3.5 DOUBLE-ANGLE AND HALF-ANGLE FORMULAS

DOUBLE ANGLE FORMULAS

By substituting $\alpha = \beta = \theta$ into the $\sin(\alpha+\beta)$, $\cos(\alpha+\beta)$ and $\tan(\alpha+\beta)$ formulas we get:

$\sin(2\theta) = 2\sin\theta\cos\theta$

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

If we use the identity that $\cos^2\theta + \sin^2\theta = 1$, then we can substitute $\cos^2\theta = 1 - \sin^2\theta$ into the $\cos(2\theta)$ formula for another variation:

$\cos(2\theta)$ $= (1 - \sin^2\theta) - \sin^2\theta =$ **$1 - 2\sin^2\theta$**

Substituting $\sin^2\theta = 1 - \cos^2\theta$ gives the following:

$\cos(2\theta)$ $= \cos^2\theta - (1 - \cos^2\theta) =$ **$2\cos^2\theta - 1$**

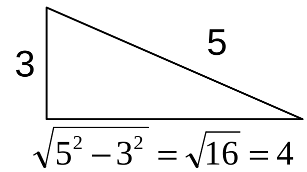
$$\tan(2\theta) = \frac{2\tan q}{1 - \tan^2 q}$$

Example 1

If $\sin\theta = 3/5$, and $\pi/2 < \theta < \pi$, find the exact value of:

a) $\sin(2\theta)$

θ is in Quadrant II so $\cos\theta$ is < 0.



We know $\sin\theta$, and from the right triangle and the fact that θ is in Quadrant II, $\cos\theta = -4/5$

$\sin(2\theta) = 2\sin\theta\cos\theta = 2(3/5)(-4/5) = -24/25$

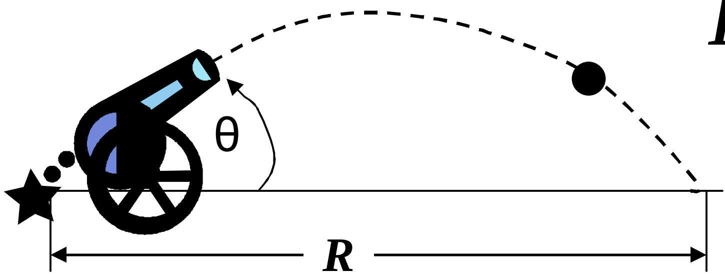
b) $\cos(2\theta)$ $= 1 - 2\sin^2\theta$

$= 1 - 2(3/5)^2 = 1 - 2(9/25) = 1 - 18/25 = 7/25$

Now do # 12 a *and* b

Example 4

An object is propelled upward at an angle θ to the horizontal with an initial velocity of $\boldsymbol{v_0}$ feet per second. If air resistance is ignored, the range, R, this horizontal distance that the object travels, is given by



$$R = \frac{1}{16} v_0^2 \sin q \cos q$$

A) Show that $R = \frac{1}{32} v_0^2 \sin\left(2q\right)$

B) Find the angle θ for which R is a maximum

a) Use double angle formula -> sin(2θ) = 2 sin θ cos θ

Divide both sides by 2

½ sin(2θ) = sin θ cos θ

Substitute this into the Range formula

$$R = \frac{1}{16} v_0^2 \frac{1}{2} \sin(2q) = \frac{1}{32} v_0^2 \sin(2q)$$

b) The maximum Range will occur at the max sin value which is

1. For what angle θ will sin(2θ) = 1?

   $\sin^{-1}(\sin 2\theta) = \sin^{-1}(1)$

   $2\theta = \pi/2_{\text{ rad}} = 90°$

   θ = 45° will give a maximum range of $R = \frac{1}{32} v_0^2 (1)$

Variations of the Double Angle Formula

$$\sin^2 q = \frac{1-\cos(2q)}{2}$$

$$\cos^2 q = \frac{1+\cos(2q)}{2}$$

$$\tan^2 q = \frac{1-\cos(2q)}{1+\cos(2q)}$$

HALF-ANGLE FORMULAS

$$\sin\frac{a}{2} = \pm\sqrt{\frac{1-\cos a}{2}}$$

$$\cos\frac{a}{2} = \pm\sqrt{\frac{1+\cos a}{2}}$$

$$\tan\frac{a}{2} = \pm\sqrt{\frac{1-\cos a}{1+\cos a}}$$

We need to know which Quadrant $\alpha/2$ is in order to choose the + or – answer.

We can use these formulas to find the exact value of trig functions of angles that when doubled become a more familiar angle from our Unit Circle.

Examples 5 and 6 will be done in class.

HOMEWORK for 3.5

p.262 #7-27 EOO #57-83 EOO