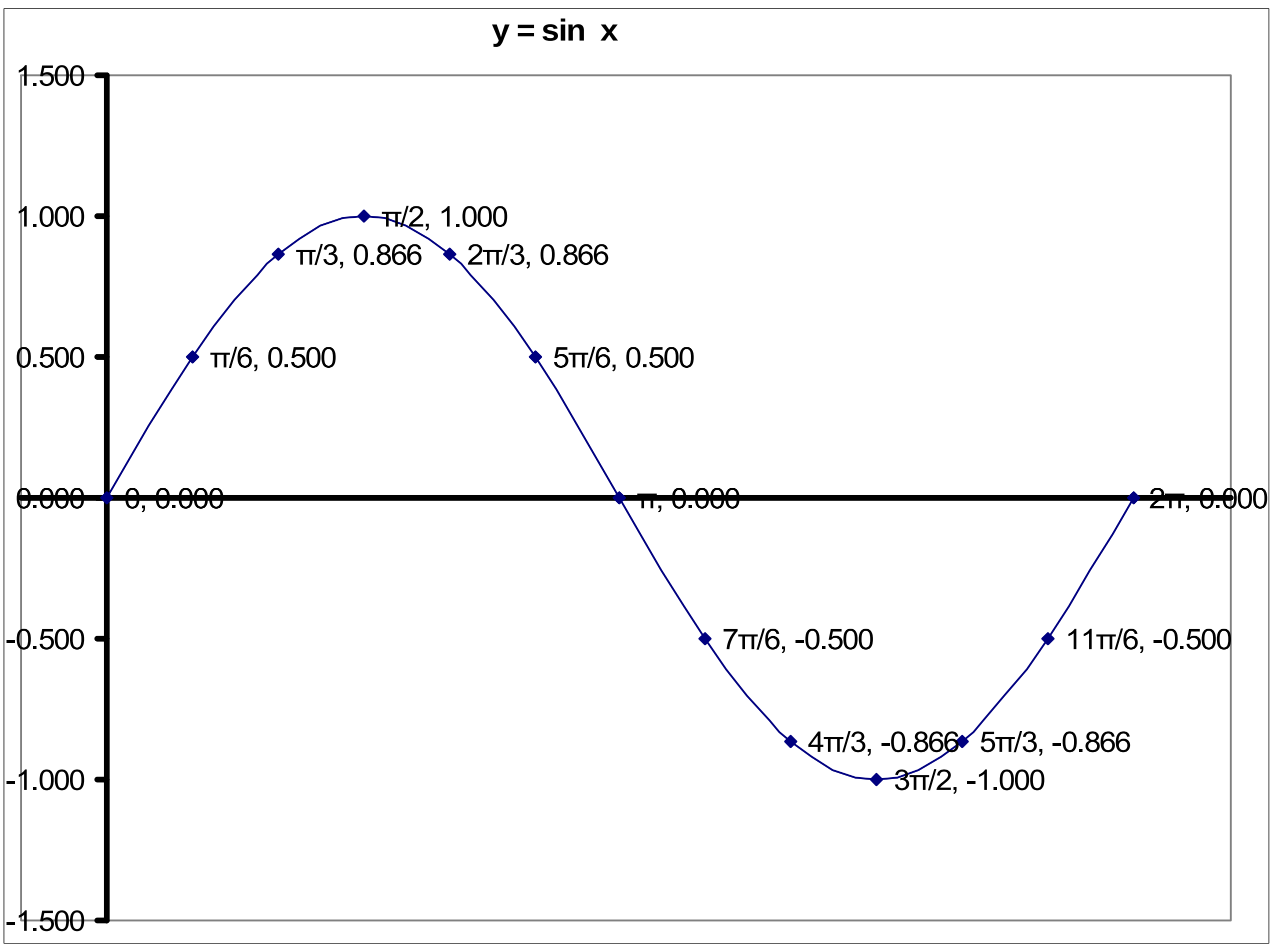
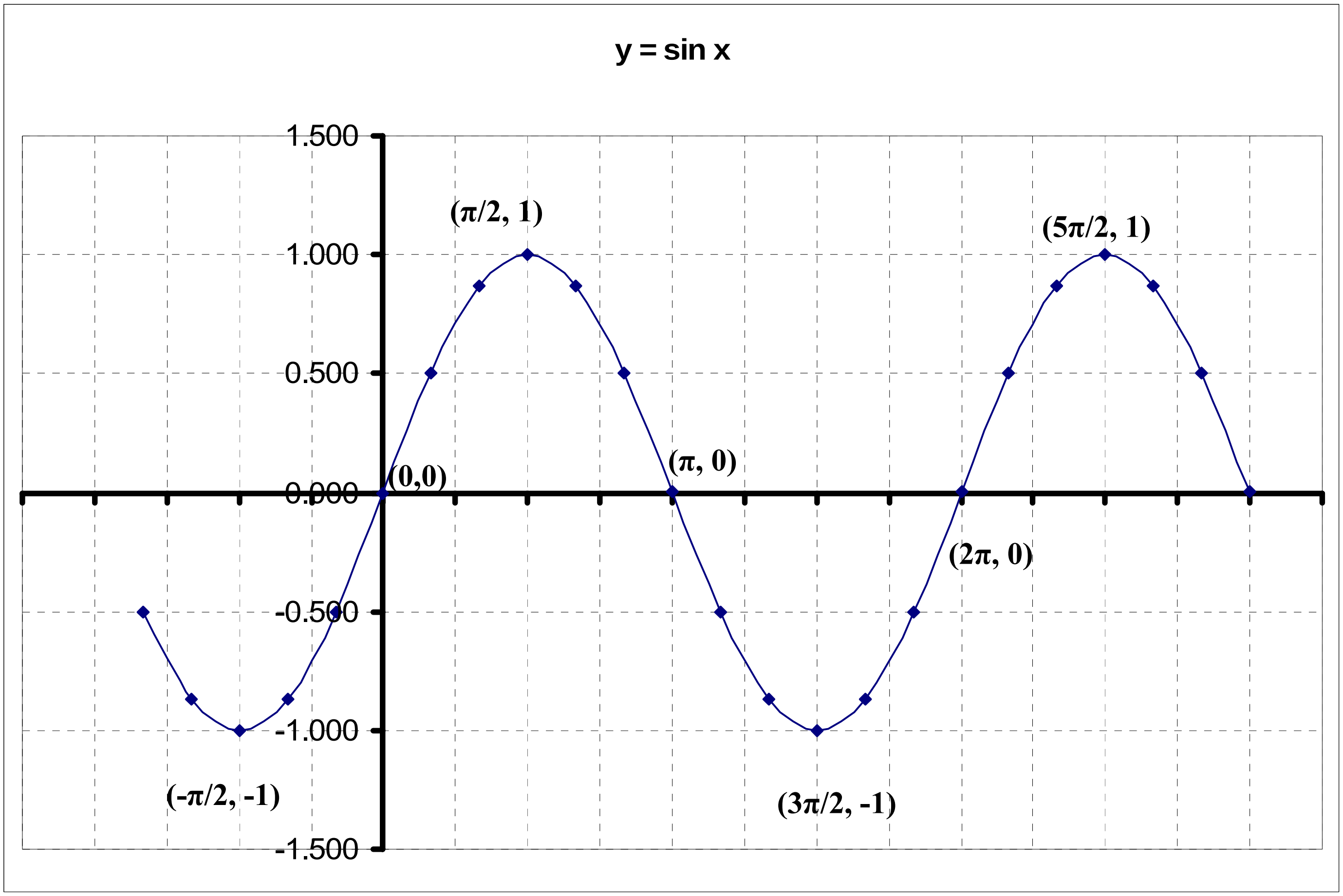
2.6 Graphs of the Sine and Cosine Functions

