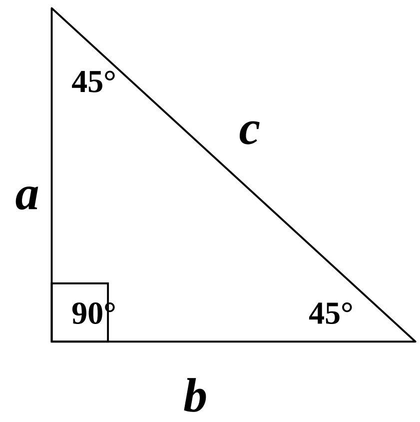
2.3 Computing the Values of Trig Functions of Given Angles



Functions of π/4 = 45°

If one acute angle of a right triangle is 45°, then the other acute angle must also be 45°, since all the angles of a triangle must add up to 180° (45°+ 45° + 90°= 180°). This makes it an isosceles right triangle with the 2 legs being of equal length (*a*=*b*)

Since trig functions are defined by ratios of lengths and not the actual lengths themselves, we can choose any number for ***a*** to calculate the trig functions for π/4 radians. We'll let ***a***=***b***=***1***.
From the Pythagorean Theorem, the hypotenuse c =

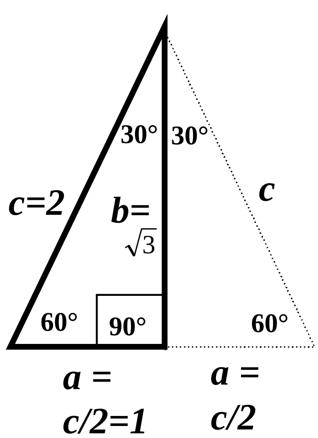
$$\sqrt{a^2+a^2}=\sqrt{1^2+1^2}=\sqrt{2}$$

Now that we know all three sides we can easily calculate all the trig functions.

Trig Fctn	Definition	Exact Value
sin(π/4)	*b/c=**opposite/hyp**otenuse*	*b/c*= $\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$
cos(π/4)	*a/c=**adjacent/hyp**otenuse*	*a/c*= $\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$
tan(π/4)	*b/a=**opposite/adj**acent*	*b/a*= ***1/1 = 1***
csc(π/4)	*c/b=hypotenuse/opposite*	*c/b*= $\frac{\sqrt{2}}{1}=\sqrt{2}$
sec(π/4)	*c/a=hypotenuse/adjacent*	*c/a*= $\frac{\sqrt{2}}{1}=\sqrt{2}$
cot(π/4)	*a/b=adjacent/opposite*	*a/b*= ***1/1 = 1***

Functions of π/3 = 60° and π/6 = 30°

As before, we can choose any length for a side and derive more info from there. Since 30° is half of 60°, then one of the legs is half the hypotenuse, so we'll choose c =2, so *a* = c/2 =1.



$$b=\sqrt{c^2-a^2}=\sqrt{2^2-1^2}=\sqrt{3}$$

Trig Fctn	Definition	Exact Value
sin(60°) =cos(*30°*)	*b/c=**opposite/hy**potenuse*	*b/c*= $\frac{\sqrt{3}}{2}$
cos(60°) =sin(*30°*)	*a/c=**adjacent/h**ypotenuse*	*a/c*= ½
tan(60°) =cot(*30°*)	*b/a=**opposite/a**djacent*	*b/a*= $\frac{\sqrt{3}}{1}=\sqrt{3}$
csc(60°) =sec(*30°*)	*c/b=hypotenuse /opposite*	*c/b*= $\frac{2}{\sqrt{3}}=\frac{2\sqrt{3}}{3}$
sec(60°) =csc(*30°*)	*c/a=hypotenuse /adjacent*	*c/a= 2/1 = 2*
cot(60°) =tan(*30°*)	*a/b=adjacent/opposite*	*a/b*= $\frac{1}{\sqrt{3}}=\frac{\sqrt{3}}{3}$

Finding Exact Value of Trig Functions

Example 4 on p.143

Find the exact value of

a) (sin 45°)(cos 30°) = $\frac{\sqrt{2}}{2} \bullet \frac{\sqrt{3}}{2} = \frac{\sqrt{6}}{4}$

b) tan π/4 - sin π/3 = $\frac{1}{3} + \frac{1}{2} = \frac{2}{6} + \frac{3}{6} = \frac{5}{6}$

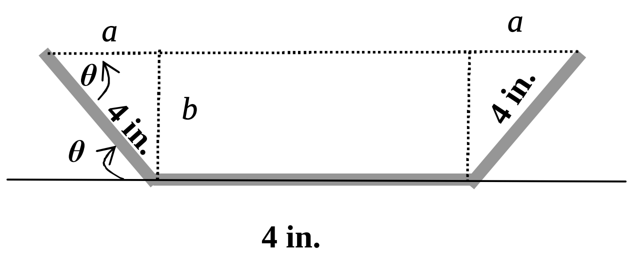
c) $\tan^2 \pi/6 + \sin^2 \pi/4$ = $1 - \frac{\sqrt{3}}{2} = \frac{2}{2} - \frac{\sqrt{3}}{2} = \frac{2-\sqrt{3}}{2}$

Now you do #19 on p. 149

Example 5 p. 144 Sometimes you have to use a calculator.

