Conservation of Momentum and Impulse

Introduction: In any event that involves moving objects, a quantity called *momentum* must be conserved. By this we mean that in a closed system, all the momentum present before the event must be accounted for after the event. In this lab we will examine this conservation and its close relative, impulse.

Theory: Isaac Newton was instrumental in defining momentum. Recall his Third Law of Motion: "For every action there is an equal and opposite reaction." Consider a collision event such as a moving object striking and rebounding off a static wall. If no permanent deformation occurs during this event it is called an *elastic* collision. Both momentum **P** and *kinetic* energy K must be conserved. These quantities are defined below:

$$\vec{P} = m\vec{v} \qquad K = \tfrac{1}{2}mv^2$$

Equations 1 and 2

Notice that momentum is a vector (direction matters) while kinetic energy is not. This provides complication, but in complication lies opportunity.

Leaving K alone for a while, consider how momentum can change over time. Mass can vary as in a rocket, or velocity can vary. Invoking Newton again: "An object in constant motion in a straight line, or at rest, stays in constant motion in a straight line unless acted upon by an **external** force." Therefore, if momentum changes (but mass doesn't), an external force must be the culprit. This is easy to see:

$$\frac{d\vec{P}}{dt} = m\frac{d\vec{v}}{dt} \quad \text{which yields} \quad d\vec{P} = \vec{F}dt$$

Equations 3 and 4

The quantity in the right box is called impulse, and you can easily see that for a given force, a change in momentum depends on how long the force is applied.

Back to our definition of an elastic collision. A collision is elastic if there is no permanent deformation. This definition does not preclude temporary deformation. A rubber ball, when colliding with a wall, changes shape, but in the process of deforming it temporarily stores its kinetic energy of motion in its molecular structure. We call stored energy *potential* energy, U. More importantly, the force that causes the deformation is not constant. It may follow roughly Hooke's Law, in which case the plot of F(t) during this event looks something like this:

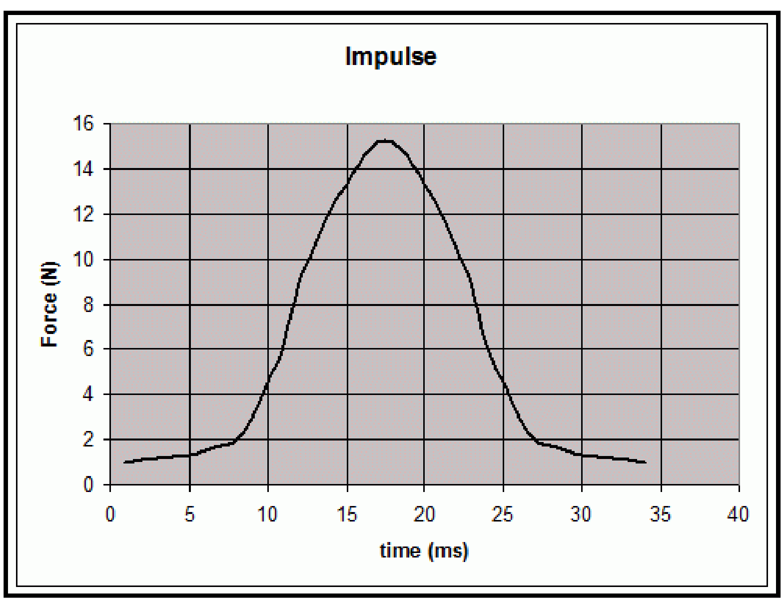

Figure 1

This is a typical shape. It easily could be thinner or broader. (If it was actually a Hooke's law plot, what would it look like? Hint: very similar.) Significantly, if one had a graph such as this, and could integrate the area under the curve, one could find the change in momentum of a bouncing rubber ball or during the elastic collision of a moving object with a stationary one. The above curve is not a well defined function, but numerical methods would serve adequately. One could estimate the impulse to be approximately 18 blocks of 2N X 5ms, or 0.18Ns. If the object has a mass of, say 1kg, the change in velocity would be 0.18 m/s. One could also calculate the initial and recoil (final) speeds to be 0.09 m/s. Can you do this calculation?