x	x	y = sin x
0	0	=SIN(B2)
π/6	=PI()/6	=SIN(B3)
π/3	=PI()/3	=SIN(B4)
π/2	=PI()/2	=SIN(B5)
2π/3	=B5+PI()/6	=SIN(B6)
5π/6	=B6+PI()/6	=SIN(B7)
π	=B7+PI()/6	=SIN(B8)
7π/6	=B8+PI()/6	=SIN(B9)
4π/3	=B9+PI()/6	=SIN(B10)
3π/2	=B10+PI()/6	=SIN(B11)
5π/3	=B11+PI()/6	=SIN(B12)
11π/6	=B12+PI()/6	=SIN(B13)
2π	=B13+PI()/6	=SIN(B14)

x	x	y = sin x
0	0	0.000
π/6	0.524	0.500
π/3	1.047	0.866
π/2	1.571	1.000
2π/3	2.094	0.866
5π/6	2.618	0.500
π	3.142	0.000
7π/6	3.665	-0.500
4π/3	4.189	-0.866
3π/2	4.712	-1.000
5π/3	5.236	-0.866
11π/6	5.76	-0.500
2π	6.283	0.000

If we graph y = sin x by plotting points, we see the following: Going from 0 to 2π, $sin(x)$ starts out with the value 0, then rises to 1 at π/2, then goes back to 0 at π. At $x > \pi$, $sin(\theta)$ goes from 0 to -1 at 3π/2, then back to 0 at 2π. At this point the sin values repeat. The period of the sine function is 2π. To graph a more complete graph of y= sin x, we repeat this period in each direction.





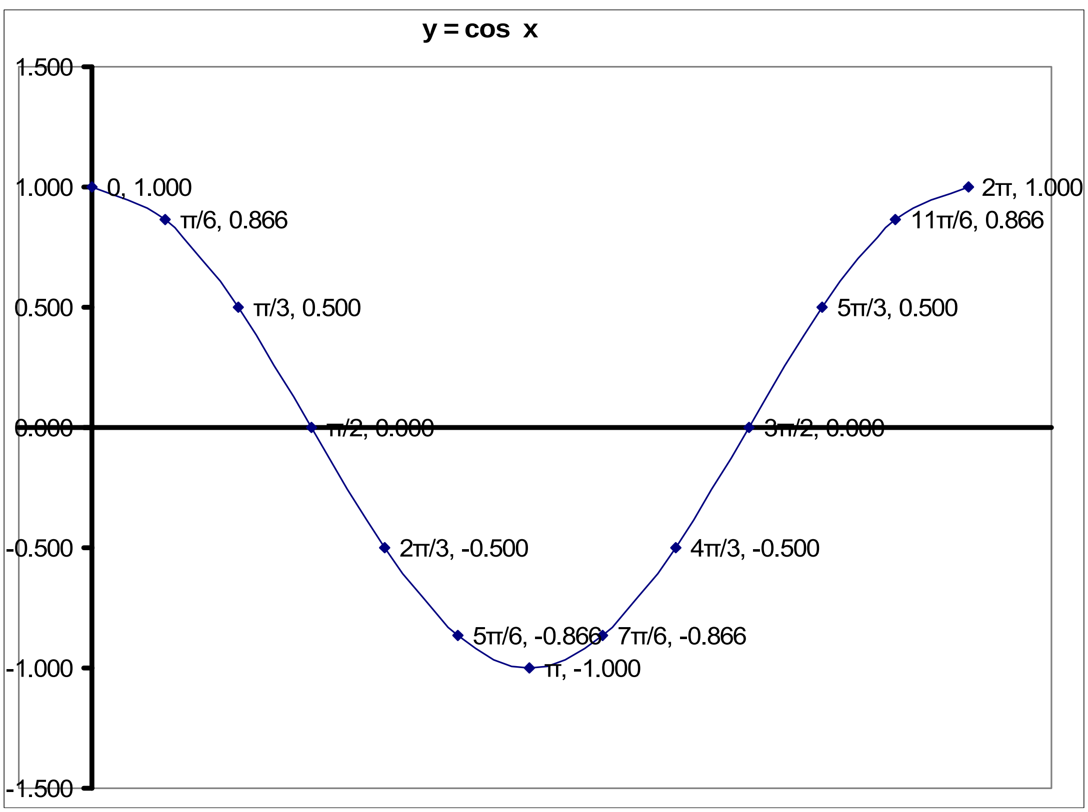
If we continued the graph in both directions, we'd notice the following:
The domain is the set of all real numbers.
The range consists of all real numbers such that $-1 \leq \sin x \leq 1$.
The functions $f(x) = \sin x$ is an odd function since the graph is symmetric with respect to the origin. ($f(-x) = -f(x)$ for every x in the domain).
The period of the sine function is 2π.
The x-intercepts are …., $-2\pi, -\pi, 0, \pi, 2\pi, 3\pi$, etc..
The y-intercepts is (0,0).
The maximum value is 1 and occurs at $x = \ldots -3\pi/2, \pi/2, 5\pi/2$, etc..
The minimum value is -1 and occurs at $x = \ldots -\pi/2, 3\pi/2, 7\pi/2$, etc..

