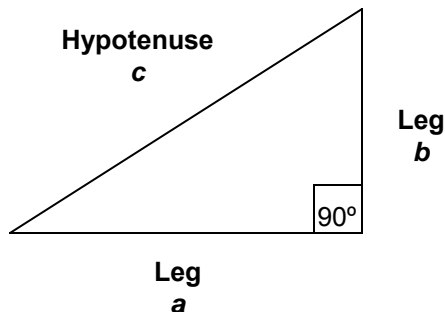


A.2 GEOMETRY REVIEW



Pythagorean Theorem (A.2 p. 582)

The Pythagorean Theorem is a statement about right triangles. A right triangle is one that contains a right angle, that is, an angle of 90° . The side of the triangle opposite the 90° angle is called the hypotenuse; the remaining two sides are called legs. In the figure on the left, we have used c to represent the length of the hypotenuse and a and b to represent the lengths of the legs.

Pythagorean Theorem:

In a right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs. Conversely, if the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs of a triangle, then the triangle is a right triangle, where the 90° angle is opposite the longest side.

$$c^2 = a^2 + b^2$$

From this we can derive the following equations:

$$c = \sqrt{a^2 + b^2}$$

$$a = \sqrt{c^2 - b^2}$$

$$b = \sqrt{c^2 - a^2}$$

Example 1

In a right triangle, one leg is of length 4 and the other is of length 3. What is the length of the hypotenuse?

c = hypotenuse

$a = 4$

$b = 3$

Using the Pythagorean theorem, we know

$$c^2 = a^2 + b^2$$

$$c^2 = 4^2 + 3^2 = 16 + 9 = 25$$

$$c = \sqrt{25} = 5$$

Now do Problem 9 on p.585

Example 2

Verifying That a Triangle is a Right Triangle

Show that a triangle whose sides are of lengths 5, 12, and 13 is a right triangle. Identify the hypotenuse.

The longest side is the hypotenuse, so if this is a right triangle, then $c = 13$. We'll let $a = 5$, and $b = 12$.

Does $c^2 = a^2 + b^2$?

$$13^2 = 5^2 + 12^2$$

$$169 = 25 + 144$$

$$169 = 169 \text{ Yes}$$

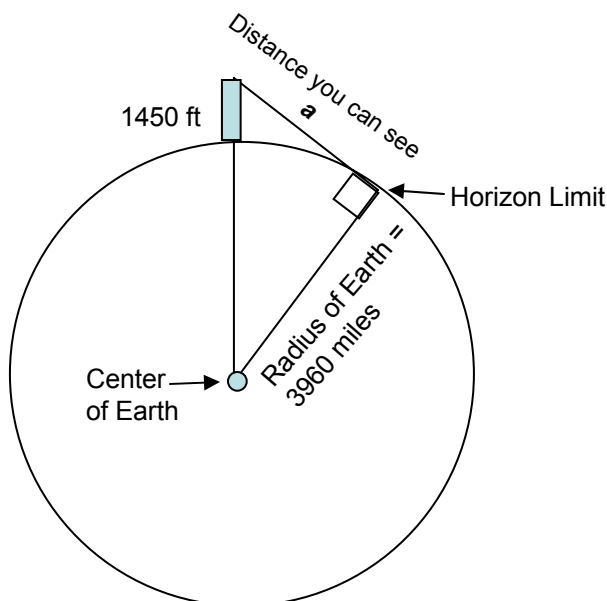
Therefore the triangle is a right triangle and the side with length 13 is the hypotenuse.

Now do Problem 17 on p. 585

Example 3

The tallest inhabited building in the world is the Sears Tower in Chicago. If the observation tower is 1450 feet above ground level, how far can a person standing in the observation tower see (with the aid of a telescope)? Use 3960 miles for the radius of Earth.

Solution: A person can only see as far as the horizon limit of the earth. That is the point at which the line segment from the top of the tower to the earth makes a right angle with the line segment from the earth to the center of the earth.



1 mile = 5280 ft.

We can now apply the Pythagorean theorem to solve the problem.