And just where would you obtain such a graph? In an airtrack cart collision with a force transducer, of course! With the equipment described later you can measure in incident and recoil speeds of the cart and see if your impulse calculations agree. Finding the same value two different ways in an experiment is a cornerstone of the scientific method.

However, this is *not* a closed system, and the subject of this lab, *conservation* of momentum, cannot be established. Why? Because the external force applied by the transducer to change the momentum of the cart must have its own reaction force, namely the friction between the track itself. This in turn finds that the friction between the track and tabletop produces another action/reaction pair, and thus the system is not closed.

Not to worry. Using the same apparatus we can have two carts collide elastically to show that momentum is conserved in a closed system. Suppose one cart of mass m_1 is moving toward the right with a speed of v_1. It impacts a stationary cart of mass m_2, one that has zero momentum. The equation describing this is:

$$m_1 v_1 + m_2 0 = m_1 v'_1 + m_2 v'_2$$

Equation 5

Primes denote factors after the event has occurred, not spatial derivatives. Recall that momentum is a vector by virtue of its velocity factor. Therefore the directions of the two carts after the collision are dependent on their masses, i.e. if a very light cart smacks a heavy one, the light cart will recoil backwards from the collision. If the situation is reversed, both will head off in the same direction after the event. And of course, kinetic energy is conserved: K = K'.

Now consider the situation where both carts are moving initially towards each other:

$$m_1 v_1 + m_2 v_2 = m_1 v'_1 + m_2 v'_2$$

Equation 6

This time, not only does mass matter, but so does initial speed. Again, kinetic energy is conserved. Finally, consider the circumstance where the two carts stick together. This is called a perfectly inelastic collision.

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) v'$$

Equation 7

Notice how the math graphically represents the event: the two masses are now one, appropriately bound up in a parenthesis. Here, kinetic energy is NOT conserved, since some was lost in the sticking process.

Task:

- To verify the initial and final speeds of an elastic collision with a cart and a force transducer via speed measurements and an impulse graph
- To verify that momentum and kinetic energy are both conserved in elastic collisions between two aircarts
- To verify that momentum is conserved but kinetic energy is not in a perfectly inelastic collision between two aircarts.

Equipment:

- Pasco Capstone Interface (PCI)
- Photogates
- Force transducer
- airtrack
- aircarts

- various clamps and rods

Procedure:

Equipment Setup:

1. Carefully place the airtrack on your table. It will most likely hang over the edge, so be careful as you walk around the lab.
2. NEVER slide the aircarts on the track with the blower off!
3. Put two photogates in their stands.
4. At the end of the airtrack away from the blower build an inverted "J" to hold the force sensor using a table clamp, long rod, two right angle clamps and two short rods. This in and of itself is an engineering aptitude test. Mount the sensor horizontally with the "tare" button up, hook at the cart's spring height
5. Position a Photogate about 30cm from the Force Sensor so that it bridges the track and is blocked when the cart passes underneath (the red LED lights).
6. Plug the Force Sensor into the first DIN jack and the Photogate into the first 1/4" phone jack.

Pasco Data Studio Setup:

1. After logging onto the student account, open the PCI.
2. The program should open with the 850 interface. If it doesn't, check to make sure you've turned the interface and it is plugged into its own port. You can then let the PCI search for the interface. If there are still problems politely ask the instructor for assistance.
3. Hardware: the following sensors must be set up:
    a. Force sensor:
        i. At the bottom center of the screen choose a sample rate of 1000 Hz (1kHz).
        ii. 

        iii. On the sensor itself press the tare button.
    b. Photogate: Start with a single Photogate in input "1"--later on you will add another and delete the Force Sensor.
    c. Recall the Conservation of Energy and Efficiency lab. As you did then with the photogates, choose Blocked then choose Unblocked. This will start the timing when the "fin" of the aircart blocks the LED and stop it when the it exits.
    d. In each case below, knowing the length of the cart "fin" and the time the photogate is blocked, the speed of the aircart can be calculated.

