
*Problem Set for Unit 3:
Measures of Spread, Measures of Relative Standing &
The Normal Distribution*

1. The age distribution of 20 subjects in a medical study is given in the stem-and-leaf plot below.

(a) Find the median age.

3	2
	7 9
4	0 1 1 1 2 2 3 4
	5 5 5 6 7 7 9
5	0 1

(b) Find the first Quartile Q1 for this age data.

Note: 3|2 represents 32

(c) Find the third Quartile Q3 for this age data

(d) Find the range for this age data.

(e) Find the interquartile range for this age data.

(f) According to the 1.5 IQR Rule are there any outliers for this age data. Show work.

(g) Draw a box-and-whisker plot for this data.

2. From the statistics listed in the box, choose all that apply.

- Interquartile Range
- Mean
- Median
- Mode
- Outlier
- Percentile
- Quartile
- Range
- Standard Deviation
- z-score

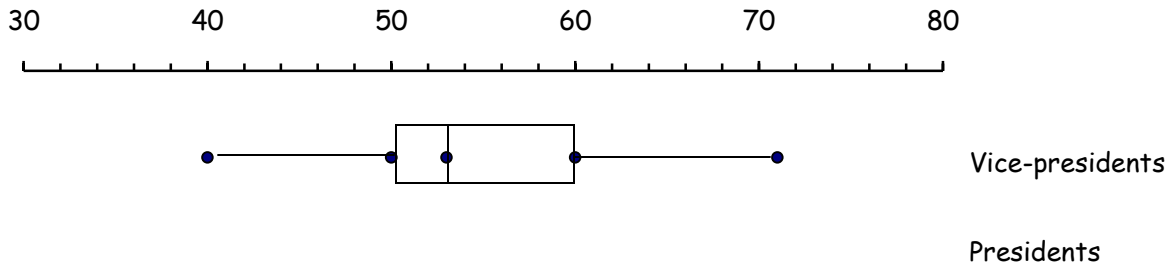
- (a) Which of the statistics are measures of central tendency?
- (b) Which of the statistics are measures of spread or dispersion?
- (c) Which of the statistics are measures of relative standing?
- (d) Which of the statistics are mean based statistics?
- (e) Which of the statistics are median based statistics?
- (f) Which of the statistics can be determined by a box-and-whisker plot?
- (g) Which of the statistics are resistant statistics?

3. (a) Make box-and-whisker plots on the same number line for the following test scores. [8TH → Pg. 502 # 8 & 9; 7TH → Pg. 470 # 7]

- Class 1: 57, 58, 59, 60, 62, 72, 75, 76, 76, 79, 80,
80, 81, 86, 86, 86, 87, 93, 93, 93
- Class 2: 66, 67, 68, 75, 77, 77, 79, 82, 83, 84, 85,
85, 87, 87, 90, 90, 92, 92, 92, 95

- (b) Which class performed better on the test? Explain.
- (c) What score is above 75% of the scores in class 1?
- (d) What score is at the 50th percentile of class 2?
- (e) What score is at the 25th percentile of class 1?

4. The following box-plot shows the ages of the 20 twentieth-century vice-presidents at their inaugurations.



- (a) The youngest vice-president was Richard Nixon. How old was he at his inauguration?
- (b) What was the median age at inauguration?
- (c) What percentage of the vice-presidents were over 60?
- (d) What percentage of the vice-presidents were between 50 and 60?
- (e) Al Gore was 44 at his inauguration. Complete this sentence: There were at least vice-presidents who were older at the time of their inaugurations.
- (f) Are any of the vice-presidents outliers? How can you tell?

President	Age at Inauguration
T. Roosevelt	42
Taft	51
Wilson	56
Harding	55
Coolidge	51
Hoover	54
F. Roosevelt	51
Truman	60
Eisenhower	62
Kennedy	43
Johnson	55
Nixon	56
Ford	61
Carter	52
Reagan	69
Bush	64
Clinton	46

The table gives the ages of the 17 twentieth century presidents at the time they took office.

- (g) Record the data from the table in a stem-and-leaf plot.
- (h) Make a box-plot of the ages of the presidents below the box plot for the vice presidents above.

