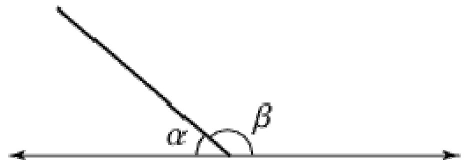
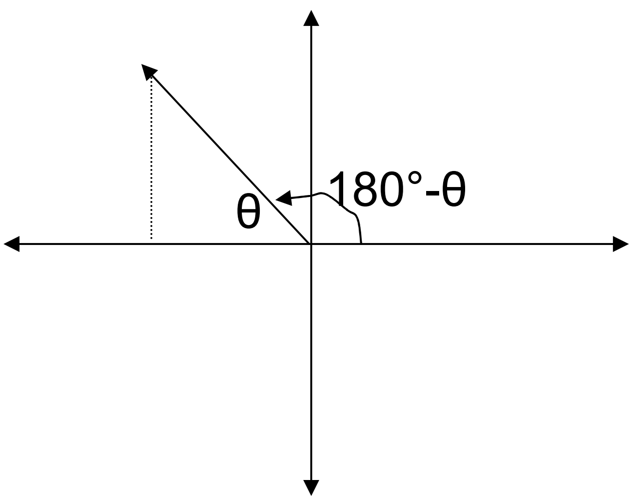
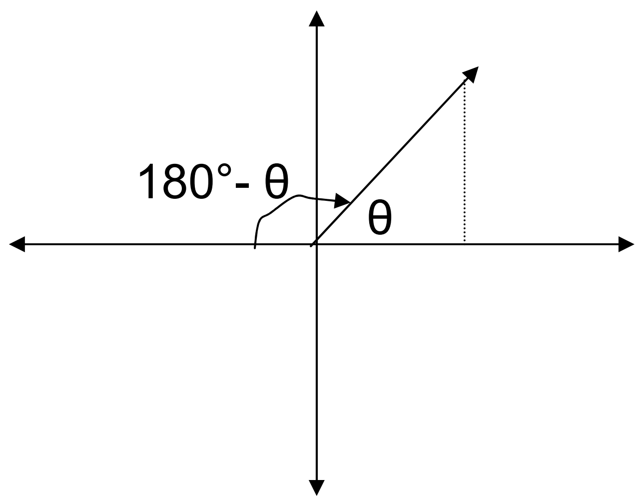
Supplementary Angles



Two angles α and β that together form a straight angle, i.e., α + β =π, (or in degrees α + β = 180°)are said to be **supplementary**.





Notice in the examples above, the right triangles formed by θ are exactly the same, just flipped horizontally. Since the triangle on the left is in Quadrant II, the cos, sec, tan, and cot will all be negative, but the sin and csc will be positive, just as the triangle on the right. Also, the ratios will all be the same as the triangle on the right.

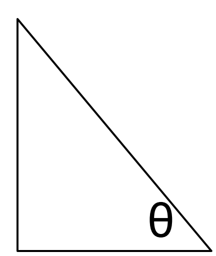
So we have these properties of Supplementary Angles