Impulse

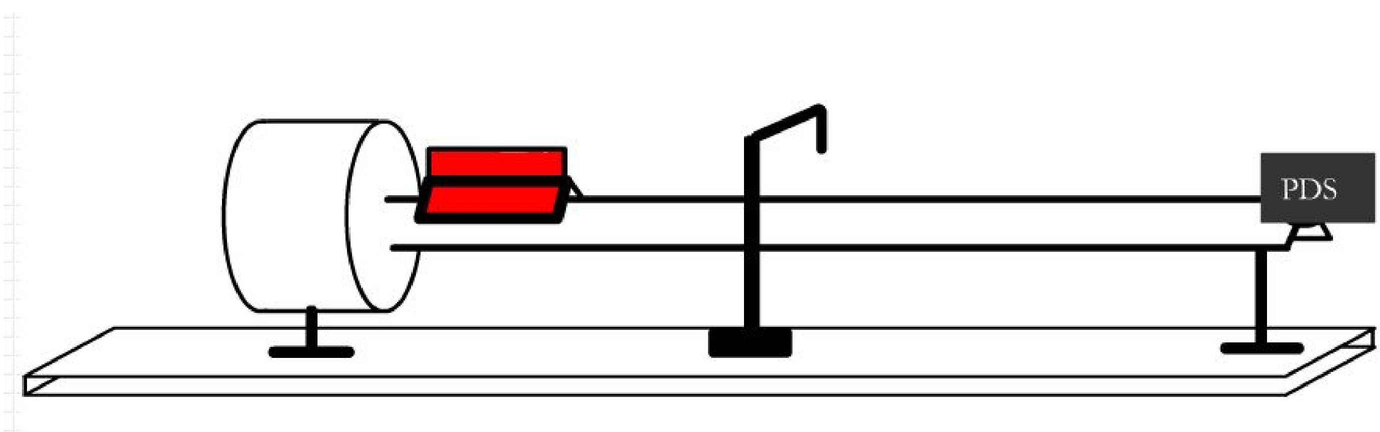

Figure 2

1. From the right-hand Display menu choose TABLE, and select Timer 1 for display.
2. Choose GRAPH and select Force for display.
3. You are ready for a run. Click the big RECORD button, and with the air on push the lighter cart toward the sensor, medium speed. After the collision and the recoil through the Photogate hit STOP and turn off the air.
4. You'll have two time intervals, and cart length. Record the two elapsed times in the table (right-hand column). From this it's easy to calculate speed.
5. Drag the graph and look at the pulse. Should see a nice pulse as in Figure 1. Drag the x-axis on the graph to zoom in to the area of interest and print it (for counting squares and find the domain of the impact). Also

export this data for your report. Later, to find the area, in Excel sum the delta T's times the Force value: very nice Simmons rectangles! Use the printed graph to zero in on where the pulse occurs.

Stationary Impact

If a lighter mass hits a heavier one, which direction is its final velocity compared with its initial velocity?. The other two scenarios will yield different results.

