Fourier Series--Sawtooth Wave

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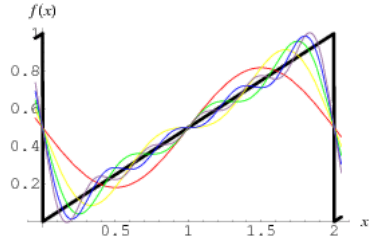

Consider a string of length $2L$ plucked at the right end and fixed at the left. The functional form of this configuration is

$$f(x)=\frac{x}{2L}. \tag{1}$$

The components of the Fourier series are therefore given by

$$a_0=\frac{1}{L}\int_0^{2L}\frac{x}{2L}\,dx \tag{2}$$
$$=1 \tag{3}$$
$$a_n=\frac{1}{L}\int_0^{2L}\frac{x}{2L}\cos\left(\frac{n\pi x}{L}\right)dx \tag{4}$$
$$=\frac{[2n\pi\cos(n\pi)-\sin(n\pi)]\sin(n\pi)}{n^2\pi^2} \tag{5}$$
$$=0 \tag{6}$$
$$b_n=\frac{1}{L}\int_0^{2L}\frac{x}{2L}\sin\left(\frac{n\pi x}{L}\right)dx \tag{7}$$
$$=\frac{-2n\pi\cos(2n\pi)+\sin(2n\pi)}{2n^2\pi^2} \tag{8}$$
$$=-\frac{1}{n\pi}. \tag{9}$$

The Fourier series is therefore given by

$$f(x)=\frac{1}{2}-\frac{1}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi x}{L}\right) \tag{10}$$
$$=\frac{1}{2}+\frac{i}{2\pi}\ln\left(-e^{-i\pi x/L}\right) \tag{11}$$
$$=\frac{1}{2}-\frac{1}{2\pi}\arg\left(-e^{-i\pi x/L}\right). \tag{12}$$

SEE ALSO:
Fourier Series, Fourier Series--Square Wave, Fourier Series--Triangle Wave, Sawtooth Wave

REFERENCES:
Arfken, G. *Mathematical Methods for Physicists, 3rd ed.* Orlando, FL: Academic Press, pp. 762-763, 1985.

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