

Solar Energy

I. OBJECTIVES

The primary goal of this experiment is to develop insight into the production and measurement of *solar radiant energy* by using a homemade *radiometer* to measure the *solar constant*. The primary challenge in conducting this experiment is dealing with the unavoidable, adverse effects of *systematic error*; that is, conditions during data acquisition that lead to erroneous results even though the measurements themselves look very good.

In this experiment, you will:

- use *critical thinking* in designing your experiment;
- pay constant attention to environmental conditions;
- become acquainted with *numerical curve fitting techniques*;
- approximate complicated natural phenomena with *analytic models*.

II. PRE-CLASS PREPARATION

A. CONCEPTS. Review material in your lecture notes and text concerning *stellar nucleosynthesis* and the nature of *electromagnetic radiation*. Be able to use the terms *conduction*, *convection*, and *radiation* to describe the flow of heat. Carefully read **Section III** about the solar constant.

B. APPARATUS. You will assemble a simple, homemade radiometer from the supplies listed in **Section IV**. You must design this radiometer to minimize heat transfer to and from the environment, so be able to explain and justify your use of black paint, aluminum foil, plastic film, styrofoam, and other materials. Consider discussing the design of your radiometer with your partner *before class*.

C. MEASUREMENTS. Make a list of the quantities that you will measure and understand how each measurement fits into the overall goal of the experiment. You will have to improvise ways to make several measurements, including the inside diameter of the radiometer at water level (**Step 3** in **Section V**) and the solar zenith angle (**Steps 5** and **8** in **Section V**).

D. MATHEMATICS. Be able to follow the algebraic steps used in **Section III** to derive the expression for the solar constant in Equation 2.

E. DATA ANALYSIS. Review the graphing and curve fitting techniques that you will use in **Sec-**

tion VI and understand the role of Equation 3 in calculating the solar constant. Refer to the **Graphing** appendix for additional assistance.

III. WHAT IS THE SOLAR CONSTANT?

The Sun contributes 99.98% of Earth's energy budget. The next largest heat source ($< 0.02\%$) is the decay of *long-lived radioisotopes* in Earth's interior. Thanks to solar energy, Earth's surface is $\approx 250\text{ }^{\circ}\text{C}$ warmer than it would be if our planet depended only on internal sources of heat. Solar energy keeps most of Earth's surface in the temperature range where H_2O is liquid, maintaining conditions necessary for life. The amount of solar energy reaching Earth is therefore one of the most important factors in understanding the habitability of this planet.

Scientists describe the incident solar energy by a quantity called the *solar constant* (I_{SC}). The *solar constant* I_{SC} is the radiant energy emitted by the sun at all wavelengths, received at the top of Earth's atmosphere per unit time per unit area when Earth is at its mean distance (1 AU) from the sun. The solar constant is expressed either in units of energy received per unit area per unit time, such as calories per square centimeter per minute ($\text{cal}/\text{cm}^2\cdot\text{min}$), or in units of power per unit area, such as watts per square meter (W/m^2). A *calorie* is the amount of energy required to raise the temperature of 1 gram of H_2O by 1°C . The solar constant is not actually a true constant because solar energy production varies by a few tenths of a percent over many years. Scientists continue to search for evidence of these variations due to their possible influence on global climate change.

It may appear that the solar constant, as defined above, can only be measured from a spacecraft outside Earth's atmosphere. In practice, scientists can measure the solar constant from any location, but they must apply *correction factors* to *normalize* their results to the conditions specified in the definition. You will take similar steps to make your value of the solar constant measured on the UA mall consistent with the definition.

This experiment is conducted by monitoring the rise in temperature of a known quantity of water exposed to sunlight. The radiant energy carried by solar *photons* is absorbed in the water and *transformed* into thermal energy. This energy causes water molecule *velocities* to increase (*i.e.*, their *kinetic energy* increases) which is observed as a *temperature* increase.

