
Problem Set for Unit 4: Probability

1. A deck of cards consists of 16 numbered cards as shown. Suppose that one card is selected at random. What is the probability that the card is

- (a) white?
- (b) a multiple of 3?
- (c) white *and* a multiple of 3?
- (d) white *or* a multiple of 3?

1	8	9	16
2	7	10	15
3	6	11	14
4	5	12	13

- (e) If you are told that the card is white, what is the probability that it is a multiple of 3?
- (f) What are the odds in favor of choosing a multiple of 4?
- (g) Give an example of an event that has probability of 0.
- (h) Give an example of an event that has a probability of 1
- (i) Give an example of two events that are mutually exclusive.
- (j) A card is drawn at random and not replaced. What is the probability that the next card drawn is white? Discuss the possibilities.

2. A poll of 3200 consumers was conducted to determine cola preference. Of the 3200 consumers polled 640 preferred cola A and the rest preferred cola B. The poll included 1800 males of which 360 preferred cola A.

(a) Complete the following table:

		GENDER		
		Male	Female	Total
COLA PREFERENCE	Cola A			
	Cola B			
	Total			

(b) Find the probability (as a decimal) that one consumer selected at random from this sample

- i)* is a female.
- ii)* is a female and prefers cola A.
- iii)* is a female given that the person prefers cola A.
- iv)* prefers cola A given that the person is female

(c) Are the events “female” and “prefers cola” independent events? Justify your answer.

(d) Give an example from this situation of mutually exclusive events.

3. A family wants to have five children. To determine the probability that they will have at least four of the same sex perform the following simulation.

Randint(0, 1, 5) where 0 = girl and 1 = boy

- (a) Give an example of a result where the outcome would have at least 4 children of the same sex.
- (b) Run this simulation 20 times and count the number of times a family has at least four children of the same sex.
 of 20 families had at least 4 children of same sex.
- (c) According to your results, what would you estimate the probability to be that a family of five children would have at least four children of the same sex.
- (d) Collect data from classmates (or generate more data yourself) and complete the following cumulative frequency table.

# of simulations	# with at least 4 same sex	Cumulative frequency	Relative Cumulative Frequency
20			
20			
20			
20			
20			
20			
20			
20			
20			
20			
20			

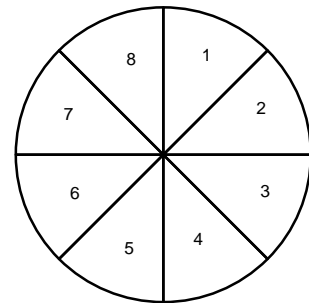
- (e) Use your table to create a line graph of your results. Carefully label your axes.

- (f) What does the line graph of cumulative relative frequencies tell you about the probability of having at least four children of the same sex in a family of five children?

4. There are three red balls and five blue balls in a box. Without looking in the box you randomly draw two balls, one after the other, without replacement.
(8th → p. 569 #20; 7th → p. 553 # 18.)

- (a) What is the sample space for this experiment?
- (b) What is the probability that the first ball drawn is red?
- (c) Given that we know the first ball drawn is red, what is the probability that the second ball drawn is also red?
- (d) What is the probability of drawing a blue ball second if the first ball was red?
- (e) What is the probability of drawing a blue ball second if the first ball was blue?

5. A spinner is spun whose central angles are all 45° .



- (a) What is the probability that it lands on 5?
- (b) What is the probability that it lands on 5, if you know that it lands on an odd number?
- (c) What is the probability that it lands on 5, if you know that it lands on a number greater than 3?
- (d) What is the probability that it lands on 5, if you know that it does not land on 7 or 8?
- (e) What is the probability that it lands on 5, if you know that it lands on a factor of 10?

6. (a) A card is drawn at random from a standard 52-card deck. (8th → p. 569 # 13; 7th → #11 p 556)

- i. What are the odds in favor of drawing a face card?
- ii. What are the odds against drawing a diamond?
- iii. What are the odds in favor of drawing the ace of spades?
- iv. What are the odds against drawing a 2, 3, or 4?

