Measurement and Uncertainty Review

No measuring instrument is perfect: any measuring tool will have two limitations: accuracy and precision.

Accuracy is how close to defined standards a tool can be. For instance if an odometer says this distance: |---------------| is a mile, well, it obviously isn't. While it isn't necessary for us to confirm that every instrument provided by the school conforms with the National Bureau of Standards, it's something you should think about when given equipment in your future career.

Precision is a measure of the finest gradation on the instrument's measuring scale. If a meter stick is marked only in meters, it is not as precise as one marked in centimeters, which in turn is not as precise as one marked in millimeters. Obviously the more precise the tool is the better the measurement, down to dimensions where quantum uncertainty is a factor. However, the finest gradation reveals an inherent weakness: **your measurement can be no better than one-half the smallest gradation**. This is a fundamental truth about measurement, and **this course requires an explicit statement of uncertainty for every result**.

How to do this? As a simple example, let's say you are determining the area of a table top. You measure with a meterstick marked down to the millimeter scale and find the table top is 89.4 cm by 159.8 cm. You would obtain, ignoring significant figures for the moment, 14,286.12 cm^2. The worst case (and you are always looking for the worst case) would be if your measurements were 0.5mm off. The table may actually be 89.45 cm by 159.85 cm for an area of 14,298.5825 cm^2. The **uncertainty** of your **calculation** would be 12.4625 cm^2, or about 0.87%. (N.B. this has nothing to do with percent difference or percent error.) With such a small uncertainty you should be very confident in your determination, easily lost when the proper number of significant figures (i.e. 3, limited by the three places in 89.4 cm) is used.

But suppose your meterstick was only marked in meters; you could at best say the table was 100 cm by 200 cm with an uncertainty in measurement of 50 cm. You assume the worst, that each measurement is low by 50 cm. Your area would be 20,000 cm^2 with an uncertainty of calculation of 17,500 cm^2! (Be sure you can replicate this uncertainty mathematically.) If you are thinking that, "heck, I could obviously *see* that the table isn't 1.5m by 2.5m". True, and suppose you are measuring the wavelength of light, down in the 500 nm range – you won't be able to "*see*" the obvious down there.

We are illustrating the principle; don't get lost in the weeds here. For every result you provide this semester, you must qualify it by an uncertainty, either +/- a value or a per cent. You can either maximize the measurement value by the uncertainty in measurement or minimize it, whatever makes the worst case scenario. Here's another example: supposed you are measuring a disk to try to determine pi. With the "precise" meterstick above, you measure the diameter of the disk to be 7.3 cm and the circumference (by carefully rolling the disk along the meterstick) to be 22.9 cm. Pi would be 3.13698630137 (again, silly with so many digits). With measurement uncertainty, you'd use 22.95 cm and 7.25 cm. Why? Because you want to maximize the result to find the worst case. The uncertainty in your result would be 0.0285309400094 (when will the over-digititus end?)