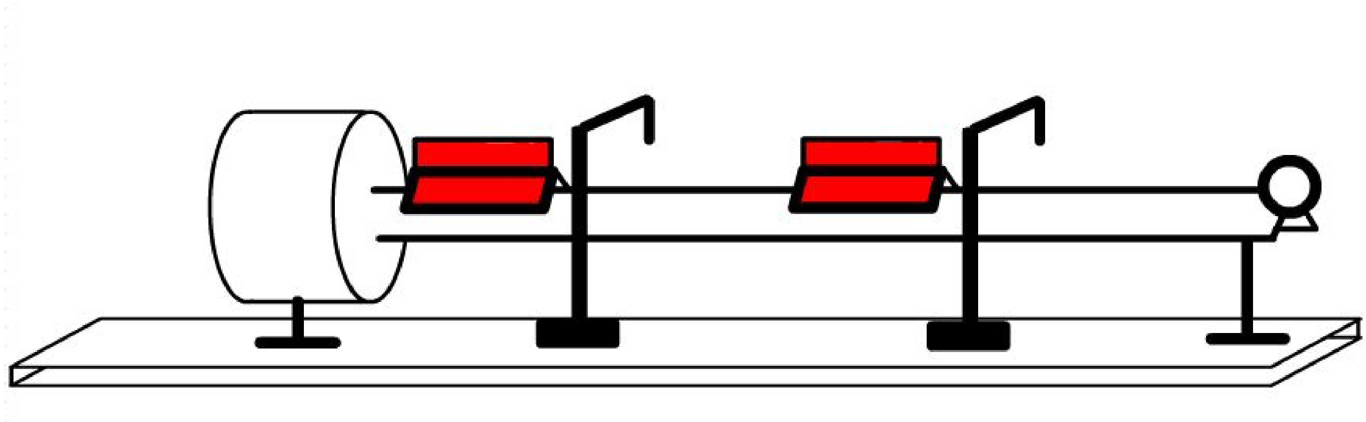

Figure 3

1. First, you need to remove the Force Sensor from the display and add another Photogate. Plug the second Photogate into input "2".
2. Go to Hardware and add the new photogate. Set this up like the other: blocked and unblocked.
3. Be sure the springs are both either "loops" or "hooks" so that they don't stick together.
4. Choose two TABLE displays for the two timers.
5. Place one aircart between the Photogates, stationary, hit START and push the other (of roughly equal mass) towards it. It must go through Photogate 1 before it collides with the stationary cart.
6. Turn off the air after you have only two elapsed times. You need to record one elapsed time for the incoming cart and one for the impacted cart.
7. Repeat with significant additional mass on the impacted cart. Be aware of which photogate is recording which time: this is the scene depicted above, and you will get two times for the impacting cart and only one for the impacted cart.
8. Repeat with significant additional mass on the impacting cart. Be aware of which photogate is recording which time. Here there will be one time for the impacting cart through the first photogate and then another for it when it passes through the second gate. There should be only one time for the impacted cart: don't allow it to rebound!
9. After recording the elapsed times, delete ALL data runs between experiments (or save them to thumb drive).

Moving Impact

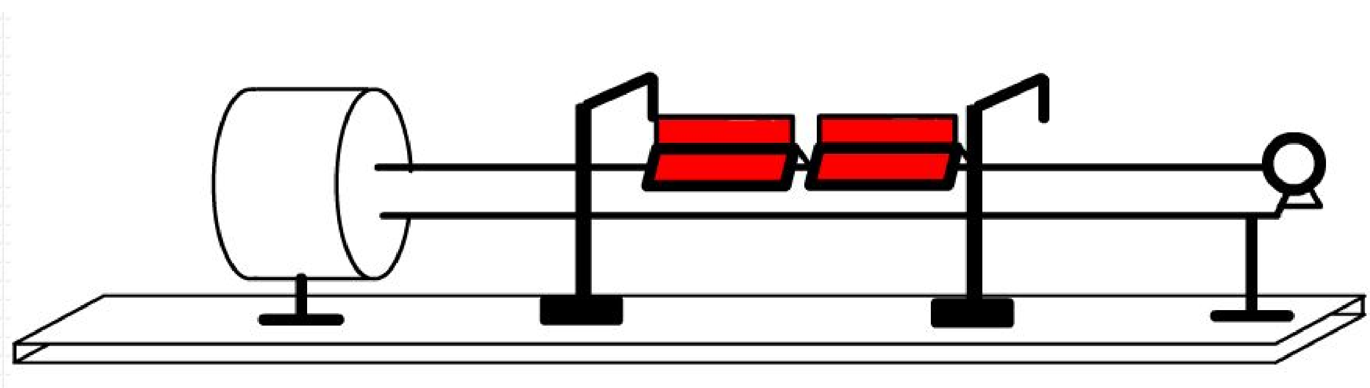

Figure 4

1. Repeat the procedure for the Stationary Impact, but this time initially push both carts towards each other.
2. Both carts will have the same mass.
3. Both carts will have two elapsed time intervals each, one for before the collision and one after.