(b) In each part, you are given the probability of event E. Find the odds in favor of event E and the odds against event E? (8th → p. 569 # 15; 7th → #13 p 553)

i. $P(E) = \frac{3}{5}$

ii. $P(E) = \frac{1}{4}$

iii. $P(E) = \frac{5}{6}$

(c) In each part, you are given the following odds in favor of event A. Find $P(A)$. (8th → # 16 p. 569; 7th → # 14 p 553)

- i. 9 : 1
- ii. 2 : 5
- iii. 1 : 3

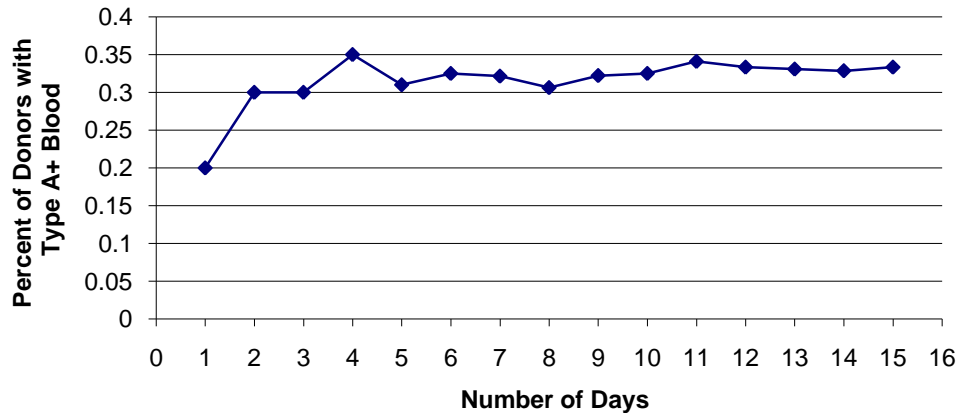
7. An experiment consists of flipping three coins.
- (a) What is the sample space if we record the sequence of flips?
 - (b) What is the sample space if we just record the number of heads that appear?
 - (c) Which of the samples spaces listed above in (a) or (b) is a uniform sample space? Explain.
 - (d) What is the probability that exactly two heads are tossed?
 - (e) What is the probability that at least two heads are tossed?
 - (f) What is the complement of the event “at least one head is tossed”?
 - (g) If I flip three coins and tell you that one of the coins is a head, how does that change the probability that exactly two of the coins are heads?
 - (h) If the first two flips resulted in heads, what is the probability that the third flip is also a head?

8. A group of 100 voters were classified in two ways: party affiliation and attitude toward a certain environmental proposal. The results are shown in the following table.

		<i>Attitude</i>		
		<i>Favor</i>	<i>Indifferent</i>	<i>Opposed</i>
<i>Affiliation</i>	<i>Democrat</i>	27	15	18
	<i>Republican</i>	13	10	17

- (a) What is the probability that the selected member will oppose the proposal?
- (b) What is the probability that the selected member will not favor the proposal?
- (c) What is the probability that the selected member will be Democrat?
- (d) What is the probability that the selected member will be a Democrat *or* one who opposes the proposal?
- (e) What is the probability that the selected member will be a Democrat *and* one who opposes the proposal?
- (f) Given that the selected member is a Democrat, what is the probability the member will be opposed to the proposal?
- (g) Are the events “member is a Democrat” and “member is opposed” mutually exclusive? Explain.
- (h) Are the events “member is a Democrat” and “member is opposed” independent? Explain

9. The following graph shows the **cumulative relative frequency** of A+ donors that come into a blood bank every day for 15 days. The blood bank gets a total of 20 donors per day.



- How many A+ donors were there the first day?
- On average how many A+ donors per day were there after 4 days?
- How many patients are represented in the graph above?
- The cumulative relative frequency graph levels out about where? What does this mean?
- If the blood bank needs ten A+ donors tomorrow, is the bank likely to get them? Explain your answer.
- After a recent emergency the blood bank decides to actively recruit donors. If it needs approximately 50 A+ donors, how many people should they recruit? Explain how you got your answer.
- What mathematical principle does this problem illustrate?

10. Sally has a snack pack of Reese's Pieces. Her pack contains 10 brown candies, 6 yellow candies and 4 orange candies. An experiment consists of choosing one candy from her pack.

(a) What is the sample space for this experiment?

