

(12.5 points) 1. The weights of 39 boxes of cereal are measured. The **sample standard deviation** is 0.37 ounces. The **sample mean** is 15.9 ounces. We are concerned that the mean weight of all boxes may be under 16 ounces.

(a) State the null and alternative hypotheses.

$$H_0: \mu = 16, \quad H_1: \mu < 16$$

(b) Test the null hypothesis at the 5% level, using either p-value or classical approach, showing calculation of the test statistic.

$$n = 39, \quad s = 0.37, \quad \bar{x} = 15.9, \quad df = 38, \quad \alpha = 0.05$$

$$t^* = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{15.9 - 16}{0.37/\sqrt{39}} = \frac{-0.1(\sqrt{39})}{0.37} = -1.687837297$$

$$p\text{-value} = t\text{cdf}(-E99, t^*, 38) = 0.0498174749 \leq 0.05 = \alpha$$

Reject H_0 .

There is enough evidence at the 5% level to say the mean weight of the boxes is under 16 oz.

(12.5 points) 2. The weights of 21 loaves of bread are measured. The **sample standard deviation** is 0.6 ounces. We are concerned that the weights of the loaves may have a standard deviation greater than 0.5 ounces.

(a) State the null and alternative hypotheses.

$$H_0: \sigma = 0.5, \quad H_1: \sigma > 0.5$$

(b) Test the null hypothesis at the 10% level, using either p-value or classical approach, showing calculation of the test statistic.

$$n = 21, \quad df = 20, \quad s = 0.6, \quad \sigma = 0.5$$

$$\chi^2* = \frac{(n-1)s^2}{\sigma^2} = \frac{20(.6)^2}{(.5)^2} = 28.8$$

$$p\text{-value} = \chi^2\text{cdf}(28.8, E99, 20) = 0.0917727417 \leq 0.10 = \alpha$$

Reject H_0 .

There is sufficient evidence at the 10% level to say the weight of the loaves has a standard deviation greater than 0.5 oz.

- (12.5 points) 5. One class, of 35 students, averages 77 on a test with **variance**, $s_1^2 = 16$.
 Another class, of 31 students, averages 81 on the same test, with $s_2^2 = 25$.

(a) Compute the **estimated standard error** of the difference of sample means.
 (You have to identify exactly what this is, here.)

$$\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} = \sqrt{\frac{16}{35} + \frac{25}{31}} = \boxed{1.12409718}$$

(store this)

(b) Are the averages significantly **different** at the 2% level?
 Show calculation of the test statistic. May use TI test.

$$t^* = \frac{77 - 81}{1.12409718} = -3.558411204$$

$$\begin{cases} H_0: \mu_1 = \mu_2 \\ H_1: \mu_1 \neq \mu_2 \end{cases}$$

↑ use stored value on Ans on TI.

2-Samp T Test gives $df = 57.38205774$

$$\begin{aligned} p\text{-value} &= 2 \cdot tcdf(-E99, -3.558411204, df) \\ &= 2(3.7841975E-9) = 7.56839... \times 10^{-9} \\ &= 0.0007568394999 \leq 0.02 = \alpha \end{aligned}$$

Reject H_0 . The averages differ significantly at the 2% level.

- (12.5 points) 6. As in the previous problem, one class, of 35 students, averages 77 on a test with **variance**, $s_1^2 = 16$. Another class, of 31 students, averages 81 on the same test, with **variance**, $s_2^2 = 25$. Is the sample variance of the 2nd class significantly higher than that of the ~~second~~ ^{first} class, at the 10% level?

Show calculation of the test statistic. $H_0: \sigma_1 = \sigma_2$, $H_1: \sigma_1 < \sigma_2$

$$F^* = \frac{16}{25} = 0.64, \quad p\text{-value} = Fcdf(0, 0.64, 34, 30) = 0.1041819866 > 0.10 = \alpha$$

$$\text{or } F^* = \frac{25}{16} = 1.5625, \quad p\text{-value} = Fcdf(1.5625, E99, 30, 34) = 0.1041819866 > 0.10 = \alpha$$

Fail to reject H_0 .

There is not enough evidence at the 10% level to say the variance of the 2nd population is greater than that of the 1st population.

(12.5 points) 8. (a) Complete the table, and find

x	y	x ²	y ²	xy
2	3	4	9	6
4	2	16	4	8
5	4	25	16	20
6	9	36	81	54
17	18	81	110	88

$$n = 4$$

$$(b) SS(x) = \sum x^2 - \frac{(\sum x)^2}{n}$$

$$= 81 - \frac{17^2}{4} = \boxed{8.75}$$

$$(c) SS(y) = \sum y^2 - \frac{(\sum y)^2}{n}$$

$$= 110 - \frac{18^2}{4} = \boxed{29}$$

$$(d) SS(xy) = \sum xy - \frac{(\sum x)(\sum y)}{n}$$

$$= 88 - \frac{17(18)}{4} = \boxed{11.5}$$

(e) Find r, the coefficient of linear correlation.

$$r = \frac{SS(xy)}{\sqrt{SS(x) \cdot SS(y)}} = \frac{11.5}{\sqrt{(8.75)(29)}} = \boxed{0.7219295437}$$

(f) Find the equation of line of best fit in the form $y = mx + b$

$$\begin{aligned} \sum y &= m \sum x + nb \\ \sum xy &= m \sum x^2 + b \sum x \end{aligned} \rightarrow \begin{aligned} 17m + 4b &= 18 \\ 81m + 17b &= 88 \end{aligned} \rightarrow \begin{aligned} m &= \frac{46}{35} \\ b &= -\frac{38}{35} \end{aligned}$$

$$\text{Also } m = \frac{SS(xy)}{SS(x)} = \frac{11.5}{8.75} = \frac{46}{35}$$

$$\boxed{y = \frac{46}{35}x - \frac{38}{35}}$$

(g) Predict y if $x = 3$

$$\hat{y} = \frac{46}{35}(3) - \frac{38}{35} = \boxed{\frac{20}{7}} \text{ or } \boxed{2\frac{6}{7}} \text{ or } \boxed{2.857142}$$