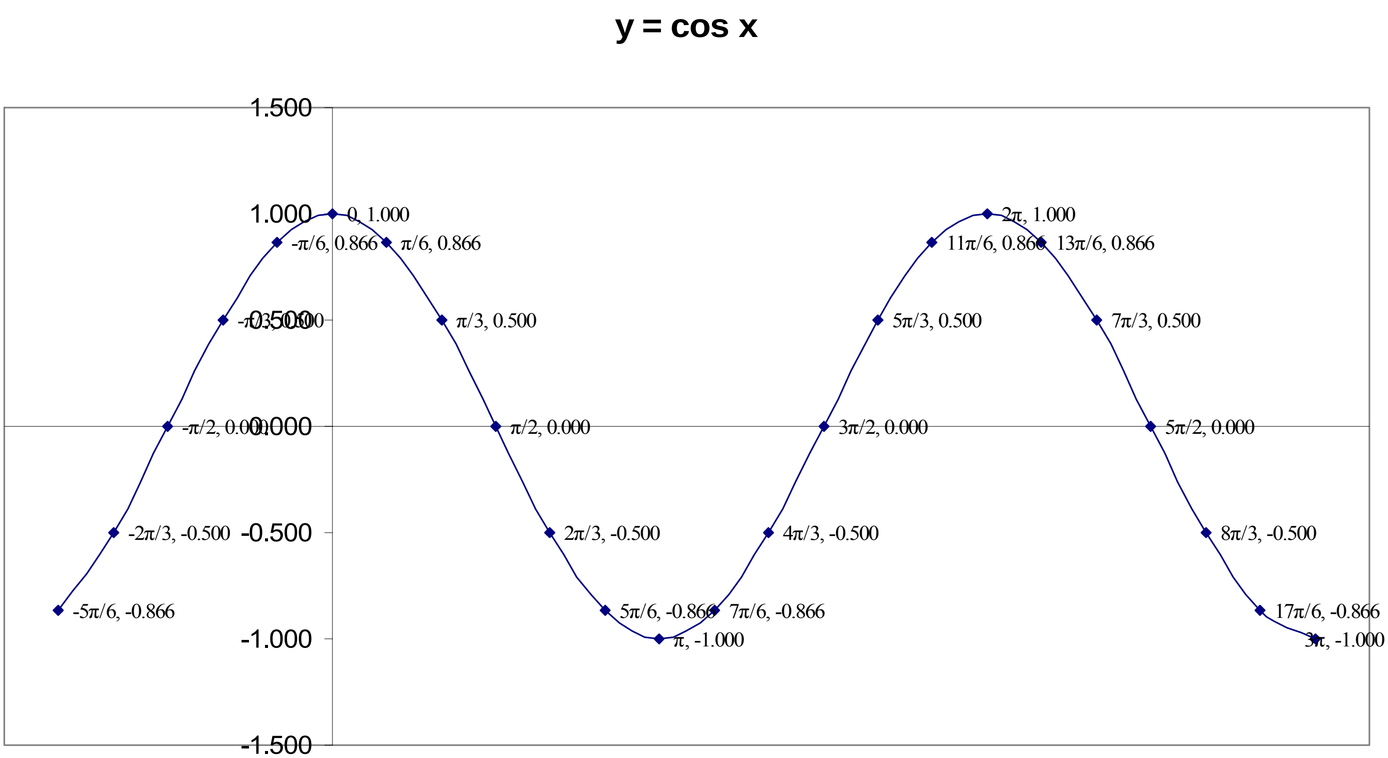
Example 1 on p. 155
How do you graph $y= \sin (x-\pi/4)$?
Notice that this function is similar to $y= \sin x$, with $(x-\pi/4)$
Instead of x. Therefore, this is just a horizontal shift to the RIGHT by $\pi/4$
[show on TI-83]

x	x	y = cos x
0	0	=COS(B2)
π/6	=PI()/6	=COS(B3)
π/3	=PI()/3	=COS(B4)
π/2	=PI()/2	=COS(B5)
2π/3	=B5+PI()/6	=COS(B6)
5π/6	=B6+PI()/6	=COS(B7)
π	=B7+PI()/6	=COS(B8)
7π/6	=B8+PI()/6	=COS(B9)
4π/3	=B9+PI()/6	=COS(B10)
3π/2	=B10+PI()/6	=COS(B11)
5π/3	=B11+PI()/6	=COS(B12)
11π/6	=B12+PI()/6	=COS(B13)
2π	=B13+PI()/6	=COS(B14)