Perfectly Inelastic Impact

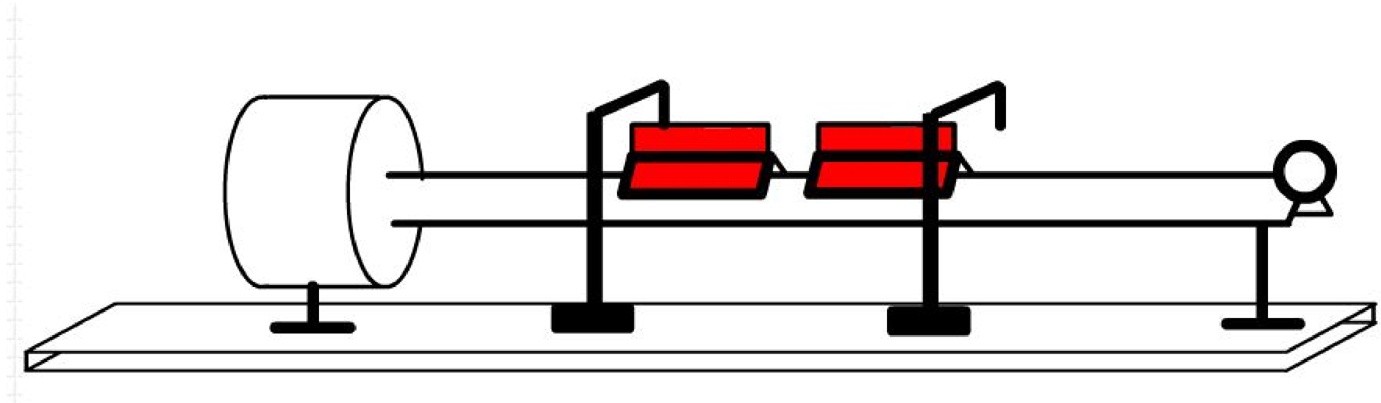

Figure 5

1. In this case turn one of the carts around so that hooks face loops.
2. The two carts should have equal masses.
3. Have one cart initially stationary between the photogates, letting the other impact it.
4. Notice how the combined carts, as they pass through the right-hand photogate, causes a blink. That is because there is a gap between the two carts. Assuming they are the same length, the time recorded by the PCI should be very close.

Analysis

Summarize the following results (1-4) in this clear, concise table, filling in the red cells.

quantity	**Del v from kinematics**	**Del v from impulse**	**Total Momentum**			**Total Kinetic Energy**		
Impulse			**Before**	**After**	**% Change**	**Before**	**After**	**% Change**
Stationary Impact								
Equal Masses								
Lighter impactor								
Heavier Impactor								
Moving Impact								
Perfectly Inelastic Impact								

1. Impulse: Show that the area under the F(t) curve, when divided by the mass of the cart, is its change in velocity. Use the imported data in an Excel file for this calculation. The printed graph from the PDS can help zero in on exactly when the impact occurred.
2. Stationary Impact: Show that energy and momentum are conserved in each instance.
3. Moving Impact: Show that energy and momentum are conserved.
4. Perfectly Inelastic Impact: Show that momentum is conserved and kinetic energy is *not*.
5. Algebraically verify the following statement from the theory section: "... the directions of the two carts after the collision are dependent on their masses, i.e. if a very light cart smacks a [stationary] heavy one, the light cart will recoil backwards from the collision. If the situation is reversed, both will head off in the same direction after the event."

Question:

Which quantity varies more, percentage-wise, for a single mass undergoing a collision, kinetic energy or momentum? Justify your answer.