

1. Given the frequency table

x	2	3	4	5	6	7	Sums
f	15	20	35	20	8	2	$n = \sum f = 100$
xf	30	60	140	100	48	14	$\sum xf = 392$
x^2f	60	180	560	500	288	98	$\sum x^2f = 1686$
cf	15	35	70	90	98	100	-----

(4 points) find (a) the **depth** of the median = $\frac{n+1}{2} = \frac{101}{2} = 50.5$,
 and the **median** = $\frac{4+4}{2} = 4$

(4 points) (b) the **mean** = $\frac{\sum xf}{\sum f} = \frac{392}{100} = 3.92$

(1 point) (c) the **mode** = 4. (most frequent value of x.)

(2 points) (d) the **midrange** = $\frac{\text{high}+\text{low}}{2} = \frac{7+2}{2} = 4.5$

(5 points) (e) the **midquartile** = $\frac{Q_1+Q_3}{2} = \frac{3+5}{2} = 4$.
 $.25 \cdot 100 = 25$, $d(Q_1) = 25.5$, $.75 \cdot 100 = 75$, $d(Q_3) = 75.5$
 $15 < 25 < 26 < 35$, average 3 and 3 for Q_1 ,
 $70 < 75 < 76 < 90$, average 5 and 5 for Q_3 ,

(4 points) (f) the **depth** of the 35th percentile, and the **35th percentile**
 $.35 \cdot 100 = 35$, $d(P_{35}) = 35.5$,
 $P_{35} = \frac{X_{35}+X_{36}}{2} = \frac{3+4}{2} = 3.5$.

(6 points) (g) the **sample variance**
 $s^2 = \frac{\sum x^2f - \frac{(\sum xf)^2}{100}}{99} = \frac{1686 - \frac{(392)^2}{100}}{99} = \frac{3734}{2475} = 1.5086868686...$

(2 points) (h) the **sample standard deviation**.
 $s = \sqrt{1.5086868686....} = 1.228286151$

(5 points) 2. We want to include at least 60% of an unknown distribution within k standard deviations of its mean. Find the smallest k that will guarantee this.

$$0.60 = 1 - \frac{1}{k^2}, \quad \frac{1}{k^2} = 1 - 0.60 = 0.40, \quad k^2 = \frac{1}{0.40} = 2.5,$$

$$k = \sqrt{2.5} = 1.58113883, \text{ approximately}$$

(5 points) 3. X has a distribution with mean 75 and standard deviation 20.
 Find the **z-score** for $X = 90$. $Z = \frac{90-75}{20} = \frac{15}{20} = 0.75$

(2 points) 4. (a) Complete the table
 (2 points) (b) $SS(x) = 110 - \frac{20^2}{4} = 10$
 (2 points) (c) $SS(y) = 219 - \frac{29^2}{4} = 8.75$
 (2 points) (d) $SS(xy) = 136 - \frac{20 \cdot 29}{4} = -9$
 (2 points) (e) Find r , the **coefficient of linear Correlation**.

x	y		y^2	xy
3	9	9	81	27
4	8	16	64	32
6	7	36	49	42
7	5	49	25	35
$\sum x$	$\sum y$	$\sum x^2$	$\sum y^2$	$\sum xy$
20	29	110	219	136

$$r = \frac{SS(xy)}{\sqrt{SS(x) \cdot SS(y)}} = \frac{-9}{\sqrt{10(8.75)}} = -0.9621404709$$

(5 points) (f) Find the **equation** of the $m = \frac{SS(xy)}{SS(x)} = \frac{-9}{10}$

line of best fit in the form

$$y = mx + b \quad \sum y = m \sum x + nb$$

$$29 = 20m + 4b = 20(-0.9) + 4b, \quad 4b = 47, \quad b = 11.75.$$

$$y = -0.9x + 11.75$$

(2 points) (g) Predict y if $x = 10$. $y = -0.9(10) + 11.75 = 2.75$

4. One card is drawn at random from a standard deck of 52 cards. Events are
 $C =$ club, $R =$ red card, $L =$ card lower than a six. $G =$ **not** a face card.
 From these events list a **pair** of events that are

(2 points) (a) **mutually exclusive** R & C

(2 points) (b) **independent** R & G

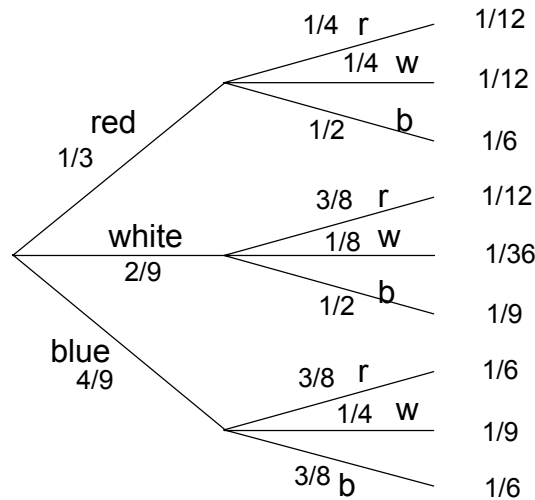
(2 points) (c) **neither** independent not mutually exclusive. L & G

5. A natural number from 1 to 30, inclusive, is drawn at random. (Each of the 30 numbers has probability $1/30$ of being drawn.) Let E be the event "the number is even", and F be the event "the number is a multiple of 7". Find

- (2 points) (a) $P(E) = 15/30 = 1/2$
- (2 points) (b) $P(F) = 4/30 = 2/15$
- (2 points) (c) $P(E \cap F) = 2/30 = 1/15$
- (5 points) (d) $P(E \cup F) = (15 + 4 - 2)/30 = 17/30$
- (5 points) (e) $P(F | E) = P(E \cap F)/P(E) = 2/15$
- (3 points) (f) if E and F are independent or not. (explain)
 $P(F | E) = P(F)$. **Independent.**

6. An urn contains three red marbles, two white marbles and four blue marbles. We draw two marbles, one at a time, without replacement.

- (3 points) (a) What is the probability of getting a white marble on the second draw, given that we got a blue one on the first draw? $2/8 = \mathbf{1/4}$
- (5 points) (b) In the **tree diagram** below, label each branch with its conditional probability, and each branchtip with its probability.



7. We get 60% of our parts from LA, 30% from SF and 10% from OC.
 Only 0.5% of LA parts are defective. 1.2% of SF parts are defective.
 1.4% of OC parts are defective.

(8 points)

(a) Make a complete Bayes' Theorem table. Let X be the parts source.

X	P(X)	P(D X)	P(X ∩ D)	P(X D)
LA	0.60	0.005	0.0030	0.375
SF	0.30	0.012	0.0036	0.450
OC	0.10	0.014	0.0014	0.175
	1.00		0.0080	1.000

.003/.008 = 3/8
 36/80 = 9/20
 14/80 = 7/40

(2 points)

(b) What is the probability that a randomly chosen part is defective? 0.008

(2 points)

(c) What is the probability a part came from SF if it is defective? 0.45