You will be converting *solar radiant energy* to *thermal energy*. Thermal energy is difficult to store because while losses due to *conduction*, *convection*, and *radiation* can be minimized, they cannot be eliminated. Think of a Thermos of hot coffee. The liquid remains hot for many hours because the flask is designed to minimize loss of thermal energy, but in several days the liquid is cold because heat leaked out. Similarly, a cold liquid eventually warms up as heat leaks in from the outside environment. Your radiometer is similar to a Thermos, so *you must plan and conduct this experiment and analyze your data knowing that heat will inevitably leak in or out of your radiometer*.

Heat and thermal energy are not the same quantities. *Heat* is energy transferred between a system and its environment because of a temperature difference. *Thermal energy* is the amount of kinetic

energy of the atoms and molecules in a system.

The relation between the increase in thermal energy in the cup (ΔQ) and the increase in temperature (ΔT) is given by Equation 1, where m is the mass of water in the cup. Note how the capital Greek letter Δ (delta) is used to represent “change in.” The constant c in Equation 1 is the *specific heat capacity* (not to be confused with the speed of light), which is the amount of energy required to raise the temperature of 1 g of a substance by 1°C. Equation 1 does not explicitly contain the solar constant I_{SC} , but it is the fundamental starting point for deriving Equation 2 which does contain the solar constant.

EQUATION 1	EQUATION 2
$\Delta Q = mc\Delta T$	$I_{SC} = \frac{mc}{L_{Atm} \cdot A} \left[\frac{\Delta T}{\Delta t} \right]$

To begin the derivation, divide both sides of Equation 1 by the time interval Δt during which the water temperature rises by ΔT .

$$\boxed{\frac{\Delta Q}{\Delta t} = mc \left[\frac{\Delta T}{\Delta t} \right]} \quad (i)$$

Now divide Equation (i) by the surface area A of the water.

$$\boxed{\frac{\Delta Q}{\Delta t \cdot A} = \frac{mc}{A} \left[\frac{\Delta T}{\Delta t} \right]} \quad (ii)$$

The purpose of these first two steps is to create the expression on the left hand side that is dimensionally equal to the solar constant, that is, a change in thermal energy per unit time per unit area. Now redefine the left hand side of Equation (ii) with the new variable $I_{Surface}$ to represent the solar constant measured on the UA mall.

$$\boxed{I_{Surface} = \frac{mc}{A} \left[\frac{\Delta T}{\Delta t} \right]} \quad (iii)$$

The amount of sunlight at Earth’s surface is always less than that at the top of the atmosphere by the numerical factor L_{Atm} because the atmosphere itself absorbs some of the sunlight. Represent this loss as follows:

$$\boxed{I_{Surface} = L_{Atm} \cdot I_{SC} \quad L_{Atm} < 1} \quad (iv)$$

Substitute $I_{Surface}$ from Equation (iii) into Equation (iv) and solve for I_{SC} to obtain Equation 2. The expression for evaluating L_{Atm} is given in Equation 3 (Section VI. B.). Note that units are not usually written out explicitly during a derivation, but units and conversion factors become impor-

tant as soon as numerical values are substituted into an equation.

IV. THE APPARATUS

Materials

- Styrofoam cup
- Black paint
- Aluminum foil
- Thermometer
- Graduated cylinder
- Water
- Plastic film
- Stopwatch
- Miscellaneous supplies

Before going outside to take measurements, you must first construct your *radiometer* from the list of materials given above. Your radiometer should be able to trap solar photons while minimizing conduction, convection, and radiation of heat to and from the surrounding environment.

V. THE LAB SESSION

Steps 1, 2, and 3 may be done in any order. Decide which order makes the most sense for your setup and assembly.

STEP 1: Assemble your radiometer, designing it to minimize heat transfer to its surroundings. Be prepared to *answer questions* on why you used the materials you chose and why you assembled the radiometer in the way you did. Consider the properties of all materials. Consider methods of assembly that minimize heat conduction, convection, and radiation.