5. Julie is in Lecture 001 and her statistics midterm test score was 75 out of 100. John is in Lecture 002 and his score was 68 out of 100. For Julie’s lecture, the mean midterm test score was 70 and the standard deviation was 5. For John’s lecture, the mean midterm was 62 and the standard deviation was 4.

- (a) Find the standard score for Julie based on the scores in her lecture and complete the sentence.

Julie scored standard deviation(s) above the mean test score of

- (b) Find the standard score for John based on the scores in his lecture and complete the sentence.

John scored standard deviation(s) above the mean test score of

- (c) Relative to their respective lectures, had the higher test score.
- (d) Can you determine the mean of the test scores for the two lectures combined? If yes, find it. If no, explain what additional information you need to compute it.

6. Consider the normal distribution given by the graph below

- (a) Approximate the mean

$\mu = \dots\dots\dots$

- (b) Approximate the standard deviation

$\sigma = \dots\dots\dots$

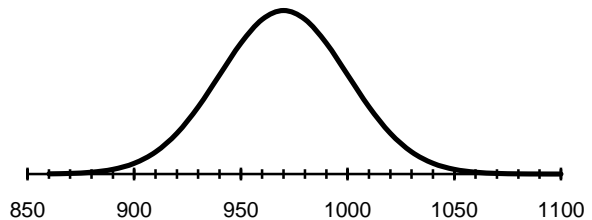
- (c) The area under a normal distribution is

- (d) The median for this distribution is

- | | |
|--|--|
| <input type="checkbox"/> Greater than the mean | <input type="checkbox"/> Less than the mean |
| <input type="checkbox"/> Equal to the mean | <input type="checkbox"/> Can't tell from the curve |

- (e) 95% of all students who take the ACT test have scores between and

- (f) On the same graph, draw a normal distribution with a larger mean but a smaller standard deviation.



7. The lengths of the first twenty words in Calinger’s Classics of Mathematics are given below.

12 8 4 9 3 9 2 11 2 2 11 11 7 2 9 6 2 3 4 5

- (a) Calculate the mean and sample standard deviation for this data.

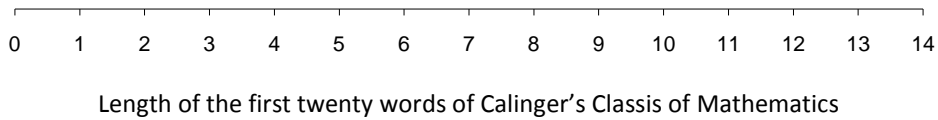
$$\bar{x} = \dots\dots\dots \quad s_x = \dots\dots\dots$$

- (b) Calculate one standard deviation below the mean and one standard deviation above the mean.

$$\bar{x} - s_x = \dots\dots\dots \quad \bar{x} + s_x = \dots\dots\dots$$

- (c) What percent of the **actual data** fall within one standard deviation of the mean? (i.e. you will need to count the number of data above that fall between the values you calculated in (b).)

- (d) Construct a line plot (dot plot) below by placing an x above the number line to represent each value.



- (e) Give two reasons how you know that this data is NOT normally distributed.

8. In the population of 1200 students who took the Graduate Record Exam in a particular year, the “analytical ability” scores were normally distributed with a mean of 520 and a standard deviation of 10.

- (a) Draw the distribution of scores. Label the horizontal axis.

- (b) What proportion of these students had scores between 510 and 540?
- (c) Approximately how many students scored higher than 540?
- (d) What scores fall at the bottom and top of the middle 95%?

- (e) Indicate the 95th percentile on your graph. Approximately what score corresponds to the 95th percentile?

- (f) What score on the Graduate Record Exam corresponds to a standard score -1.2 ?

9. Here are Mr. Emery’s class scores for two tests. (See 8th → #25 pg. 503; 7th → # 21 pg. 471)

(a) Find the mean and standard deviation for each test.

