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# Fourier Series--Square Wave

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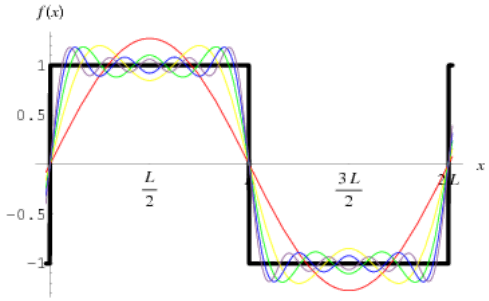


Consider a square wave $f(x)$ of length $2L$. Over the range $[0, 2L]$, this can be written as

$$f(x) = 2[H(x/L) - H(x/L - 1)] - 1, \tag{1}$$

where $H(x)$ is the Heaviside step function. Since $f(x) = f(2L - x)$, the function is odd, so $a_0 = a_n = 0$, and

$$b_n = \frac{1}{L}\int_0^{2L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx \tag{2}$$

reduces to

$$b_n = \frac{2}{L}\int_0^{L} f(x)\sin\left(\frac{n\pi x}{L}\right)dx \tag{3}$$

$$= \frac{4}{n\pi}\sin^2\left(\tfrac{1}{2}n\pi\right) \tag{4}$$

$$= \frac{2}{n\pi}[1 - (-1)^n] \tag{5}$$

$$= \frac{4}{n\pi}\begin{cases} 0 & n \text{ even} \\ 1 & n \text{ odd.} \end{cases} \tag{6}$$

The Fourier series is therefore

$$f(x) = \frac{4}{\pi}\sum_{n=1,3,5,\ldots}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi x}{L}\right). \tag{7}$$

**SEE ALSO:**
Fourier Series, Fourier Series--Sawtooth Wave, Fourier Series--Triangle Wave, Gibbs Phenomenon, Square Wave

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