If $\theta \leq 180°$

$\sin(\theta) = \sin(180° - \theta)$

$\cos(\theta) = -\cos(180° - \theta)$

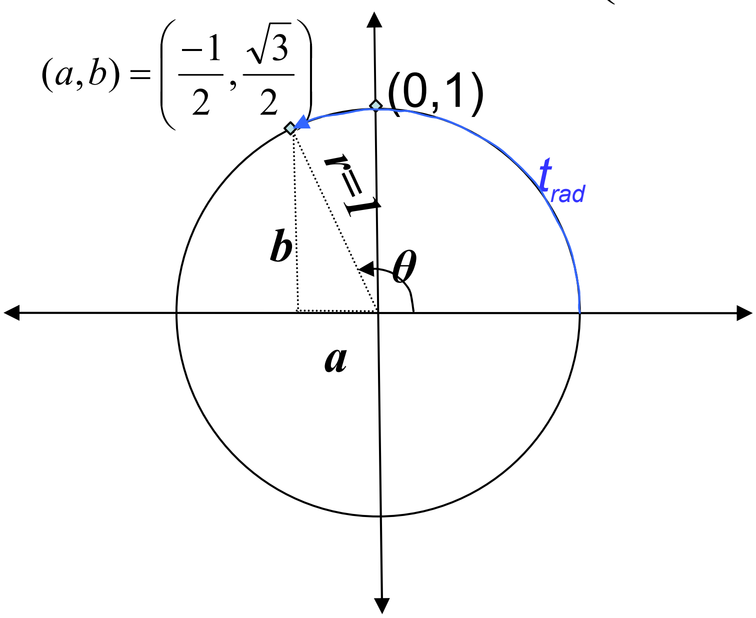
$\tan(\theta) = -\tan(180° - \theta)$



2.5 Finding the Value of the Trig Functions Using a Point on the Unit Circle

A unit circle is a circle with its center at the origin (0,0) and a radius of 1. The equation for this circle is $x^2+y^2 = 1$.

Example 1 on p. 144
Find the value of sin *t*, cos *t*, tan *t*, csc *t*, sec *t*, and cot *t* for a point P = (a,b) = $\left(\frac{-1}{2}, \frac{\sqrt{3}}{2}\right)$ on a unit circle.



cos t = a/1 = a= -½

sin t = b/1 = b= $\frac{\sqrt{3}}{2}$

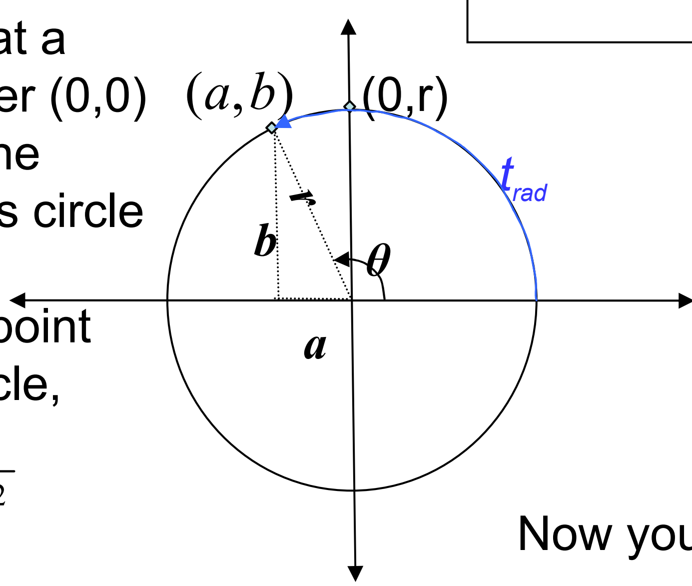
tan t = b/a = $\frac{\sqrt{3}}{2} \Big/ \frac{-1}{2} = \frac{\sqrt{3}}{-1} = -\sqrt{3}$

cot t = a/b = $\frac{-1}{2} \Big/ \frac{\sqrt{3}}{2} = \frac{-1}{\sqrt{3}} = \frac{-\sqrt{3}}{3}$

sec t = 1/a = 1 / -½ = -2

csc t = 1/b = $1 \Big/ \frac{\sqrt{3}}{2} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