x	x	y = cos x
0	0	1.000
π/6	0.524	0.866
π/3	1.047	0.500
π/2	1.571	0.000
2π/	2.094	-0.500
5π/	2.618	-0.866
π	3.142	-1.000
7π/	3.665	-0.866
4π/	4.189	-0.500
3π/	4.712	0.000
5π/	5.236	0.500
11π	5.76	0.866
2π	6.283	1.000

If we graph y = cos x by plotting points, we see the following:
Going from 0 to 2π, *cos*(x) starts out with the value 1, then decreases to 0 at π/2, then continues decreasing to -1 at π. At $x > \pi$, *cos*(x) goes from -1 to 0 at 3π/2, then continues increasing to 1 at 2π. At this point the cos values repeat. The period of the cosine function is 2π. To graph a more complete graph of y= cos x, we repeat this period in each direction.





If we continued the graph in both directions, we'd notice the following:
The domain is the set of all real numbers.
The range consists of all real numbers such that $-1 \leq \cos x \leq 1$.
The functions $f(x) = \cos x$ is an even function since the graph is symmetric with respect to the y-axis. [$f(-x) = f(x)$ for every x in the domain].
The period of the cosine function is 2π.
The x-intercepts are …., $-\pi/2$, $\pi/2$, $3\pi/2$, $5\pi/2$, etc…
The y-intercepts is (0,1).
The maximum value is 1 and occurs at x = … -2π, 0, 2π, etc..
The minimum value is -1 and occurs at x = … $-\pi$, π, 3π, etc..

TRANSFORMATIONS

$y = A\sin(\omega x + h) + v$ or $y = A\cos(\omega x + h) + v$

$|A|$ = amplitude = the biggest value of a periodically changing value.

If $A < 0$, the graph is reflected on the x-axis.

$-|A| \leq A\sin x \leq |A|$ and $-|A| \leq A\cos x \leq |A|$

[since $\sin x \leq 1$ and $\cos x \leq 1$]

ω=Frequency = number of periods per time. The number of periods per second is measured in hertz (hz for short). We have a frequency of 1hz if a wave has exactly one period per second. If we have 5 periods per second, we have a frequency of 5hz.

Period = the length between two points which are surrounded by the same pattern.

For sin and cos functions, period = 2π

For tan and cot functions, period = π

The new period of this function will either compress or stretch by a factor of $1/\omega$.

If the $0 < \omega < 1$, the new period is *longer* than the original function.

If the $\omega > 1$, the new period is *shorter* than the original function.

For sin and cos:

T = period of new function = $2\pi/\omega$

h = horizontal displacement.

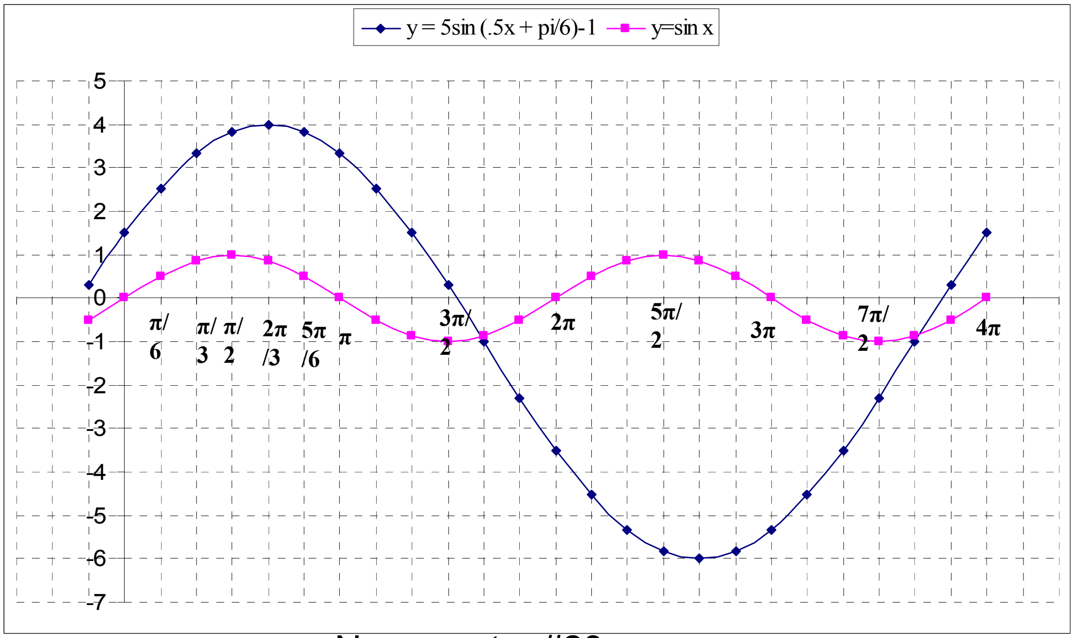
(+) If $h > 0$, graph is shifted to the LEFT.