Test 1: $\bar{x} = \dots\dots\dots$ $s_x = \dots\dots\dots$

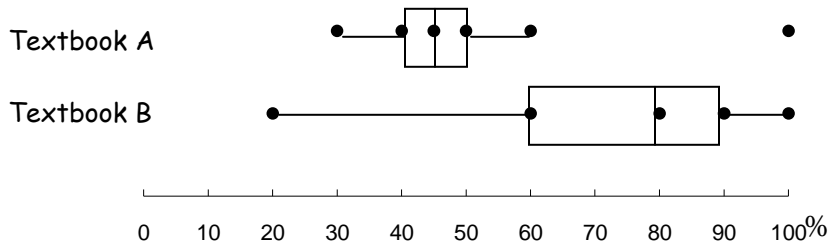
Test 2: $\bar{x} = \dots\dots\dots$ $s_x = \dots\dots\dots$

Student	Test 1 Score	Test 2 Score
Lora	85	89
Juan	72	93
Marisol	89	96
Veronica	75	65
Victor	79	79
Michelle	86	60

(b) Find Lora’s standard score for each test.

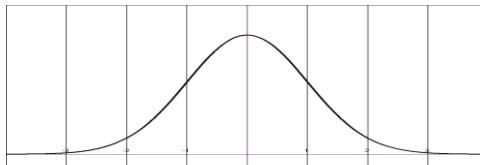
(c) On which test did Lora do better relative to the entire class? Explain why.

10. The following boxplot shows the final exam scores (in percent) in algebra for students using two different textbooks.



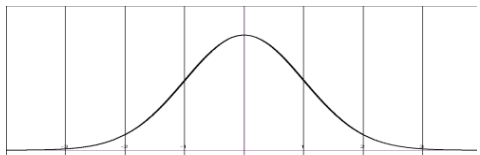
- (a) The lowest score for students using Textbook A is
- (b) The proportion of the students using Textbook A that scored less than 50% is
- (c) Complete this sentence: Half of the students using Textbook B got percent or more on the final exam.
- (d) Which textbook gave students scores that varied less? Explain your answer.
- (e) Which textbook do you think is better? Explain your answer.

11. (a) The graph of one normal distribution is given below. On the same axes draw another normal distribution with the same mean score but a smaller standard deviation. (See 8th →pg. 504 # 27; 7th →# 23 pg. 472)



Which of these graphs has the higher “peak” – the one with the smaller standard deviation or the one with the larger standard deviation?

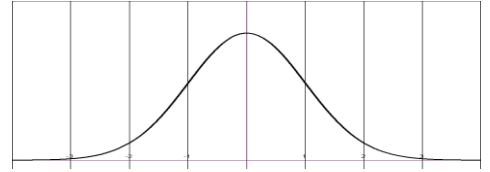
(b) The graph of one normal distribution is given below. On the same axes draw another normal distribution with a different mean score but the same standard deviation.



Which of these graphs is farther right – the one with the smaller mean score or the one with the larger mean score?

12. Suppose that adults have IQ scores that are normally distributed with a mean of 100 and a standard deviation of 15 (as on the Wechsler test).

(a) Janice has an IQ of 90. Label the axis and indicate Janice's score on the graph.



(b) How many standard deviations above or below the mean is Janice? What is this value called?

(c) What percent of adults would be expected to have an IQ of less than 115?

(d) The requirement for membership in the Mensa organization is an IQ of 131.5 or greater. Shade this region on the graph. Approximately what percentile must one be in order to qualify for Mensa?

(e) Normal range for IQ is said to be between 90 and 110. Show this area on the graph provided and estimate approximately what percentage of adults would be classified as "normal".

(f) The lower quartile would be between and

(g) Cerritos College has 22,000 students, about how many students at Cerritos College could be expected to have an IQ of 115 or higher?

(h) What assumption did you make in answering (g)? What might be wrong with that assumption?

13. (a) Suppose a class test has scores with a normal distribution. [8th → pg. 503 # 16]
- i. If you have a score that is 1 standard deviation above the mean, what percent of the rest of the class has a score below yours? What is your z -score?

 - ii. If you have a score that is 2 standard deviations above the mean, what percent of the rest of the class has a score below yours? What is your z -score?

