Math 80 – Spring 2009
Sample Problems for Midterm III – Chapters 6 – 8

1. (a) Quadratic equations are equations of the form ……. where a, b, and c are real numbers and $a \neq 0$.

   (b) There are many techniques for solving quadratic equations. List three of them.

   ……..

   ……..

   ……..

   (c) The zeroes of a quadratic function are given by the quadratic formula. What is the quadratic formula?

   (d) What is the name of the expression under the radical sign in the quadratic formula?

   (e) Solve the following equation by using the quadratic formula. Round answers to three decimal places.

   $$4 - x = 4(x^2 + 2x) - 10$$

   (f) What is the ***discriminant*** and how is it related to the x-intercepts of a quadratic function?

2. An arrow is shot vertically into the air. The height (in feet) of the arrow after *t* seconds is given by $h(t) = -16t^2 + 112t$.

   (a) How long before the arrow returns to the ground? Show what you did.

   (b) At what time did the arrow reach maximum height? What is the maximum height? Show or explain what you did.

3. Skill Practice. Answer the following expressions algebraically.

   (a) Write the quadratic function in standard form: $y = -4\ x-6\ ^2 + 12$

   (b) Find the zeroes of $y = 12 - 4x^2$.

   (c) Find the *y*-intercept of $y = -4\left|\ 2x - 5\ \right| + 12$

   (d) Write $x^2 + 8x + 4$ in vertex form by completing the square.

   (e) i. Simplify $5 + 2\sqrt{2}\ -\ 3 - 2\sqrt{2}$ ii. Simplify $5 + 2\sqrt{2}\ \ 3 - 2\sqrt{2}$

   (f) Simplify $9x^{-6\ \ -3/2}$.

(g) Write $\sqrt{x}\ \sqrt[3]{x}$ using exponents. Simplify using properties of exponents. Write your result in radical notation.

(h) Simplify $\dfrac{-6+\sqrt{-144}}{3}$

(i) Expand and simplify $1+i^{\,3}$.

(j) Use your calculator to find $\dfrac{3+2i}{3-i}$ in fraction form. …….

Then show how to obtain this result by multiplying both the numerator and the denominator by the conjugate of the denominator and simplify.

(k) True or False: $\sqrt{a^2+b^2}=a+b$. Justify your answer.

(l) True or False: $a+b^{\,2}=a^2+b^2$. Justify your answer.

(m) Simplify the following expressions. Write your final answer without negative exponents.

i. $x^{-3}\cdot x^{-3}$ *ii.* $x^{-3}+x^{-3}$ *iii.* $x^{-3\ -3}$ *iv.* $\dfrac{x^3}{x^{-3}}$

v. $100x^{4\ 1/2}$ *vi.* $x^{2\ -1/2}$ *vii.* $16x^{2\ -1/2}$ *viii.* $16x^{-2\ -1/2}$

ix. $2a^{-1\ -3}$ *x.* $81^{-3/4}$ *xi.* $\dfrac{3x^{-1}}{x}-\frac{1}{2}x^{\,-2}$ *xii.* $\dfrac{a^{5/2}b}{2a^{1/2}b^{\,-1}}$

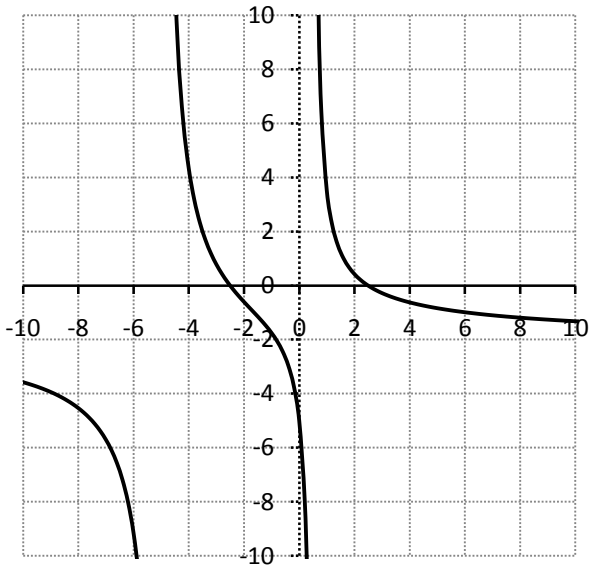
4. Consider the rational function $f(x) = \dfrac{25 - 4x^2}{2x^2 + 9x - 5}$

(a) What is the domain of this function?

