

- Algebra
- Applied Mathematics
- Calculus and Analysis
- Discrete Mathematics
- Foundations of Mathematics
- Geometry
- History and Terminology
- Number Theory
- Probability and Statistics
- Recreational Mathematics
- Topology
- Alphabetical Index
- Interactive Entries
- Random Entry
- New in MathWorld
- MathWorld Classroom
- About MathWorld
- Contribute to MathWorld
- Send a Message to the Team
- MathWorld Book

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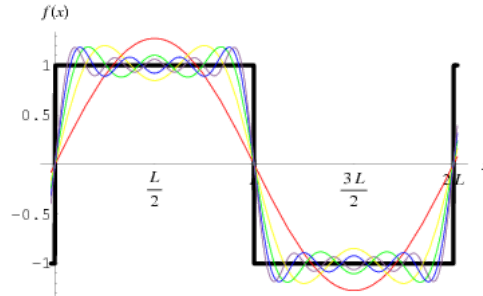
13,594 entries
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Created, developed, and nurtured by Eric Weisstein at Wolfram Research

Calculus and Analysis > Series > Fourier Series >

Fourier Series--Square Wave

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Consider a square wave $f(x)$ of length $2L$. Over the range $[0, 2L]$, this can be written as

$$f(x) = 2[H(x/L) - H(x/L - 1)] - 1, \tag{1}$$

where $H(x)$ is the Heaviside step function. Since $f(x) = f(2L - x)$, the function is odd, so $a_0 = a_n = 0$, and

$$b_n = \frac{1}{L} \int_0^{2L} f(x) \sin\left(\frac{n\pi x}{L}\right) dx \tag{2}$$

reduces to

$$b_n = \frac{2}{L} \int_0^L f(x) \sin\left(\frac{n\pi x}{L}\right) dx \tag{3}$$

$$= \frac{4}{n\pi} \sin^2\left(\frac{1}{2}n\pi\right) \tag{4}$$

$$= \frac{2}{n\pi} [1 - (-1)^n] \tag{5}$$

$$= \frac{4}{n\pi} \begin{cases} 0 & n \text{ even} \\ 1 & n \text{ odd.} \end{cases} \tag{6}$$

The Fourier series is therefore

$$f(x) = \frac{4}{\pi} \sum_{n=1,3,5,\dots}^{\infty} \frac{1}{n} \sin\left(\frac{n\pi x}{L}\right). \tag{7}$$

SEE ALSO:

[Fourier Series](#), [Fourier Series--Sawtooth Wave](#), [Fourier Series--Triangle Wave](#), [Gibbs Phenomenon](#), [Square Wave](#)

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Fourier series square wave (2π)

THINGS TO TRY:

- = Fourier series square wave (2π)
- = representations square wave(x)
- = sum_(k=0)^infinity sin(2(1+2 k) pi x)/(1+2 k)



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