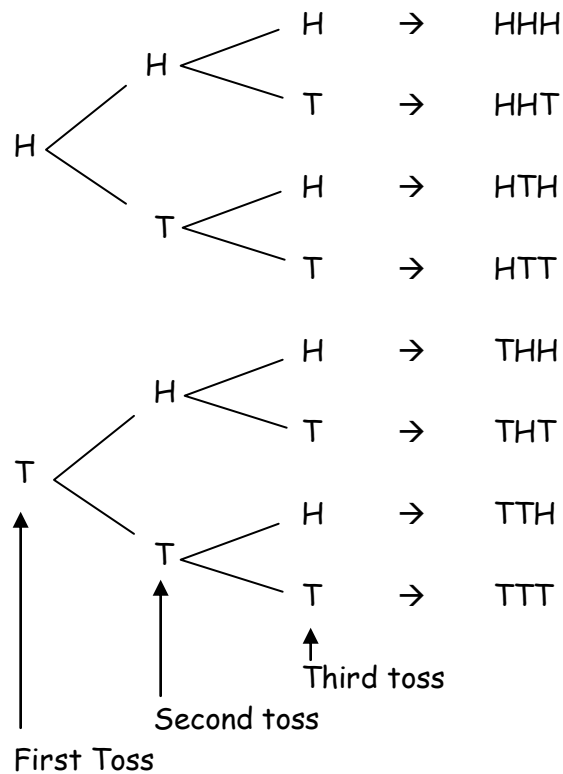


**Binomial Experiment:** A binomial experiment consists of  $n$  trials, where each trial has exactly two possible outcomes. In each trial, the probability for each outcome remains constant. That is each trial is *independent* of the other trials.

***Example:*** Toss a fair coin three times and record the number of heads that occur. There are two outcomes for each toss (heads or tails) and the probability of heads is the same for each toss ( $\frac{1}{2}$ ). A tree diagram can be used to help list all possible outcomes and eventually help to find the probability distribution for this experiment.



Thus the **Binomial Distribution** for this experiment is

Number of Heads	Frequency	Probability
0	1	1/8
1	3	3/8
2	3	3/8
3	1	1/8

**Counting Principle:** The counting principle states that if a first event can occur in  $a$  ways and a second event can occur in  $b$  ways, then the two events can occur together in  $a \cdot b$  ways.

*Note:* The counting principle tells how you can find the number of outcomes when there is more than one way to put things together. If you start with 2 choices and each of these choices leads to 3 choices, then you can put things together in  $2 \times 3$  ways.

*Example 1:* A team shirt comes in 4 colors, and in long or short sleeves. How many choices of shirts are there?

$$\begin{aligned} \# \text{ of choices} &= \# \text{ of colors} \times \# \text{ sleeve lengths} \\ &= 4 \quad \times \quad 2 \\ &= 8 \end{aligned}$$

*Example 2:* Six people are running in the class elections. The person with the most votes will be President. The person who comes in second will be Vice President. How many different teams of President and Vice President can be elected?

$$\begin{aligned} \text{Number of} & & \text{Choices for} & & \text{Choice left for} \\ \text{different pairs} & = & \text{President} & \times & \text{Vice President} \\ & = & 6 & \times & 5 \\ & = & 30 & & \end{aligned}$$

There are only 5 choices left for Vice President because 1 of the 6 people has to be President.

There are 30 different teams that can be elected.

**Factorial:** Take any whole number, call it  $n$ . Multiply all of the counting numbers from 1 through  $n$ . The product is called  $n$  factorial, written  $n!$ .

*Example:* Find the number of easy you can arrange 5 people in a line. To find the number of possible arrangements for a number of items, you find the factorial of the number of items.

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$$

So, 5 people can be arranged in 120 different ways.

**Combination:** A combination is a group of items or events. If you change the order of items or events, you do not change the group, and you do not make a new combination. So, a nickel, a dime, and a quarter is the same combination of coins as a dime, a quarter, and a nickel. Either way, the combination is worth 40¢.

