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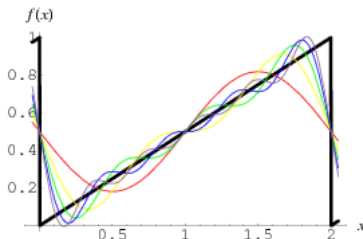
13,594 entries
 Last updated: Tue Sep 27 2016

Created, developed, and nurtured by Eric Weisstein at Wolfram Research

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Fourier Series--Sawtooth Wave

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Consider a string of length $2L$ plucked at the right end and fixed at the left. The functional form of this configuration is

$$f(x) = \frac{x}{2L}. \tag{1}$$

The components of the [Fourier series](#) are therefore given by

$$a_0 = \frac{1}{L} \int_0^{2L} \frac{x}{2L} dx \tag{2}$$

$$= \frac{1}{L} \int_0^{2L} \frac{x}{2L} \cos\left(\frac{n\pi x}{L}\right) dx \tag{3}$$

$$a_n = \frac{1}{L} \int_0^{2L} \frac{x}{2L} \cos\left(\frac{n\pi x}{L}\right) dx \tag{4}$$

$$= \frac{[2n\pi \cos(n\pi) - \sin(n\pi)] \sin(n\pi)}{n^2 \pi^2} \tag{5}$$

$$= 0 \tag{6}$$

$$b_n = \frac{1}{L} \int_0^{2L} \frac{x}{2L} \sin\left(\frac{n\pi x}{L}\right) dx \tag{7}$$

$$= \frac{-2n\pi \cos(2n\pi) + \sin(2n\pi)}{2n^2 \pi^2} \tag{8}$$

$$= -\frac{1}{n\pi}. \tag{9}$$

The Fourier series is therefore given by

$$f(x) = \frac{1}{2} - \frac{1}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi x}{L}\right) \tag{10}$$

$$= \frac{1}{2} + \frac{i}{2\pi} \ln\left(-e^{-i\pi x/L}\right) \tag{11}$$

$$= \frac{1}{2} - \frac{1}{2\pi} \arg\left(-e^{-i\pi x/L}\right). \tag{12}$$

SEE ALSO:

[Fourier Series](#), [Fourier Series--Square Wave](#), [Fourier Series--Triangle Wave](#), [Sawtooth Wave](#)

REFERENCES:

Arfken, G. *Mathematical Methods for Physicists, 3rd ed.* Orlando, FL: Academic Press, pp. 762-763, 1985.

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fourier series--sawtooth wave 


THINGS TO TRY:

- = fourier series--sawtooth wave
- = {25, 35, 10, 17, 29, 14, 21, 31}
- = eigenvectors {{1,0,0},{0,0,1}, {0,1,0}}

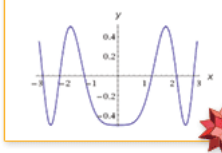
Interactive knowledge apps from Wolfram Demonstrations Project

 **Approximation of Discontinuous Functions by Fourier Series**
 David von Seggern

Wolfram|Alpha Online Integral Calculator

integrate x sin(x^2) 

$$\int x \sin(x^2) dx = -\frac{1}{2} \cos(x^2) + \text{constant}$$



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