(9 points) 1. The maximum error of the estimate of a population mean in a 92% confidence interval is to be under 4.0. The population standard deviation is 18.0. Find the minimum **sample size** that will do.

$\alpha = 1 - .92 = .08, \quad \frac{\alpha}{2} = .04, \quad z(\frac{\alpha}{2}) = z(.04) = 1.75$

$$n \geq \left[z\left(\frac{\alpha}{2}\right)\frac{\sigma}{E}\right]^2 = \left[1.75\left(\frac{18}{4}\right)\right]^2 = (7.875)^2$$

$$= 62.015625$$

$\boxed{\text{The sample size should be at least 63.}}$

(12 points) 2. The weights of 144 "16 ounce" cakes are measured. The **sample standard deviation** is 0.9 ounces. The sample **mean** is 16.2 ounces. We are concerned that the cakes we are baking may **differ** from 16 ounces.

(a) State the null and alternative hypotheses.

$H_0: \mu = 16$

$H_a: \mu \neq 16$

(b) At what α level would we reject the null hypothesis? (To nearest 0.1%.)

$n = 144$ is a "large" sample size

$$z^* = \frac{16.2 - 16}{0.9/\sqrt{144}} = \frac{0.2}{0.075} = 2.666\ldots$$

$$P(|z| \geq z^*) = 2P(z \geq 2.666\ldots)$$

$$\approx 2(0.0038) = 0.0076 = \boxed{0.76\%} \text{ or } \approx \boxed{0.8\%}$$

Using t distribution with $df = n-1 = 143$ (TI program)

$$P(|t| \geq t^*) = 2P(t \geq 2.666\ldots) \cong 2(0.0043) = 0.0086$$

$$= \boxed{0.86\%} \text{ or } \approx \boxed{0.9\%}$$

(Refering to typical values of α we can reject H_0 at the 1% level.)

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(12 points) 3. If the **sample mean** is 45, for a sample of size 16, and the **sample** standard deviation is 6, find the 98% **confidence interval** for the population mean. (The population is normally distributed.)

$\bar{x} = 45$, $n = 16$, $= 6$, $\alpha = 0.02$ $\frac{\alpha}{2} = 0.01$ $df = 16 - 1 = 15$

$$E = t\left(df, \frac{\alpha}{2}\right)\frac{s}{\sqrt{n}} = t(15, 0.01)\left(\frac{6}{\sqrt{16}}\right) = 2.60\,(1.5) = 3.9$$

$$\bar{x} \pm E = 45 \pm 3.9$$

$\boxed{\text{98\% confidence interval is } [41.1, 48.9]}$

(10 points) 4. In a random sample of 50 apples, 35 are ripe. Find the 95% confidence interval for the proportion of ripe apples in this population of apples.

$p' = \frac{x}{n} = \frac{35}{50} = 0.7$, $q' = 0.3$

$n = 50$

$x = 35$

$\alpha = 0.05$

$\frac{\alpha}{2} = 0.025$

$$E = z(0.025)\sqrt{\frac{0.7(0.3)}{50}} = 1.96\sqrt{0.0042} \approx 0.1270225177.$$

$\pm E \doteq 0.7 \pm 0.12702$

95% confidence interval is approximately.

$[0.57298, 0.82702]$

(9 points) 5. The maximum error of the estimate is to be 0.05 in a 95% confidence interval for the proportion. What should the minimum sample size be? (Assume we cannot estimate p and q.)

$$0.05 \geq E = z(0.025)\sqrt{\frac{0.5(0.5)}{n}}$$

$$0.05\sqrt{n} \geq 1.96\,(0.5)$$

$$\sqrt{n} \geq 19.6$$

$$n \geq (19.6)^2 = 384.16$$

$\boxed{\text{The sample size should be at least 385.}}$

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(12 points) 6.

Student	Chris	Rosa	Kevin	Sean	Tom
Test One	78	60	82	90	71
Test Two	87	75	78	90	73
d	9	15	-4	0	2
d^2	81	225	16	0	4

$d = X_2 - X_1$

$\Sigma d = 22$

$\Sigma d^2 = 326$

$n = 5$

$\bar{d} = \frac{\Sigma d}{n} = 4.4$

$s_d = \sqrt{\frac{\Sigma d^2 - \frac{1}{n}(\Sigma d)^2}{n-1}} \cong 7.569676347$

Test the hypothesis that there is no improvement in average score from test one to test two, based on the above sample of five students, at the 10% level of significance.

$H_0: \mu_1 \geq \mu_2$ or $\mu_2 - \mu_1 \leq 0$ or $d \leq 0$

$H_a: \mu_1 < \mu_2$ or $\mu_2 - \mu_1 > 0$ or $d > 0$

$$t^* = \frac{4.4 - 0}{s_d/\sqrt{n}} = \frac{4.4}{3.385262176} = 1.299751621$$

$P(t > 1.299751621) = 0.1318 > 0.10 = \alpha$

$df = 4$

Fail to reject H_0. No strong evidence of improvement.

(12 points) 7. The weights of 14 pound cakes are measured.
The **sample standard deviation** is 0.4 ounces.
We are concerned that the weights of the pound cakes in may have a standard deviation greater than 0.3 ounces.

(a) State the null and alternative hypotheses.

$H_0: \sigma \leq 0.3$

$H_a: \sigma > 0.3$

(b) Test the null hypothesis at the 5% level and at the 2% level.

$n = 14$

$s = 0.4$

$df = 13$

$$\chi^{2*} = \frac{13(.4)^2}{(.3)^2} = 23.11111$$

$P(\chi^2 > 23.\bar{1}) = 0.0404$

$0.02 < .0404 < .05$

Reject H_0 at the 5% level but not at the 2% level.

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(12 points) 8. In the year 1996, 45% of our 600+ employees had at least one sick day. We don't have all the data for 1997 yet, but we checked 60 employees at random and 35 had at least one sick day. Has there been a signicant increase in the proportion of employees with at least one sick day, at the 5% level?

$n = 60$ $\quad p = .45$

$x = 35$ $\quad q = .55$

$p' = \frac{35}{60} = 0.58\bar{3}$

$q' = 0.41\bar{6}$

$\alpha = .05$

$$z^* = \frac{0.58\bar{3} - 0.45}{\sqrt{\frac{(.45)(.55)}{60}}} = 2.075997184$$

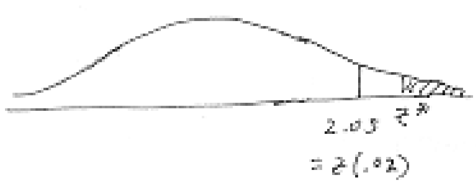
$P(z > 2.075997184) \cong 0.0189 < .05 = \alpha$

Reject Ho. There has been a significant increase.

(6 points) 9. We rejected a null hypothesis at the 2% level in a right-tailed test, using z, and the p-value approach. What is the comparison that would be made in the classical approach? (State exactly what is compared, with any numerical values that are involved.)

$P(z > z^*) \leq 0.02 = \alpha$

$z(0.02) = 2.05$



The comparison in the classical approach is $z^* \geq 2.05$

(6 points) 10. We are told that a null hypothesis is not rejected in a test using $\alpha = 5\%$. Do we know what would happen at the 2% level? At the 10% level?

The p-value exceeds 0.05.
It also exceeds 0.02, so we fail to reject at the 2% level.
We don't know how p compares with 0.10. We may reject, or fail to reject, depending on the exact p-value.

(100 points, total.)