a) Find cos 48° b) Find csc 21° c) Find tan π/12

Example 6 on p.145
Constructing a Rain Gutter



A 12" wide aluminum sheet is bent up 4" from each end to make a rain gutter.
A) Express the area of the rain gutter opening as a function, $A(\theta)$, where θ is the angle that the sheet is bent up.
B) Find the area for $\theta = 30°, 45°, 60°$ and $75°$

A) Express Area as a Function of θ . Area = $A(\theta)$

Total Area = area of the 2 right triangles + middle rectangle.
We'll let ***a*** = base of triangles, ***b***=height of the triangles and the rectangle.
Area = ½****a*****b + ½****a********b*** + 4****b***.

length of rectangle, height of rectangle

If we draw a line from one end of the rain gutter to the other, we have 2 parallel lines: the base of the rain gutter, and the top. From geometry, we know that if a line intersects 2 parallel lines, then the angles from the parallel lines to the intersecting line are the same.

We only know the lengths of 1 side of each triangle, and the length of the rectangle. How do we get the rest?

We are told to put everything in terms of θ. Notice there is a relationship between ***b*** and θ, and ***a and*** θ.

$\sin(\theta) = b/4 \qquad \cos(\theta) = a/4$

Cross multiplying gives $4\sin(\theta) = b$ and $4\cos(\theta) = a$

We can now use this for ***b and a.***

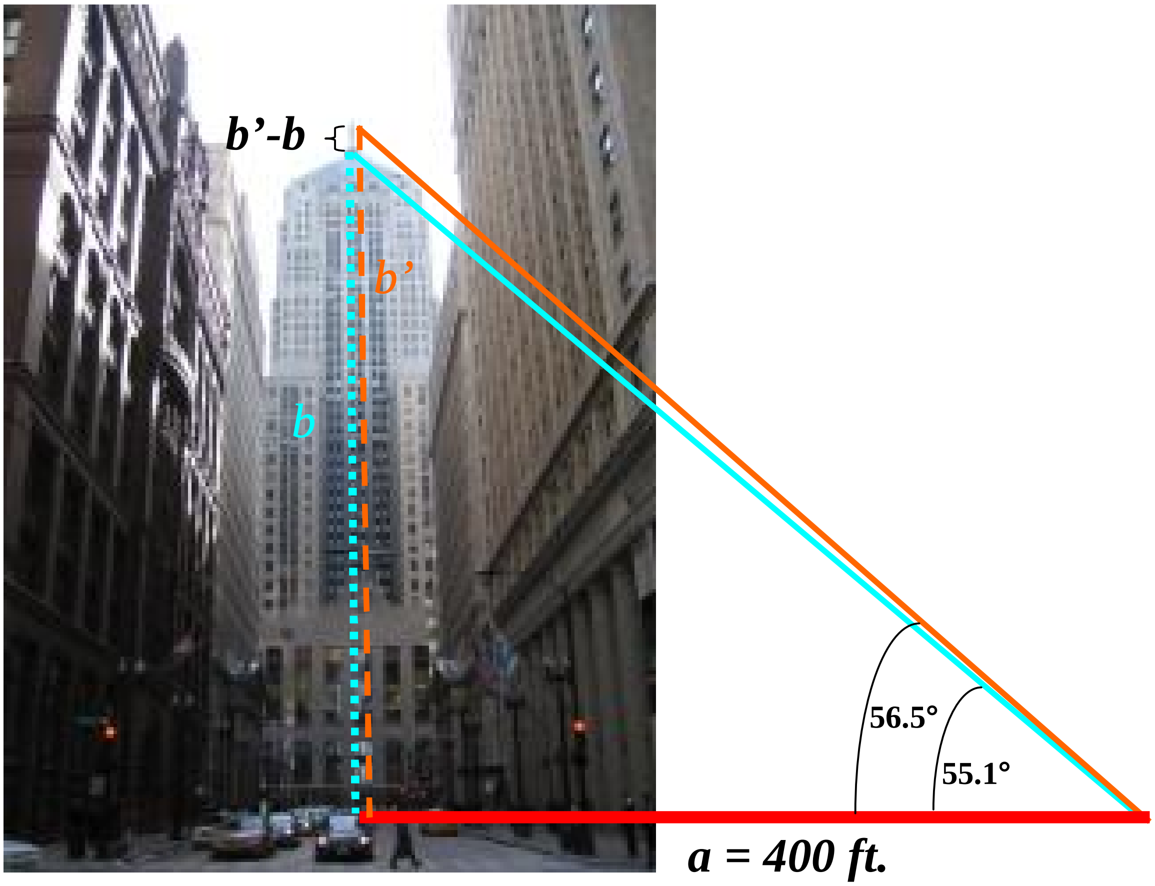
Area = ½****a*****b + ½****a********b*** + 4****b***.

Area = $½ * 4\cos(\theta) * 4\sin(\theta) + ½ * 4\cos(\theta) * 4\sin(\theta) + 4 * 4\sin(\theta)$

Simplifying we get:

Area $= A(\theta) = 16\cos(\theta)\sin(\theta) + 16\sin(\theta)$

Example 9 p. 148



$$\tan 55.1° = \frac{b}{400} \quad b = 400\tan 55.1° \approx 573$$

$$\tan 56.5° = \frac{b'}{400} \quad b' = 400\tan 56.5° \approx 604$$

Height of statute is approx. $604 - 573 = 31$ feet

Homework
p.149 #18-45 ETP
p.150-151 #59-75 ODD