

SOLUTIONS

INTERMEDIATE ALGEBRA TEST TWO SAMPLE QUESTIONS. ACTUAL TEST MAY DIFFER. SHOW ALL CALCULATIONS AND SIMPLIFY ANSWERS. PAGE 1 OF 4.

1. Simplify and write with only positive exponents
- (5 points) (a) $(2x^{-3}y^4)^{-4} = \left(\frac{2y^4}{x^3}\right)^{-4} = \left(\frac{x^3}{2y^4}\right)^4 = \frac{(x^3)^4}{2^4(y^4)^4} = \frac{x^{12}}{16y^{16}}$
- (10 points) (b) $\frac{(a^3b^{-4})^{-3}}{(a^{-2}b^{-1})^{-2}} = \frac{(a^3)^{-3}(b^{-4})^{-3}}{(a^{-2})^{-2}(b^{-1})^{-2}} = \frac{a^{-9}b^{12}}{a^4b^2} = \frac{b^{12}b^{-2}}{a^9a^4} = \frac{b^{10}}{a^{13}}$
- (2 points) 2. Find the degree of the polynomial $3x^2yz - 2xy^2z^2 + z^5$
 $\begin{matrix} 2+1+1=4 & 1+2+2=5 & 5 \\ \hline 5+4 \end{matrix}$
- (6 points) 3. Given the two polynomials $5x^2 - 2x + 1$ and $2x^2 - 5x + 6$ find their difference. $5x^2 - 2x + 1 - (2x^2 - 5x + 6) = 5x^2 - 2x + 1 - 2x^2 + 5x - 6 = 3x^2 + 3x - 5$
4. Find the products
- (5 points) (a) $(3x + 5)(4x - 9) = 3x(4x) + 3x(-9) + 5(4x) + 5(-9) = 12x^2 - 27x + 20x - 45 = 12x^2 - 7x - 45$
- (10 points) (b) $(2x + 5)(2x^2 - 5x + 3) = 2x(2x^2 - 5x + 3) + 5(2x^2 - 5x + 3) = 4x^3 - 10x^2 + 6x + 10x^2 - 25x + 15 = 4x^3 - 19x + 15$
- (10 points) 5. Expand $(3x + 4)^3$ and simplify.
 $= (3x)^3 + 3(3x)^2(4) + 3(3x)(4)^2 + (4)^3 = 27x^3 + 108x^2 + 144x + 64$
- (5 points) 6. Write the product in scientific notation. $(4.5 \times 10^{25}) \times (8 \times 10^{-87}) = (4.5 \times 8) \times (10^{25-87}) = 36 \times 10^{-62} = 3.6 \times 10^1 \times 10^{-62} = 3.6 \times 10^{-61}$
- (10 points) 7. Find the quotient using synthetic division
- $$\begin{array}{r|rr}
 x+1 & 2x & -5 \\
 \hline
 & 2x^2 & -3x & -5
 \end{array}
 \qquad
 \begin{array}{r}
 -1 \mid 2 \quad -3 \quad -5 \\
 \hline
 2 \quad -2 \quad 5 \\
 -5 \quad 0
 \end{array}$$
- (8 points) 8. Factor $x^2 + 9x + 20 = (x+4)(x+5)$
 $4+5=9$
 $4 \cdot 5 = 20$
- (10 points) 9. Factor $12x^2 - 16x - 35 = (12x^2 - 30x) + (14x - 35)$
 $12(-35) = -420 = -30(14) = 6x(2x-5) + 7(2x-5) = (6x+7)(2x-5)$
 $-30+14 = -16$
- (10 points) 10. Factor $20x^3 + 8x^2 - 45x - 18 = (20x^3 + 8x^2) - (45x + 18)$
 $= 4x^2(5x+2) - 9(5x+2) = (4x^2-9)(5x+2)$
 $= [(2x)^2 - 3^2](5x+2) = (2x-3)(2x+3)(5x+2)$

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11. Given the parabola $y = 2x^2 - 3x - 5 = (x+1)(2x-5)$ [See #7.]

(2 points)

(a) find the (x, y)-coordinates of the point of intersection with the y-axis

Let $x=0$, $y = 2(0)^2 - 3(0) - 5 = -5$

$(0, -5)$

(8 points)

(b) find the (x, y)-coordinates of the vertex

$h = -b/2a = -(-3)/(2(2)) = \frac{3}{4}$, $k = 2(\frac{3}{4})^2 - 3(\frac{3}{4}) - 5 = -\frac{9}{8} - \frac{40}{8} = -\frac{49}{8}$

(6 points)

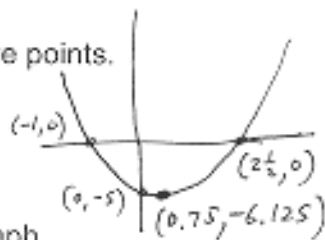
(c) complete the table

Vertex is $(\frac{3}{4}, -\frac{49}{8})$

x	-2	-1	0	1	2	3	4
y	9	0	-5	-6	-3	4	15

(7 points)

(d) then draw a graph of the function, including all of the above points.