STEP 2: Fill the radiometer cup about 1/2 to 3/4 full of water. Use a graduated cylinder to measure the volume (V) of water used. The water should either be chilled with ice or warmed with hot water so that its temperature at the start of your measurements is 5 to 10 °C below the ambient outside air temperature (T_{Amb}). *Record all measurements in the tables on the data page. Partners must copy all data to their own pages before leaving the lab.*

Calculate the mass of the water. The mass m of water in the radiometer is calculated from the measured volume V knowing that the density $\rho = m/V$ of liquid water is 1 g/cm³ (1 cm³ = 1 ml).

Volume of water = _____ ml Mass of water = _____ g

STEP 3: Measure or calculate the inside diameter of the cup at the water line using an improvised method of your choice. You should think about this before you put water in the cup. Use the diameter to calculate the surface area A of water exposed to sunlight.

Diameter = _____ cm Surface Area = _____ cm²

Cover the cup with plastic film and insert the thermometer through a small hole in the film. *Why do you cover the radiometer with plastic film?*

Read steps 4 through 9 before going outside!

STEP 4: Take the radiometer outside and set it in a location that will remain fully exposed to the Sun throughout the measurements (about 45 minutes). Prop the radiometer (see Figure 1) so that it points directly at the Sun. The rim of the cup must never cast a shadow on the surface of the water, so you may have to adjust the tilt during observations. Orient the thermometer so that it does not have to be removed to be read.

Something to think about before proceeding:

Be careful where you place the radiometer on the ground. Consider the consequences of putting it on grass, crushed rock, asphalt, or cement. Think about lengthening shadows.

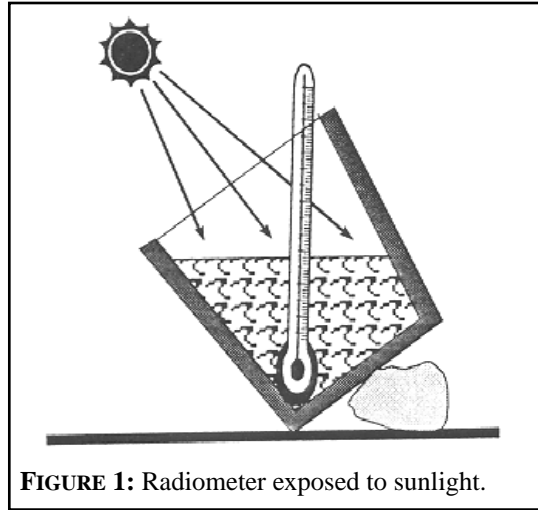


FIGURE 1: Radiometer exposed to sunlight.

STEP 5: Take your first temperature reading and start the stopwatch. You will take the next reading in 2 minutes. In between these readings, complete step 6.

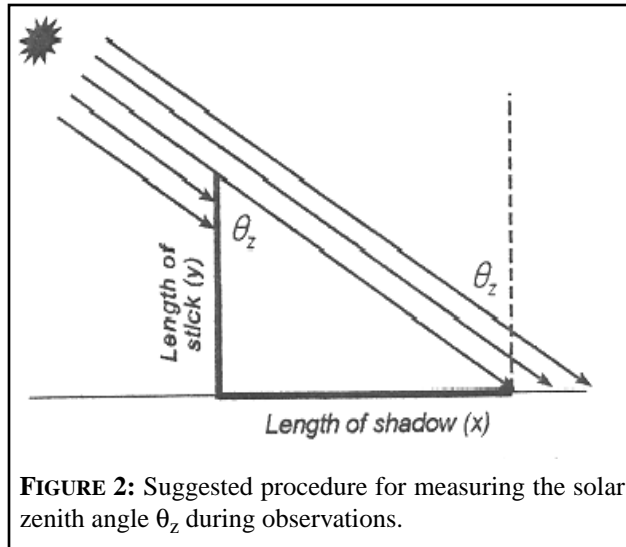


FIGURE 2: Suggested procedure for measuring the solar zenith angle θ_z during observations.