(-) If $h < 0$, graph is shifted to the RIGHT.

v = vertical displacement

(+) If $v > 0$, graph is shifted UP.

(-) If $v < 0$, graph is shifted DOWN.



Now you try #23

Example 8 on p.164

$$y = 2\sin\left(-\frac{\pi}{2}x\right)$$

From the form **y = Asin(ωx + h) + v**
ω = -π/2
(However we need ω > 0)
Let's remember that sin x is an odd function, where
f(-x) = -f(x)
So

$$y = 2\sin\left(-\frac{\pi}{2}x\right) = -2\sin\left(\frac{\pi}{2}x\right)$$

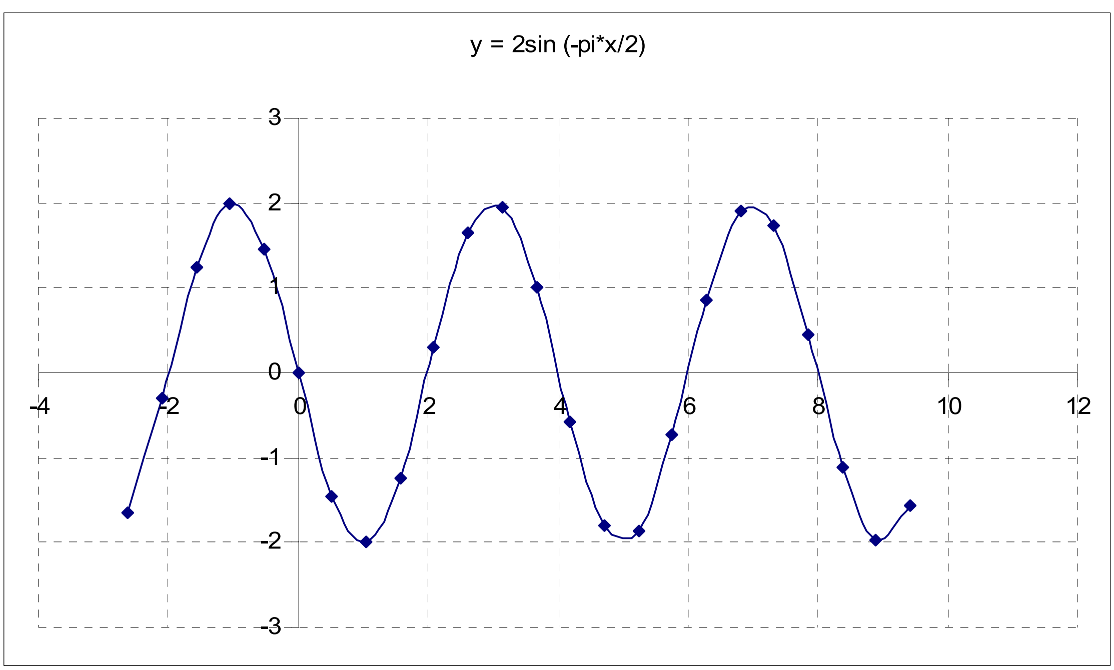
Negative sign means sin function will be reflected on the x-axis