*Example:* Sheila has 4 different shirts. She wants to pack 2 of the 4 shirts for a trip. How many different combinations of 2 shirts can she pack?

Let's say her 4 shirts can be represented by the set {red, pink, blue, green}, then all of the possible combinations of 2 shirts is equivalent to finding all subsets that contain two elements. They are

{red, pink}    {red, blue}    {red, green}  
 {pink, blue}    {pink, green}  
 {blue, green}

Thus there are 6 different possible combinations of 2 shirts.

*Notation:* You can use special notation to show the number of combinations. For the example above, we would write  ${}_4C_2 = 6$  and say "The number of combinations of 4 items taken 2 at a time is 6."

**Pascal's Triangle:** Pascal's Triangle is a special triangular tabulation of numbers. Each row in the triangle corresponds to the frequencies in a binomial probability table for  $n$  trials.

Row 0				1					
Row 1			1	1					
Row 2			1	2	1				
Row 3			1	3	3	1			
Row 4			1	4	6	4	1		
Row 5			1	5	10	10	5	1	
Row 6			1	6	15	20	15	6	1

Blaise Pascal was a French mathematician, philosopher, and scientist who lived from 1623 to 1662. Pascal, together with another French mathematician, Pierre de Fermat (1602 – 1665), studied the theory of probability and explored its applications. Although Pascal's Triangle is named after Blaise Pascal because he studied it, it most likely was invented at least 500 years before that by a Chinese mathematician and by Omar Khay Yam.

Notice that the sum of the numbers in each row is 2 times the sum of the numbers in the previous row. The sum of the numbers in Row  $n$  is  $2^n$ .

**Combinations and Pascal's Triangle:** To use Pascal's Triangle to find the number of combinations of  $n$  items taken  $k$  at a time go to the  $n^{\text{th}}$  row and the  $k^{\text{th}}$  entry in that row (remember that the 1 at the top of the triangle is always counted as row zero and that a 1 on the left hand side of the triangle is always counted as entry zero for that row).

Row 0				1						
Row 1				1	1					
Row 2				1	2	1				
Row 3				1	3	3	1			
Row 4				1	4	6	4	1		
Row 5				1	5	10	10	5	1	
Row 6				1	6	15	20	15	6	1

${}^4C_2$

Example: For finding the number of ways Shelia can pack 2 of her 4 shirts in the example above we want  ${}^4C_2$ , 4 shirts taken 2 at a time  $\rightarrow$  so go to the 4<sup>th</sup> row of Pascal's Triangle and look at the 2<sup>nd</sup> entry (remember start counting at 0).

**Permutation:** A permutation is any arrangement of items or events. If you change the order of the items or events, then you have another permutation.

*Note:* The number of permutations may be found by

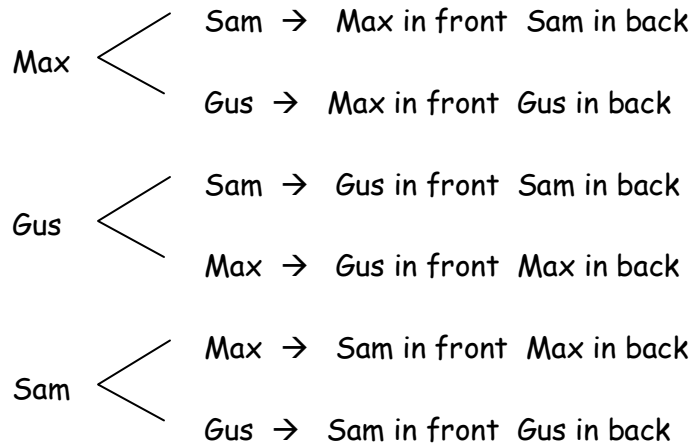
- Making a list
- Drawing a tree diagram
- Using the multiplication principle (counting principle)

**Example 1:** The word *ARE* is a permutation of the word *EAR*. The items are the same (the letters *A*, *E*, and *R*), but the order is different, The six permutations of the letters of the word *ARE* are:

**EAR ARE ERA RAE REA AER**

**Example 2:** Three friends rent and take turns riding a 2-seat bicycle. How many possible seating arrangements are there?