STEP 6: Measure the *solar zenith angle* θ_z at the **beginning** of your observations using the shadow cast by a meter stick or an improvised method of your choice. You will have to do some trigonometry to put the sun angle into degrees.

(Hint: Hold the meter stick at a right angle to the ground as shown in Figure 2. Sketch the right triangle with appropriate length measurements on each side. Use the **Math Help!** appendix to determine which trigonometric function to use.)

STEP 7: Take temperature measurements at 2 minute intervals until the water in the radiometer is at least 5 °C above T_{Amb} . (T_{Amb} will be given to you by your instructor.) Record times to the nearest second and *interpolate* temperatures to a fraction of the smallest division on the thermometer. Include an estimate of the experimental error in reading the temperature.

Something to think about while doing this step: Your measurements will be invalid if water spills from the cup during observations! If this happens, you will have to start over.

STEP 8: Indicate your assessment of atmospheric conditions during the observations.

STEP 9: Measure the solar zenith angle again at the **end** of your observations.

VI. DATA ANALYSIS

Start with Equation 2. Identify each variable in the equation (see the last page of this lab). You should have already calculated the mass of the water (m) and the surface area of the water (A). The specific heat capacity of water (c) is a constant you will find in the **Units, Conversions, and Constants** box. To understand the other variables, read the following sections.

A. TEMPERATURE PLOT ($\Delta T/\Delta t$). Plot water temperature and elapsed time as shown in Figure 3 using the **DeltaGraph** program (a help sheet will be kept at each computer in the lab). Your objective is to determine the slope $\Delta T/\Delta t$ in $^{\circ}\text{C}/\text{min}$ that best characterizes your data near T_{Amb} .

Units, Conversions, and Constants

Energy (erg, Joule, calorie)

$$1 \text{ erg} = 1 \text{ g cm}^2/\text{sec}^2 = 10^{-7} \text{ Joules} = 10^{-7} \text{ J}$$

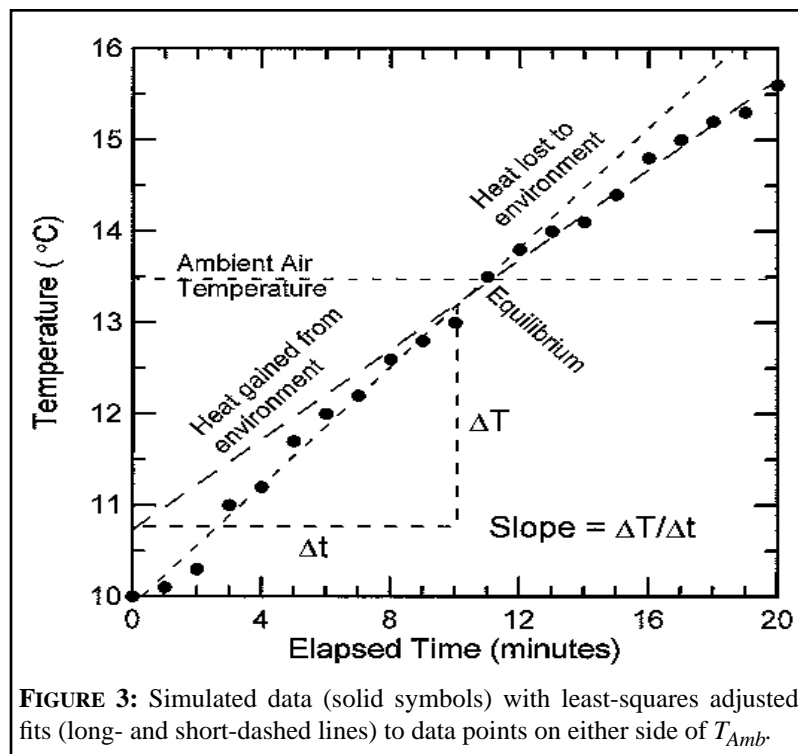
$$1 \text{ calorie} = 4.186 \text{ J}$$

Power (watts)