(b) Write the equation(s) of the any vertical asymptotes?

(c) Find the *y*-intercept algebraically.

(d) Find the *x*-intercepts algebraically.



5. (a) Perform the indicated operations and simplify: $\dfrac{7w^2 - 14w}{w^2 + 3w - 10} \div \dfrac{21w^3}{w + 5}$

(b) Given $p(x) = \dfrac{2}{x}$. Evaluate and simplify $\dfrac{p(3+h) - p(3)}{h}$.

6. Janet has 35 ounces of liquid that is 20% grape juice. Suppose she adds x ounces 75% grape juice.

   (a) Write a rational function that represents the concentration of grape juice in the final mixture.

   (b) How many ounces of liquid containing 75% grape juice must be added to raise the concentration to 30% grape juice?

7. The game commission introduces some deer into newly acquired state game lands. The population of the herd is given by $N(t) = \frac{10(5+3t)}{1+0.04t}$, $0 \le t$, where t is the time in years since the land was acquired.

   (a) What is the y-intercept and what does it mean in this situation?

   (b) Find $N(12)$ and explain what it means in this scenario.

   (c) How long will it take for the deer population to reach 400? Solve algebraically and check with table or graph.

   (d) In the long run what would you expect the population to be? Explain.

8. Use algebra to solve each of the following equations. Remember to check your answers. Round where appropriate to two decimal places.

(a) $\sqrt[3]{2x-1}+2=7$

(b) $4\left(\frac{x-2}{6}\right)^2=100$

(c) $750\left(1+\frac{0.06}{12}\right)^{12t}=1200$

(d) $\frac{2b}{b-1}-\frac{2}{b^2-1}=\frac{1}{b+1}$

(e) $2\ln(3x-1)+12=5$

(f) $\sqrt{x-3}=6-x$

(g) $3x^2-3=7-13x$

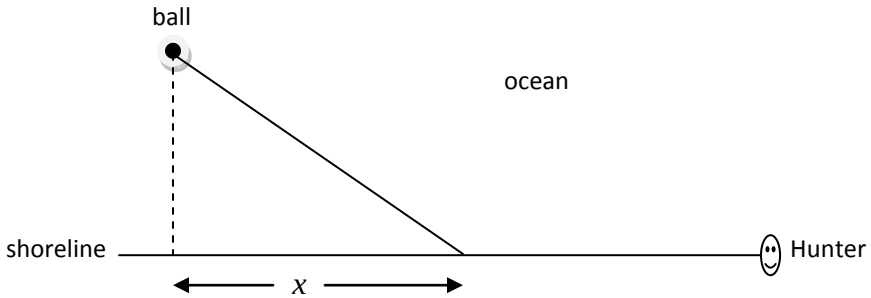
(h) $40{,}000\left(\frac{1}{2}\right)^{x/3}=7500$

(i) $4|5-3x|-10=22$

(j) $2(x-7)^2+9=1$

9. Mark and his dog Hunter are standing together on the shore at the beach. Mark throws a ball into the ocean and Hunter runs along the shoreline until some point in which he enters the water and swims to retrieve the ball. The ball enters the water 200 feet down the shoreline from where they stand and 50 feet from the shore. Hunter can run 8.8 feet per second along the shore but can only swim 3.6 feet per second in the ocean. At what point along the shore should Hunter enter the water in order to minimize his total time?

   Let x represent the distance from where Hunter enters the water to the point on the shore opposite the ball.



   (a) Write an expression in terms of x that represents

   the distance Hunter will run along the shore. ……

   the distance that Hunter swims ……...

   (b) The total time it takes Hunter to reach the ball is given by $T = \frac{d_1}{r_1} + \frac{d_2}{r_2}$

   where d_1 = the distance Hunter runs along the shore, r_1 = the rate at which Hunter runs, d_2 = the distance Hunter swims and r_2 = the rate at which Hunter swims. Write a function of T in terms of x.

   (c) At what point should Hunter enter the water in order to minimize the time it takes him to reach the ball? Document your process for answering this question.

   (d) How much time would it take for Hunter to reach the ball if he followed your suggestion?

10. In 1980, the average price of a house in a certain county was \$80,000. Prices have increased at an average rate of 5% every three years. Let $P(x)$ represent the price of a house in this county x years after 1980.