(b) Make a probability distribution function for Sally's pack of Reeses pieces.

(c) Is the sample space a uniform sample space? Explain.

(d) Describe how many more candies and what color Sally would have to add to her pack so that

$$P(\text{Brown}) = \frac{1}{4}$$

$$P(\text{Yellow}) = \frac{1}{4}$$

$$P(\text{Orange}) = \frac{1}{4}$$

11. Two fair six-sided dice are rolled and the sum of the dots on the top faces is recorded.

(a) Complete the table, showing the number of ways each sum can occur

SUM	2	3	4	5	6	7	8	9	10	11	12
WAYS	1	2	3								

(b) Use the table to find the probability of the following events

A: The sum is prime.

B: The sum is a divisor of 12.

C: The sum is a power of 2.

D: The sum is greater than 3.

(c) If you knew one of the die was a 4, does that increase, decrease or not change the probability of the sum being a divisor of 12? Give justification for your answer.

(d) Two dice are thrown (8th → p. 569 # 13; 7th → # 11 p. 553)

i. What are the odds in favor of getting a sum of 7?

ii. What are the odds in favor of getting a sum greater than 3?

iii. What are the odds of getting a sum that is an even number?

iv. What are the odds against getting a sum of 7 or 11?

12. Two four-sided dice are tossed and the **product** of their results is recorded.

(a) Complete the table.

	1	2	3	4
1				
2				
3				
4				

(b) What is the sample space for this experiment?

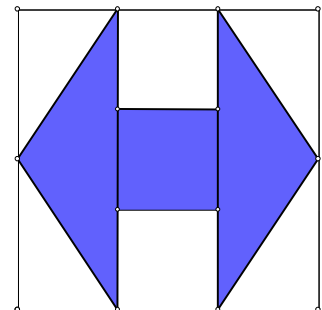
(c) Is this a uniform sample space? Explain.

(d) Find $P(12)$

(e) If the dice were tossed 200 times about how many times would you expect the product to be 12?

(f) Mac designed a game in which two four-sided dice are tossed. Player A wins if the product of the two dice is a multiple of 4 and Player B wins if the product is not a multiple of 4. Is this a fair game? Explain.

13. A dart is randomly thrown hits the dartboard shown. What is the probability that the dart lands on the shaded arrow? Explain what you did to find the probability.



14. When a thumbtack is dropped, it will land point up or point down. This experiment was repeated 60 times with the following results:

Point up: 42 times; Point down: 18 times

- (a) What is the empirical (or experimental) probability that the thumbtack will land point up?
- (b) What is the empirical (or experimental) probability that the thumbtack will land point down?
- (c) Landing “point up” and landing “point down” are
Events because
- (d) If you were to try this experiment another 60 times, would you get the same results? Explain.
- (e) Would you expect to get nearly the same results on a second trial? Why? Use mathematical language and theory in your explanation.

15. A die is made that has two faces marked with 2s, three faces marked with 3s, and one face marked with a 5. The die is thrown once. Find the following probabilities. (see # 13 pg. 515)

- (a) Getting a 2.
- (b) Not getting a 2.
- (c) Getting an odd number.
- (d) Not getting an odd number.
- (e) Write a probability distribution for this experiment.

16. A carnival game consists of tossing one four-sided die and one six-sided die. The **sum** of their faces is recorded. The player wins if the **sum** is a **multiple of three** and loses otherwise?

(a) Is this a fair game? Explain.

	1	2	3	4	5	6
1						
2						
3						
4						

(b) In order to raise money the carnival coordinators decide to charge \$5 to play the game. If a Player **wins** (i.e. *gets a multiple of 3*) the player will receive the dollar amount equal to the sum of the two dice.

If 300 people play this game, how much money can the carnival expect to make?

(c) How much should the Player pay to play this game in order for it to be a fair game?

17. A carnival game cost \$5 to play and consists at throwing a dart at the dartboard with the designated pay offs.

- \$0.50 for hitting a Gray Triangle
- \$1.00 for hitting a White Triangle
- \$10.00 for hitting the Black Square

(a) If 600 people play this game, how much can the carnival expect to make?

(b) What should the game cost if it is to be a “fair game”?