$$1 \text{ watt} = 1 \text{ J/sec}$$

Specific Heat Capacity of Water

$$c = 1 \text{ cal/g} \cdot ^{\circ}\text{C}$$



What is so important about T_{Amb} ? Examine carefully the shape of the experimental curve in Figure 3. The general rise in water temperature is due to absorbed solar radiation, but there is a subtle change in slope when the water temperature equals T_{Amb} . Since the curve is supposed to be a straight line, some explanation is necessary.

Something to think about before proceeding: Why should your data points lie on a straight line rather than some other curve?

Here is one possible explanation for observing a change in slope during the observations. Measurements below T_{Amb} were systematically **high** because heat leaked into the radiometer from its warmer surroundings, thus **increasing** the slope of the straight line (short dashes) from $t = 0$ to 11 minutes. Measurements above T_{Amb} , on the other hand, were systematically **low** because heat leaked out of the radiometer to its cooler surroundings, thus **decreasing** the slope of the line (long dashes) from $t = 11$ to 20 minutes. Only when the water was at T_{Amb} was it in **thermal equilibrium** with its surroundings, and only solar energy contributed to the temperature rise of the water. The slope at this point on the curve is therefore free of systematic error due to thermal effects.

The primary objective of your data analysis is to **extract a slope** from your graph that is based on a similar critical evaluation. Should you use one line or the average of two lines? You must interpret your curve in terms of what actually happened during the experiment to **defend your choice** of numerical values derived from it later.

B. CORRECTION FOR LOSSES IN EARTH'S ATMOSPHERE (L_{Atm}).

Sunlight interacts in complex ways with atmospheric and surface materials (see Figure 4). The amount of solar energy reaching the ground is only 60-70% of the solar energy incident on the top of the atmosphere. Some solar photons are absorbed by molecules in the atmosphere, thereby warming it. Other photons are either reflected back to space or diffusely scattered within the atmosphere by molecules and dust. The direct component of sunlight reaching the ground is either absorbed by the surface or reflected upwards interacting again with the atmosphere. Consequently, there is a very complex relation between the amount of solar energy detected on the ground and that at the top of the atmosphere. Understanding the physics of this process is beyond the scope of this lab, but the losses are too large to be neglected.

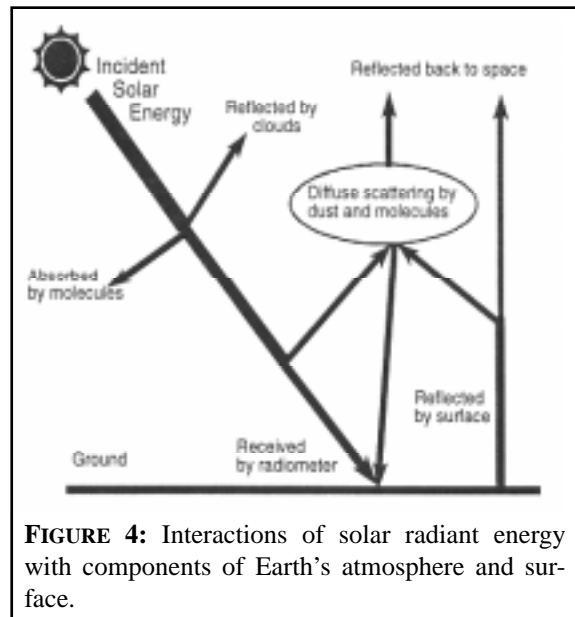


FIGURE 4: Interactions of solar radiant energy with components of Earth's atmosphere and surface.