12. Given the function $y = x^3 - 4x = x(x-2)(x+2)$

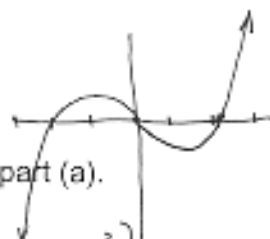
(5 points)

(a) complete the table, to find coordinates of points on the graph

x	-3	-2	-1	0	1	2	3
y	-15	0	3	0	-3	0	15

(6 points)

(b) then draw a graph of the function, including the points from part (a).



(10 points)

13. Factor $24x^4 - 81xy^3 = 3x(8x^3 - 27y^3) = 3x[(2x)^3 - (3y)^3]$
 $= 3x(2x-3y)[(2x)^2 + (2x)(3y) + (3y)^2] = 3x(2x-3y)(4x^2 + 6xy + 9y^2)$

(10 points)

14. Solve the equation $x^2 = 2x + 35$

$x^2 - 2x - 35 = 0 \rightarrow x+5=0$ or $x-7=0$
 $(x+5)(x-7) = 0 \rightarrow x = -5, 7$

(10 points)

15. Solve the equation $6x^2 + 13x - 15 = 0$ $(x+3)(6x-5) = 0$ or

$a=6, b=13, c=-15$
 $b^2 - 4ac = (13)^2 - 4(6)(-15) = 169 + 360 = 529 = 23^2$
 $x = \frac{-13 \pm 23}{12} = \frac{-36}{12}, \frac{10}{12} = \left\{ -3, \frac{5}{6} \right\}$

(10 points)

16. A stone is thrown downward at 32 ft/sec, from the top of a 308 ft cliff. Ignoring air resistance, its height above the ocean below the cliff is given by

$$s = 308 - 32t - 16t^2$$

How many seconds after it is thrown does it hit the ocean? $s=0$ when it hits

$0 = 308 - 32t - 16t^2$

$16t^2 + 32t - 308 = 0$

$4t^2 + 8t - 77 = 0$

$t = \frac{-8 \pm \sqrt{(8)^2 - 4(4)(-77)}}{2(4)}$

$t = \frac{-8 + \sqrt{1296}}{8} > 0$

$t = \frac{-8 + 36}{8} = \frac{28}{8} = 3.5$

It hits in 3.5 seconds.

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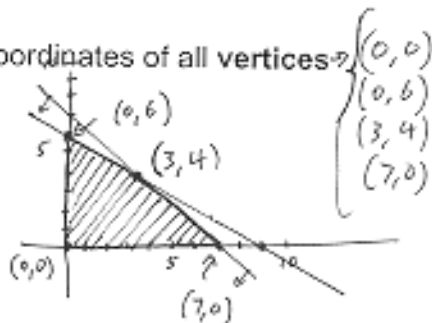
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(10 points)

$$\begin{aligned} x + y &= 7 \\ 2x + 3y &= 18 \\ -2x - 2y &= -14 \\ \hline y &= 4 \\ x &= 3 \end{aligned}$$

17. Graph the system of inequalities, giving the (x, y)-coordinates of all vertices of the solution set.

$$x \geq 0, y \geq 0, x + y \leq 7, 2x + 3y \leq 18$$



(10 points)

18. Simplify $\frac{x+2}{x^2-4x-21} + \frac{3-x}{x^2+5x+6}$

$$= \frac{x+2}{(x-7)(x+3)} + \frac{3-x}{(x+2)(x+3)} = \frac{(x+2)^2 + (3-x)(x-7)}{(x-7)(x+3)(x+2)} = \frac{x^2+4x+4 - x^2+7x+3x-21}{(x-7)(x+3)(x+2)} = \frac{14x-17}{(x-7)(x+3)(x+2)}$$

(10 points)

19. Simplify $\frac{x^2+4x}{x^2-4x-21} \cdot \frac{x^2+5x+6}{x^2-16}$

$$= \frac{x(x+4)(x+2)(x+3)}{(x-7)(x+3)(x-4)(x+4)} = \frac{x(x+2)}{(x-7)(x-4)}$$

(8 points)

20. Simplify $\frac{a^2b^4}{x^2} \div \frac{a^3b}{x^3} = \frac{a^2b^4}{x^2} \cdot \frac{x^3}{a^3b} = \frac{a^2b^4x^3}{a^3bx^2} = \frac{b^3x}{a}$

(4 points)