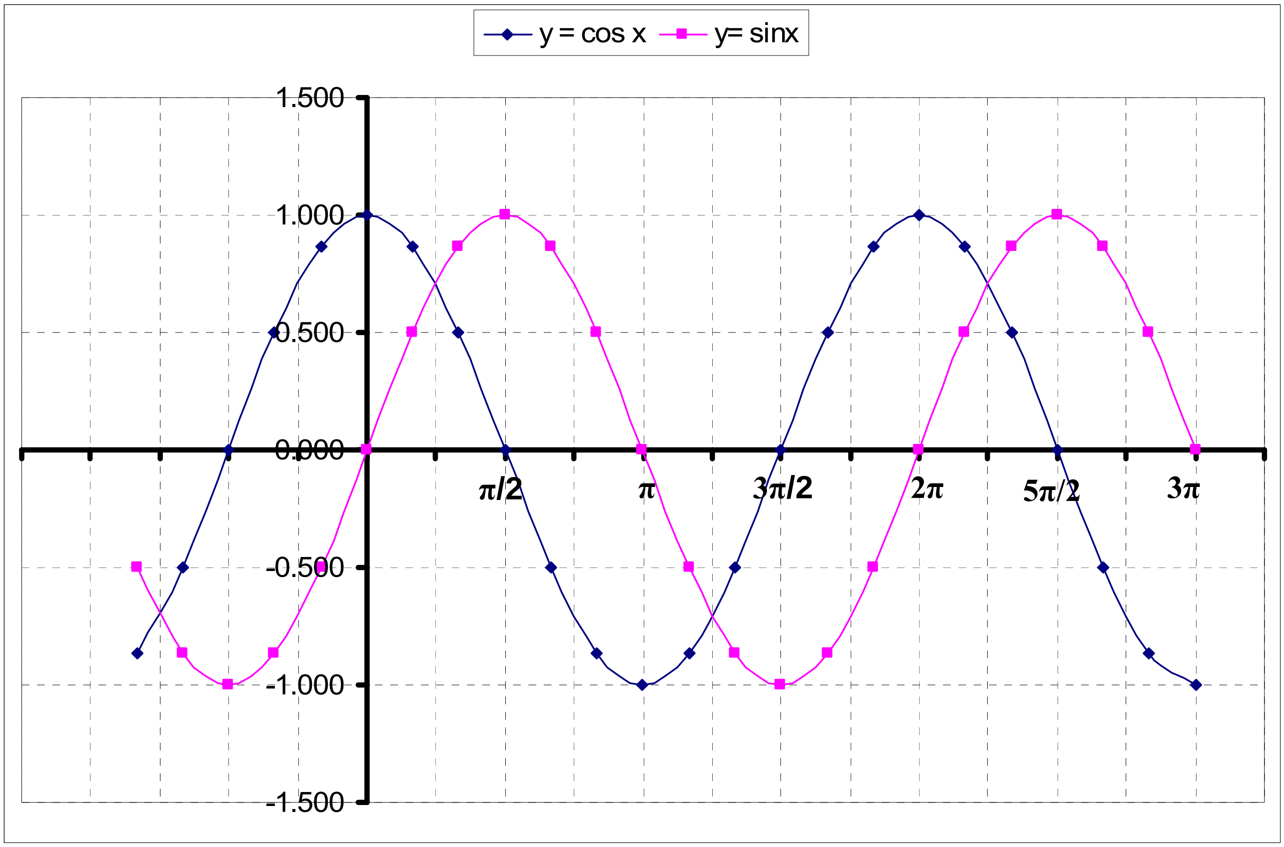
|A|=2   ω = π/2

The amplitude is |-2| = 2 so the largest value of y is 2.
The period is T = 2π/**ω** = $\frac{2\pi}{\pi/2} = 2\pi \bullet 2/\pi = 4$

h = 0, v = 0, so there is no horizontal or vertical displacement**.**
Therefore, the period starts at 0 and ends at 4.
sin x = 0 at x = π, so

$$\sin\left(\frac{\pi}{2}x\right) = 0, \text{ when } x = 2$$





Notice sin x and cos x are basically the same curves.
cos x is just sin x shifted to the **left** by π/2. Therefore,
cos x = sin (x + π/2).
Or alternatively, sin x is just cos x shifted to the **right** by π/2. So
sin x = cos (x - π/2).

Because of the similarity of cosine and sine curves, these functions are often referred to as **sinusoidal graphs.**

Homework
p. 166-169
#11* , 23*,25,27, 33*,35, 39*,43, 53,
63*, 67*, 69, 81

* Will be done in class.