To get around this difficulty you will use the results of an **atmospheric model** which expresses these complex interactions by the dimensionless **correction factor** L_{Atm} . The model calculations require three parameters: atmospheric conditions, the solar zenith angle θ_z during observations, and the altitude of the UA mall (0.76 km). Values of L_{Atm} for our altitude and two types of atmospheric conditions are listed as a function of solar zenith angle in a lookup table (Table 1). Use your average value of θ_z to interpolate the correction factor L_{Atm} for atmospheric conditions that best apply to your observations.

Something to think about before finding L_{Atm} : Why should the correction factor depend on altitude and the solar zenith angle?

Table 1: L_{Atm}

Clear Atmosphere				Hazy Atmosphere			
θ_z	L_{Atm}	θ_z	L_{Atm}	θ_z	L_{Atm}	θ_z	L_{Atm}
0	0.707	50	0.623	0	0.569	50	0.452
5	0.707	55	0.598	5	0.568	55	0.419
10	0.705	60	0.566	10	0.566	60	0.379
15	0.701	65	0.525	15	0.561	65	0.329
20	0.697	70	0.471	20	0.554	70	0.270
25	0.690	75	0.400	25	0.545	75	0.199
30	0.682	80	0.308	30	0.533	80	0.127
35	0.672	85	0.218	35	0.519	85	0.084
40	0.659	89	0.202	40	0.501	89	0.081
45	0.643			45	0.479		

C. THE SOLAR CONSTANT. You are now ready to substitute numerical values for all terms in Equation 2. Express your calculated value of the solar constant in units of W/m^2 . **Watch units and conversions!**

VII. DATA ANALYSIS

In this section, you should include the following:

- Descriptions of procedures that deviated from, or augmented, those presented in the lab write-up.
- Comments on any aspect of the experiment that adversely affected your data.
- Your temperature plot with a full description of how you arrived at the value of the slope $\Delta T/\Delta t$ that you used in Equation 2.
- An orderly summary of the calculations leading to your value of the solar constant. Start with Equation 2, indicate the numerical value that you substituted for each parameter, and show your final result, **including units**, with enough intermediate steps for others to follow your work.

VIII. DISCUSSION

A. DISCUSSION OF YOUR VALUE OF THE SOLAR CONSTANT. In this section you demonstrate the critical thinking that you applied to your value of the solar constant. Your presentation must include thoughtful responses to the questions below.

- *How does your solar constant compare with the accepted value of 1370 W/m^2 ?*

Approach. Express your difference as a percent error. The expression for percent error is found in the **Experimental Error** appendix.

Your percent error may seem high, but don't interpret this to mean that the experiment was a failure. Remember that the purpose of this experiment is not "to get the right answer," but to develop insight into how scientists cope with similar problems.

- *What factors contributed most to the error in your value of the solar constant?*

Approach. Consider the relative importance of such factors as the design of the apparatus, experimental error in the data, and conditions during the experiment. Your objective in this section is to identify all sources of error in your measurements.

- *How would you improve the experiment if you were to perform it again?*

- *What did you learn from the experiment?*

B. USING YOUR VALUE OF THE SOLAR CONSTANT. The solar constant is defined with a terrestrial perspective, but it can also be used in an astrophysical context. The following exercises demonstrate how the solar constant can be used to deduce what is happening in the Sun's unobservable interior.

- *Calculate the total power radiated from the Sun (the "solar luminosity") in watts. The power of the sun is mathematically $P = 4\pi I_{SC} R_0^2$.*

Approach. Think of an imaginary sphere with radius R centered on the Sun. All of the Sun's radiant energy must pass through the surface of this sphere. Now imagine that the imaginary sphere is placed at Earth's mean distance from the Sun where your measured value of the solar constant applies; that is, $R = R_0 = 1 \text{ AU} = 1.496 \times 10^8 \text{ km}$. The surface area of the imaginary sphere is $4\pi R_0^2$ and you know the solar power incident on an area of 1 m^2 at $R = R_0$. You need only multiply these two numbers to get total solar power. **Watch units and conversion factors!**

- *What is the loss of mass in tons per second in the Sun's interior that is required to sustain its energy production?*