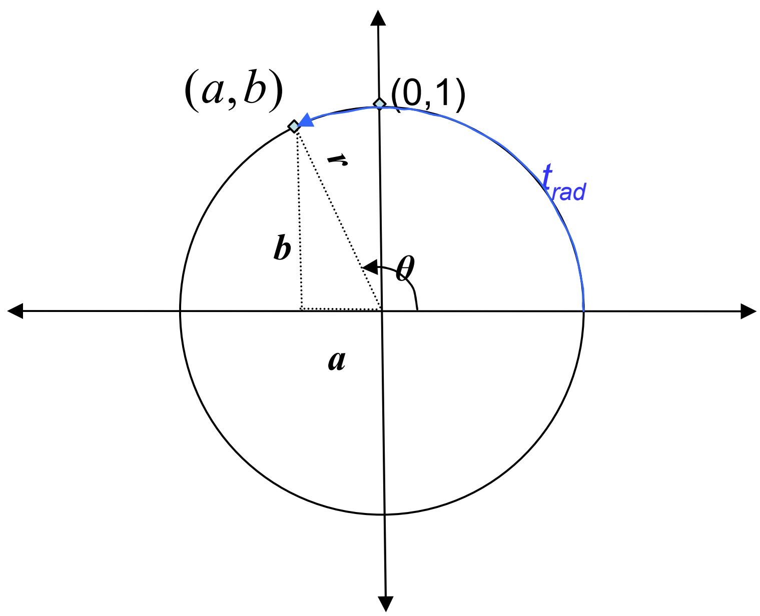
Now let's look at a circle with center (0,0) and radius r. The equation for this circle is $x^2+y^2 = r^2$.
Notice for any point (a,b) on the circle, $a^2+b^2 = r^2$.
So r = $\sqrt{a^2+b^2}$



cos t = a/r = a
sin t = b/r = b
tan t = a/b
cot t = b/a
sec t = r/a
csc t = r/b

Now you try #7 on p.152

Domain and Range of the Trigonometric Functions



Notice that for every right triangle formed inside a unit circle with one corner at the center and another corner on the circle, the hypotenuse is the radius, r.
The hypotenuse is always the longest side of a right triangle.
Therefore $|a| \leq r$, and
$|b| \leq r$.

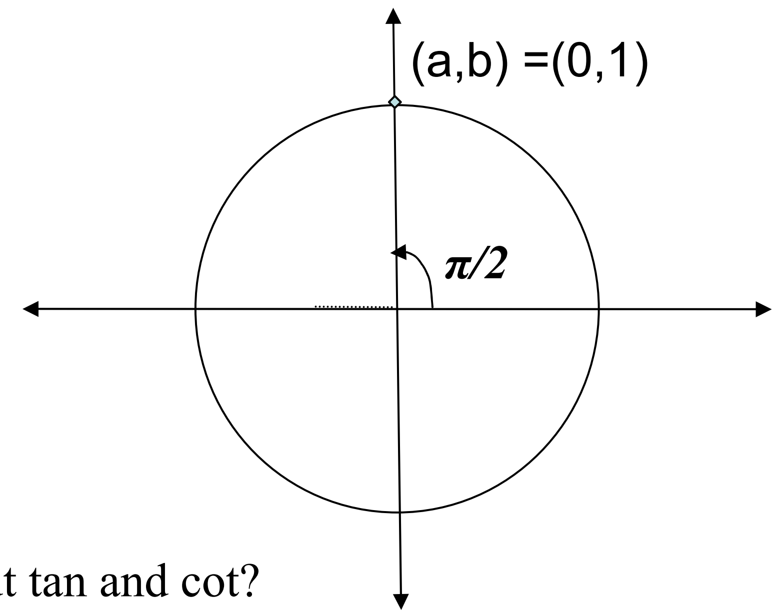
This mains that the trigonometric functions that are ratios with the hypotenuse as the denominator are always ≤ 1 and ≥ -1.
This gives us the following ranges for sin and cos:
sin (θ) = *opp/hyp*
$-1 \leq \sin(\theta) \leq 1$
sin (θ) = *adj/hyp*
$-1 \leq \cos(\theta) \leq 1$

Reciprocally:
csc(θ) ≥ 1 or csc(θ) ≤ -1
sec(θ) ≥ 1 or sec(θ) ≤ -1

The domain of sin and cos is the set of all real numbers. Any number can be used as ***θ.***
$\theta > 360°$ or $2\pi_{rad}$ is just an angle that has made more than 1 full revolution around the circle.
$\theta < 0$ is just an angle that has rotated clockwise around the circle.