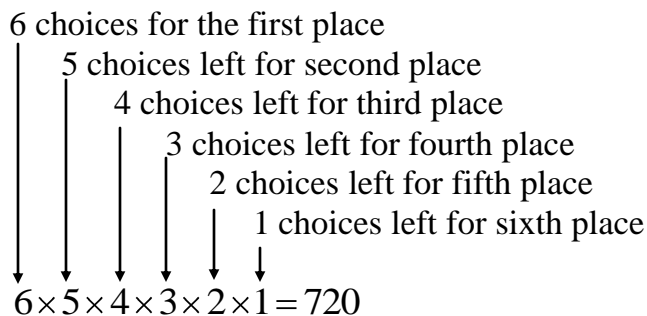
A Tree Diagram can be useful in finding all possible permutations.



So, there are 6 possible permutations for seating arrangements.

**Notation:** You can use special notation to show the number of permutations. For the 2-seat bicycle situations, we would write  ${}_3P_2 = 6$  and say “The number of permutations of 3 items taken 2 at a time is 6.”

**Example 3:** There are 6 runners in a race. How many different permutations are possible for the places in which the runners finish?

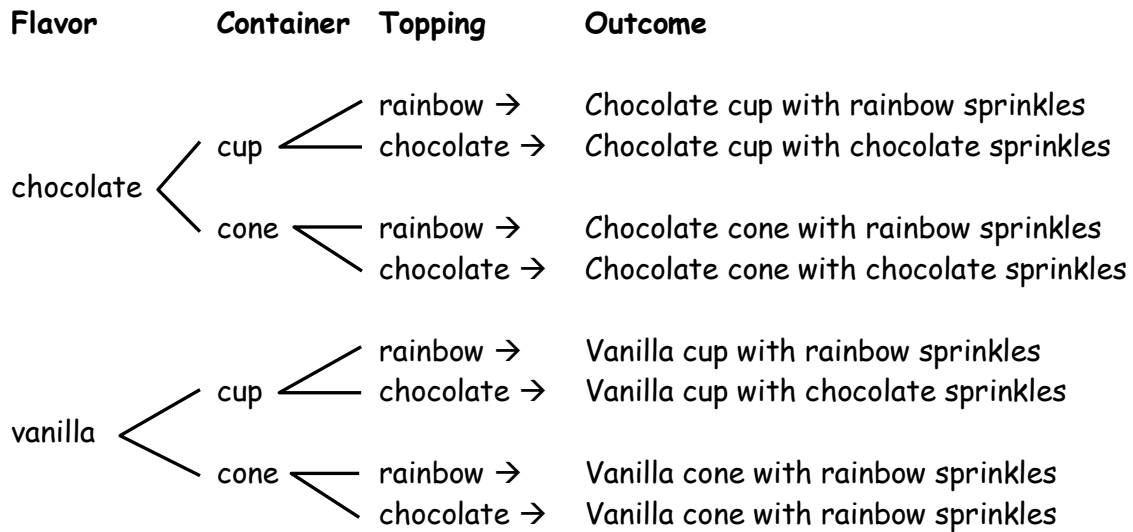


Thus there are 720 possible orders in which the runners finish.

**Tree Diagrams:** You may be able to find all possible outcomes in a sample space by drawing a tree diagram to making an organized list.

Example: An ice cream stand offers two flavors, chocolate and vanilla. You can get either a cup or a cone. You can have rainbow or chocolate sprinkles as a topping. How many possible combinations of flavors, containers, and toppings are there? What are the possible combinations?

Make a tree diagram to show all of the possible combinations (outcomes) in the sample space.



There are 8 possible combinations. The sample space contains 8 outcomes.