidence interval is } [29.540, 30.460].}$

(7 points) 10. Using a sample of size n, the endpoints of a confidence interval for the population mean are given by $\bar{x} \pm 1.88(\sigma/\sqrt{n})$.

(a) What is the value of α ? $1.88 = z\left(\frac{\alpha}{2}\right)$

$P(z \geq 1.88) \cong 0.0301 = \frac{\alpha}{2}$

$\boxed{\alpha \text{ is about } 6\%.}$

(b) What is the **confidence level**?

$c = 1 - \alpha = \boxed{94\%}$

(100 points, total.)