Since csc(θ) is the reciprocal of sin, that is, 1/ sin(θ), then csc(θ) is undefined when csc(θ) = 0. This occurs when b=0 (on the x-axis). So the domain of csc(θ) is all real numbers except for integral (integer valued) multples of *π.*
The domain of sin and cos is the set of all real numbers. Any number can be used as *θ.*
$\theta > 360°$ or $2\pi_{rad}$ is just an angle that has made more than 1 full revolution around the circle.
$\theta < 0$ is just an angle that has rotated clockwise around the circle.

Domain and Range of Tan and Cot



What about tan and cot?

tan(θ) = opp/adj = b/a

If a = 0, then tan(θ) is undefined.
Therefore the domain of tan <u>does not include</u> angles at points where a=0. Those are all odd multiples of π/2
(i.e. π/2, 3π/2, 5π/2, etc..)
The range of tan(θ) is all real numbers, because when b is large and a is small, tan(θ) can increase infinitely; and when b is small and a is large, tan(θ) can decrease infinitely.

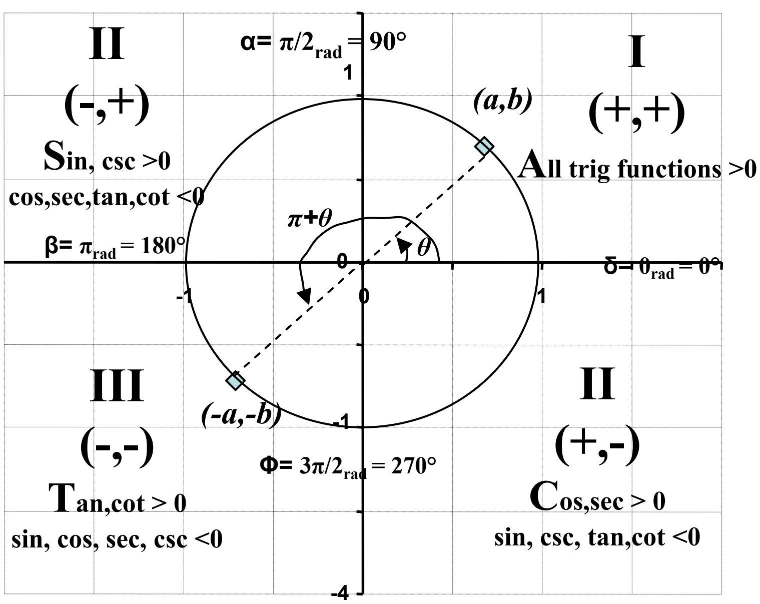
cot(θ) = opp/adj = b/a

If b = 0, then cot(θ) is undefined.
Therefore the domain of cot <u>does not include</u> angles at points where b=0. Those are all integral (integer valued) multiples of π. (i.e. 0, π, 2π, 3π, 4π, etc..)
Similarly to tan, the range of cot is all real numbers.

Period of Trig Functions

Review of Coterminal Angles:
If θ is an angle measured in degrees, then $\theta \pm 360°(k)$, where k is any integer, is an angle coterminal with θ.
If θ is an angle measured in radians, then $\theta \pm 2\pi k$, where k is any integer, is an angle coterminal with θ.
Since coterminal angles are basically equal, all their trigonometry functions are equal.
Therefore
$\sin(\theta + 2\pi) = \sin(\theta)$ and **$\cos(\theta + 2\pi) = \cos(\theta)$**

Functions that exhibit this type of behavior are called <u>periodic</u> functions.
A function f is periodic if there is a positive number p such that, whenever θ is in the domain of f, so is $\theta+p$, and $f(\theta+p) = f(\theta)$.
Let's revisit our "*ASTC*" graph:



Going from 0 to 2π, $\sin(\theta)$ starts out with the value 0, then rises to 1 at $\pi/2$, then goes back to 0 at π. At $\theta > \pi$, $\sin(\theta)$ goes from 0 to -1 at $3\pi/2$, then back to 0 at 2π. At this point the sin values repeat. The period of the sine function is 2π. (Likewise for cosine).