(a) Complete the table below:

Year	x	$P(x)$
1980	0	
1983		
1986		
1989		
1992		

(b) Write an algebraic representation for $P(x)$ using fractional exponents.

(c) Find the exponential regression model for this data. What does the regression model suggest about the growth rate of housing prices in this time period?

(d) How could you have predicted the regression model from your answer to (b)?

(e) How much will a house sell for in 2000? Document your process.

(f) When will a house be worth \$150,000? Document your process.

11. Miranda bought a truck for $40,000 with an expected half-life of 3 years.

(a) Complete the table below.

Time in years	0	3	6	9	12
Value of Truck in dollars	40,000				

(b) Write an algebraic model for this problem situation using fractional exponents. Define your variables.

(c) Find an exponential regression model for the table above.

(d) What is the annual depreciation of this truck?

(e) Explain why the algebraic model and the regression model are equivalent?

12. The interest formula for compounding *n* times a year is given by $A = P\left(1+\frac{r}{n}\right)^{nt}$.

(a) How much money will Jason accrue in 10 years if he deposits $10,000 in an account paying 6% compounded monthly?

(b) How long will it take $5000 to double in value if it is deposited into a certificate of deposit that pays 4.5% compounded quarterly?

13. (a) Write $Log_9 \ \frac{1}{27} = -\frac{3}{2}$ in exponential form. Then verify the result using what you know about fractional exponents (not your calculator).

(b) Change from exponential to logarithmic for, or vice versa. Solve for the variable.

Exponential Form	Logarithmic Form	Solution for variable
$3^4 = x$	…….	……
$e^x = 10$	…….	……
……..	$t = Log \ 60$	……
…….	$1 = Ln \ 0.5t$	……
…….	$x = Ln \ 5$	……

(c) Use your properties of logarithms to write equivalent expressions for each of the following. Simplify your expression whenever possible. Each equivalent expression will contain either an x or $\log x$ and other constants.

$\log(10x)$ = ……..…

$\log 100^x$ = ……

$\log 0.01^{3x}$ = ……..

$\log \frac{x}{10,000}$ = ……..

$\log \frac{1}{x}$ = ……..

$\log \ 10^x \cdot 1000$ = ……

14. Biologists often model population growth using the number *e* since this is continuous reproduction in populations in normal circumstances. A group of biologist studying chickadee populations in West Texas estimate that currently there are approximately half a million chickadees in the region. Based on past studies, they estimate that the population is growing at a continuous rate of 34.66% each year. Thus the chickadee population can be modeled by the function $y = 500,000e^{.3466x}$, where *x* is the number of years from now and *y* is the Chickadee population.

(a) How long does it take the chickadee population to double?

(b) Make a table that shows the chickadee population for a 5-year period.

Years	0	1	2	3	4	5
Chickadee Population						

(c) What pattern do you notice in the table? Write another model for the population growth based on this pattern.

(d) Explain why the two models you have constructed are equivalent?

15. A bit more skill practice ☺ True / False or Fill in the blank.

a) $\ln 7$ means $\log_e 7$. True False

b) The expression x(x + 4) + 7 is in factored form . True False
$(4-x)(4+x) = 16 - 8x + x^2$ True False

c) x^{-2} , $\frac{x}{x^3}$, and $x^5 x^{-7}$ are equivalent expressions. True False

d) The opposite of 4 – x is x – 4. True False

e) A rational expression is a fraction whose numerator and denominator can be factored.
True False

f) $\frac{\frac{1}{2}}{x}$ is equivalent to $\frac{x}{2}$ True False

g) The product of any complex number and its conjugate is always a real number.
True False

h) What number must be added to both sides of $x^2 - 14x = 20$ to "complete the square"? …….

i) $i^6 = -i$ True False

j) $\log_7(3x^5) = 15\log_7 x$ True False

k) The expressions $\frac{x-3}{4-x}$ and $-\frac{x-3}{x-4}$ are equivalent. True False

l) The x-coordinate of the vertex of the parabola $y = 4x^2 - 12x + 11$ is
……

m) $\log(MN) = \log(M)\log(N)$ True False

n) The expression $\frac{x^7}{y^5}$ can be simplified by subtracting exponents True False

o) The domain of f(x) = $\frac{x-6}{(2x-9)(4x+13)}$ is……..

p) The difference of squares $A^2 - B^2$ can be factored as …….

q) $6^0 = 0$ True False

r) The expression 4^{-3} represents a negative number. True False