21. For what values of x is $\frac{x+2}{x^2-4x-21}$ defined? $x^2-4x-21 \neq 0$

$$(x-7)(x+3) \neq 0$$

All real numbers except $-3, 7$.

$$x \neq -3, 7.$$

(10 points)

22. Simplify $\frac{\frac{2}{x+1} - \frac{1}{x-1}}{\frac{1}{x^2-1}} + 1$

$$= \frac{2(x-1) - (x+1)}{(x+1)(x-1)} \div \frac{1+x^2-1}{x^2-1} = \frac{x-3}{(x+1)(x-1)} \cdot \frac{x^2-1}{x^2} = \frac{x-3}{x^2}$$

(12 points)

23. Solve $\frac{x+1}{x^2-9} - \frac{x-1}{x^2+3x} = \frac{1}{x}$

$$\frac{x+1}{(x-3)(x+3)} - \frac{x-1}{x(x+3)} = \frac{1}{x}$$

LCM = $x(x-3)(x+3) \neq 0$
 Multiply by LCM
 (OK IF $x \neq 0, 3, -3$)

$$\begin{aligned} x(x+1) - (x-3)(x-1) &= (x-3)(x+3) \\ x^2+x - (x^2-4x+3) &= x^2-9 \\ x^2+x - x^2+4x-3 &= x^2-9 \\ 0 &= x^2-5x-6 \end{aligned}$$

$$0 = (x-6)(x+1)$$

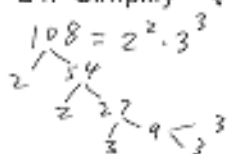
$$x = 6, -1$$

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(4 points) 24. Simplify $\sqrt{108} = \sqrt{2^2 \cdot 3^2 \cdot 3} = 2 \cdot 3 \sqrt{3} = \boxed{6\sqrt{3}}$



(10 points) 25. Rosa can mow her lawn in 30 minutes. Rosa and Claudia can mow the lawn together in 12 minutes. How long would it take Claudia to mow the lawn, working alone? Set up an appropriate equation, solve it, check, and answer with a sentence. Round to the nearest tenth of a minute if necessary.

	jobs	rate	time
Rosa	1	1/30	30
Claudia	1	1/x	x
Both	1	1/12	12

$$\frac{1}{30} + \frac{1}{x} = \frac{1}{12}$$

$$2x + 60 = 5x$$

$$60 = 3x$$

$$20 = x$$

LCB = 60x Mult. by LCB.
 Check $\frac{1}{20} + \frac{1}{20} = \frac{2+2}{60} = \frac{4}{60} = \frac{1}{15}$

Claudia can mow it in 20 minutes, alone.

(10 points) 26. A boat travels 9 miles down a river, and returns, with a round trip travel time of 6.25 hours. The speed of the current is 0.6 miles per hour. Find the speed of the boat in still water. Use proper word problem procedure, including an appropriate equation, check and sentence answer.

The speed is 3 mi/hr.
 Let it be x

	dist	rate	time
down	9	x + .6	$\frac{9}{x+.6}$
up	9	x - .6	$\frac{9}{x-.6}$

$$\frac{9}{x+.6} + \frac{9}{x-.6} = 6.25$$

$$9(x-.6) + 9(x+.6) = 6.25(x^2 - .36)$$

$$18x = 6.25x^2 - 2.25 \rightarrow 6.25x^2 - 18x - 2.25 = 0$$

Can mult by 4
 $25x^2 - 72x - 9 = 0$
 $(x-3)(25x+3) = 0$
 $x = 3, -\frac{3}{25}$
 $\frac{9}{3.6} + \frac{9}{2.4} = 2.5 + 3.75 = 6.25$

(4 points) 27. y varies inversely as x. If y is 4 when x is 3, find y when x is 5.

$$y = \frac{k}{x}, \quad 4 = \frac{k}{3}, \quad 12 = k, \quad y = \frac{12}{x} = \frac{12}{5} = \boxed{2.4}$$

(10 points) 28. Solve by completing the square: $x^2 - 10x + 6 = 0$
 Write solutions with simplified radicals.

$$x^2 - 10x = -6$$

$$x^2 - 10x + 25 = -6 + 25$$

$$(x-5)^2 = 19$$

$$x-5 = \pm \sqrt{19}$$

$$x = 5 \pm \sqrt{19}$$

(10 points) 29. Solve by using the quadratic formula: $3x^2 - 10x + 6 = 0$
 Write solutions with simplified radicals.

$a = 3, b = -10, c = 6,$ $b^2 - 4ac = (-10)^2 - 4(3)(6)$
 $= 100 - 72 = 28$

$$x = \frac{-(-10) \pm \sqrt{28}}{2(3)} = \frac{10 \pm 2\sqrt{7}}{6} = \frac{5 \pm \sqrt{7}}{3}$$

$$x = \frac{5 \pm \sqrt{7}}{3}$$