Approach. Power is a change in energy with time (call it $\Delta E/\Delta t$ here). The total solar power in watts calculated above is equivalent to the energy in joules radiated per second ($1 \text{ W} = 1 \text{ J/sec}$). The source of this energy is mass lost in the Sun's core during hydrogen fusion reactions. The energy ΔE obtained from mass loss Δm can be calculated from Einstein's equation in **Appendix A** at the end of this lab. *Watch units and conversion factors!*

- *Compare the mass lost by the Sun during its expected lifetime to its total mass.*

Approach. The expected lifetime of the sun is 10 billion years. Use your previous result to calculate how much mass the sun burns in its lifetime. Determine the fraction of the total mass of the sun ($M_{sun} = 2.0 \times 10^{27}$ tons) this represents.

- *How can the Sun lose so much mass and still exist for billions of years?*
- *Do you realize that this all started with simply taking a cup of water outside?*

APPENDIX A

The Source of Solar Energy

Energy is created in the cores of stars by nuclear reactions that fuse light elements into heavier elements (*stellar nucleosynthesis*). During these reactions mass is converted into energy by Einstein's equation $E = mc_L^2$ (where c_L is the speed of light). All stars begin their life cycle with hydrogen burning, a series of nuclear reactions in which hydrogen (${}_1\text{H}^1$, a proton) is used to create helium (${}_2\text{He}^4$). The hydrogen fusion reactions are summarized as $4 {}_1\text{H}^1 \rightarrow {}_2\text{He}^4 + \text{energy}$. The mass of 4 protons is 6.693×10^{-24} g, but the mass of one helium nucleus is only 6.645×10^{-24} g. The lost mass $\Delta m = 0.048 \times 10^{-24}$ g (0.7%) was converted to an amount of energy given by $E = mc_L^2$. The energy released in creating one helium nucleus from proton fusion is calculated below. Energy generated in the stellar core eventually reaches the outer atmosphere by conduction and convection and is then radiated to space.

Einstein's Equation:

$$\Delta E = (\Delta m)c_L^2$$

Energy produced from mass loss in creating 1 He nucleus:

$$\Delta E = (0.048 \times 10^{-24} \text{ g})(3 \times 10^{10} \text{ cm/sec})^2$$

Use the conversion factors in Section III:

$$\Delta E = 4.3 \times 10^{-5} \text{ g}\cdot\text{cm}^2/\text{sec}^2 = 4.3 \times 10^{-5} \text{ ergs} = 4.3 \times 10^{-12} \text{ J}$$

NOTE: In Appendix A, c_L is the speed of light not the specific heat capacity!

$$c_L = \text{speed of light} = 3 \times 10^{10} \text{ cm/sec} = 3 \times 10^8 \text{ m/sec}$$

NAME _____ DATE _____

PARTNER _____

Miscellaneous MeasurementsAmbient air temperature $T_{Amb} = \text{_____} \pm \text{_____} \text{ } ^\circ\text{C}$ Volume of H₂O used in cup $V = \text{_____} \pm \text{_____} \text{ ml}$ Inside diameter of cup at H₂O line $d = \text{_____} \pm \text{_____} \text{ cm}$

Solar zenith angle at beginning:

Solar zenith angle at end:

Atmospheric conditions:

- Clear
- Hazy
- Other _____

Error in temperature readings = $\pm \text{_____} \text{ } ^\circ\text{C}$

Other Observations:

Calculation of the Solar Constant

$$I_{SC} = \frac{mc}{L_{Atm} \cdot A} \left[\frac{\Delta T}{\Delta t} \right]$$

m = mass of water = _____ g

A = surface area = _____ cm² = _____ m²

c = specific heat capacity = _____ cal/g · °C = _____ J/g · °C

L_{Atm} = _____

ΔT/Δt = _____ °C/min = _____ °C/sec

Now calculate I_{sc}. Cancel units where you can.
You should express your answer in terms of **W/m²**.