- (b) In a class with test scores in a normal distribution, a teacher can “grade on a curve” using the following guideline for assigning grades:

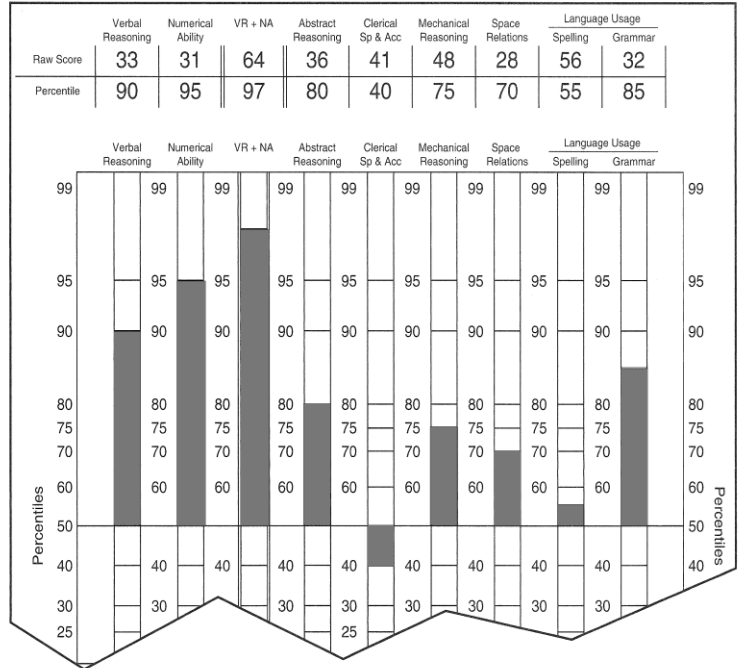
- A: $z\text{-score} > 2$
- B: $1 < z\text{-score} \leq 2$
- C: $-1 < z\text{-score} \leq 1$
- D: $-2 < z\text{-score} \leq -1$
- F: $z\text{-score} > -2$

On a 100-point test with a mean of 85 and a standard deviation of 5, find the test scores that would yield each grade. [8th pg. 503 # 17]

- (c) On the critical reasoning portion of the SAT in 2006 the mean was 503 and the standard deviation was 113. If Marcella had a score of 630 on the exam,
- i. What percentile is she in? How did you get your answer?

 - ii. What percent of all students who took the exam had a score lower than hers?

14. (a) The table and bar graph show a student’s performance on a differential aptitude test. The raw score in each category represents the number of questions that the student answered correctly. The student’s percentile score is obtained by comparing these raw scores with the scores from a sample of thousands of other students. The bar graph is a visual representation of the percentile score.



- i. The verbal reasoning score shown is at the 90th percentile. What does this mean?
- ii. In which of the nine categories is the student’s performance below the median?
- iii. What percentage of people in the sample group had better spelling scores than this student?

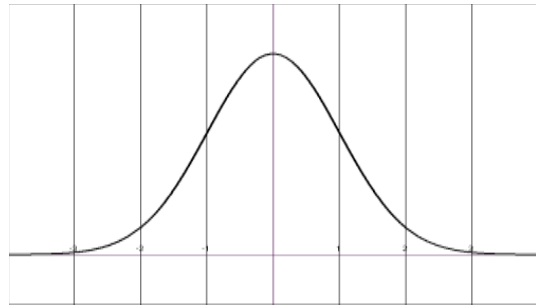
(b) The distribution of IQ scores for 20-year olds is normally distributed with a mean of 120 and a standard deviation of 20.

- i. What percent of 20-year olds have an IQ of more than 140?
- ii. Cerritos College has approximately 2600 twenty-year olds currently enrolled. Approximately how many would you expect to have an IQ between 120 and 160?
- iii. Tommy has a standard score of -1.5 . What does this mean?
- iv. What is Tommy’s IQ?
- v. What percentile is someone who is one-standard deviation above the mean?

15. The distribution exam scores in Statistics Section 001 was normally distributed with a mean of 80 and a standard deviation of 8. Matthew scores a 76 on the exam.

(a) Label the x -axis for the distribution of exam scores for Section 001.

(b) Mark the value of Matthew's score on the axis and find his z -score.



(c) What proportion of the students in Section 001 scored 64 or higher?

(d) If there were 120 students in Section 001, how many scored between 80 and 88?

(e) The professor announced that the top 2.5% of scores will receive a grade of "A". What is the minimum score needed to receive an "A"?