Going from 0 to 2π, $\tan(\theta)$ starts out with the value 0, then rises to infinity as θ *approaches* $\pi/2$. In Quadrant II, $\tan(\theta)$ rises from negative infinity (since the y-coordinates are + and the x-coordinates are -), then goes back to 0 at π. At $\theta > \pi$, $\tan(\theta)$ repeats the same process. The period of the tangent function is π. (Likewise for cotangent).
$\tan(\theta + \pi) = \tan(\theta)$ and **$\cot(\theta + \pi) = \cot(\theta)$**

Using Periodic Properties to find exact trig function values;
Even-Odd Properties

Example 2 on p.150

Find the exact value of b) tan **5π/4**

Since the period of tan is π, we just need to subtract π from 5π/4 to get a familiar angle for which we know that exact value of the tan function.

Tan (5π/4) = tan(5π/4 – 4π/4) = tan(π/4).

Recall that π/4 is the same as 45°. At 45°, the point (a,b) on the unit circle has a=b, so tan(π/4) = a/b = 1.

Now you do #13 on p.152

Even-Odd Properties

A function is even if ***f(-θ) = f(θ) for all θ in the domain of f.***

A function is odd if ***f(-θ) = - f(θ) for all θ in the domain of f.***

Even Trig Functions

cos(-*θ*) = *cos(θ)*

sec(-*θ*) = *sec(θ)*

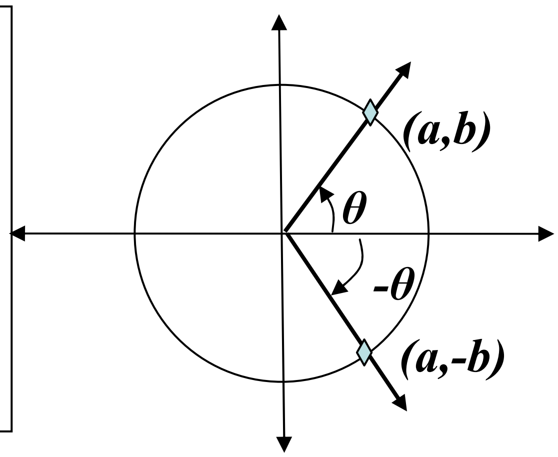
Odd Trig Functions

sin(-*θ*) = *-sin(θ)*

csc(-*θ*) = *-csc(θ)*

tan(-*θ*) = *-tan(θ)*

cot(-*θ*) = *-cot(θ)*



Example 3 on p.151

Find the exact value of

d) $\tan\left(-\frac{37\pi}{4}\right)$

The tangent function repeats its values for any integral multiple of π. Therefore, let's *add* (since the angle is negative) a multiple of π to $-\frac{37\pi}{4}$ that will give us a reference angle.

$$\frac{36\pi}{4} = 9\pi$$

$$\tan\left(-\frac{37\pi}{4}\right) = \tan\left(-\frac{37\pi}{4} + \frac{36\pi}{4}\right) = \tan\left(-\frac{\pi}{4}\right)$$

Since tan is an ODD function, we can use the property that tan(-θ) = -tan(θ).

$$\tan\left(-\frac{\pi}{4}\right) = -\tan\left(\frac{\pi}{4}\right) = -1$$

Now you do #29 on p.152 and #65 on p.153

Homework

p.141 #79,93,97

p. 152 #3,9,15,35,47,53,55, 73,75

Project 2

Practice Test (sections 2.1-2.5)

We will go over this in class in the next lecture.

You will turn it in with the Test on Feb.21 for 20 pts + 15 points of your test score (out of a possible 150).

Chapter Review p. 191-193

#1,4,25,31,43,75-86