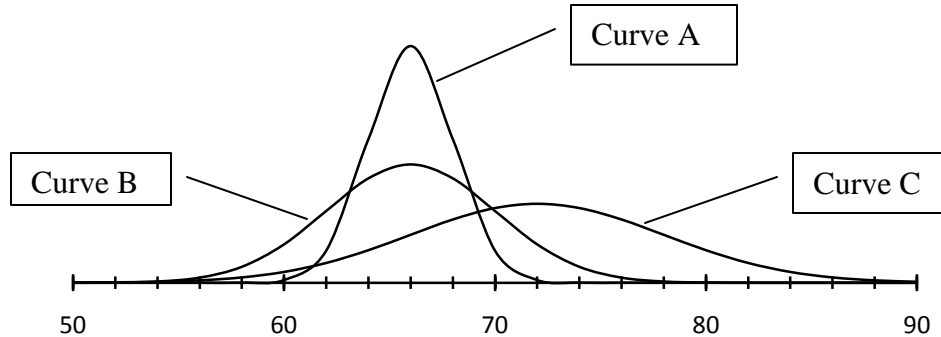
(f) If a Maggie earned a z -score of $+1.8$, what was her exam score?

(g) Is Q_3 greater than, less than, or equal to 88? Explain.

(h) In which section would Matthew be better off – in his current Section 001 or Section 002, in which the exam scores were normally distributed with a mean of 78 and a standard deviation of 6? Justify your answer.

(i) What score in Section 002 is equivalent to a classmate's score of 90 in Section 001?

17. Consider the distribution scores on three forms of a math placement test, each form given to a large group of students. The scores on each form are normally distributed. If we graph all three sets of test scores on the same axes we get the graph below.



- a. Curve has the greatest standard deviation. Why?
- b. Curve has the greatest mean? Why?

Suppose we know that on Test Form 1 the mean is 66 and the standard deviation is 4; on Test Form 2 the mean is 66 and the standard deviation is 2; and on Test Form 3 the mean is 72 and the standard deviation is 6.

c. Which curve represents which test form?

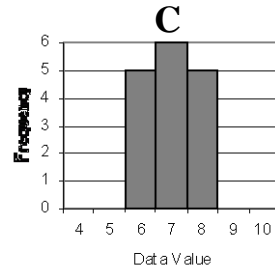
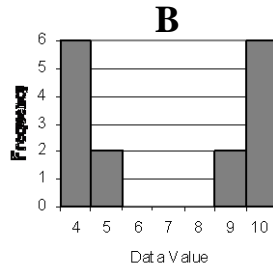
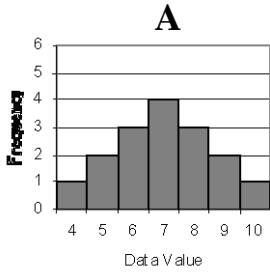
Curve A: Curve B: Curve C:

d. Looking at the mean and the standard deviation for Test Form 1, between what score values would you expect 99.7% of the test scores to fall? Does your prediction appear to agree with the graph?

99.7% of the test scores fall between and

- e. Suppose Maria took Test Form 1 and had a score of 70. Janelle took Test Form 2 and also had a score of 70. Do you think one did better than the other? Why or why not?
- f. Adrian had a score of 68 on Form 2 of the test. Julio had a score of 68 on Form 3 of the test. Who did better?
- g. Leroy had a score of 63 on Form 1 of the test and Brandon had a score of 63 on Form 3 of the test. What are the z -scores for Adrian, Julio, Leroy, and Brandon? Rank them according to their z -scores.

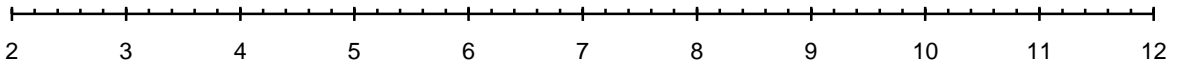
18. Draw box-and-whisker plots on the same number line for the distributions described by the histograms below.



A

B

C



19. Answer the questions based on the three distributions given below.

(a) Which distribution has the largest mean?

A



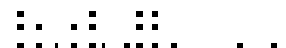
(b) Which distribution has the largest standard deviation?

B



(c) Which distribution has the least variability?

C



(d) For which distribution would the empirical rule (68-95-99.7 rule) most likely apply? Explain why.