The farthest distance you can see is one of the legs of the right triangle, which we'll call a . The hypotenuse is the radius of Earth + the height of the tower. Let's convert all units to miles.

$$1450 \text{ ft} = 1450 \text{ ft} / 5280 \text{ ft/mile} \approx .2746212 \text{ miles}$$

$$\text{Hypotenuse} = c = 3960 + .2746212 \text{ miles}$$

$$b = 3960 \text{ miles}$$

$$c^2 = a^2 + b^2$$

$$(3960.2746212)^2 = a^2 + 3960^2$$

Solve for a .

$$a^2 = (3960.2746212)^2 - 3960^2$$

$$a^2 \approx 2175.08$$

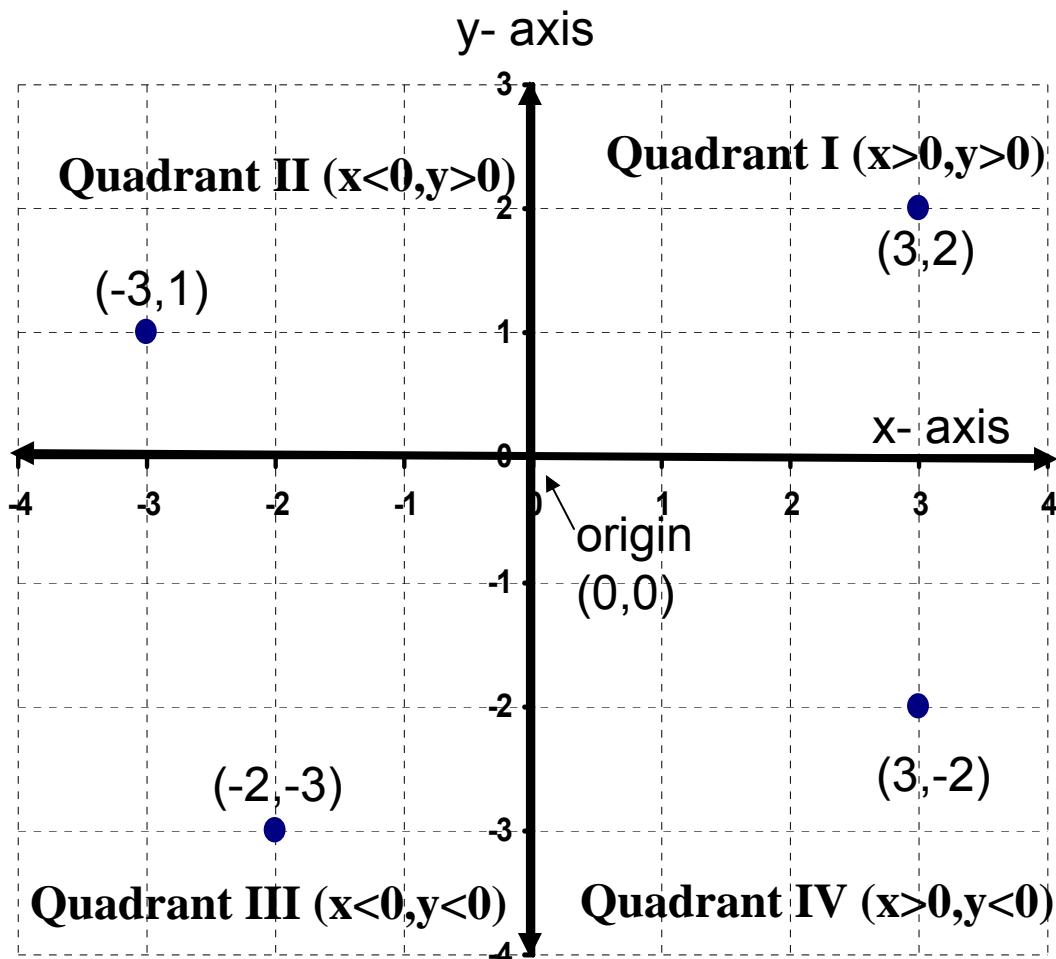
$$a \approx 46.64 \text{ miles}$$

A person can see about 47 miles from top of the tower.

SECTION 1.1 RECTANGULAR COORDINATES; GRAPHING UTILITIES

The modern Cartesian coordinate system in two dimensions (also called a rectangular coordinate system) is commonly defined by two axes, at right angles to each other, forming a plane (an xy -plane). The horizontal axis is labeled x , and the vertical axis is labeled y . All the points in a Cartesian coordinate system taken together form a so-called Cartesian plane. Equations that use the Cartesian coordinate system are called Cartesian equations.

The point of intersection, where the axes meet, is called the *origin* normally labeled O . The x and y axes define a plane that can be referred to as the xy plane. Given each axis, choose a unit length, and mark off each unit along the axis, forming a grid. To specify a particular point on a two dimensional coordinate system, you indicate the x coordinate first (abscissa), followed by the y coordinate (ordinate) in the form (x,y) , an ordered pair. The choices of letters come from the original convention, which is to use the latter part of the alphabet to indicate unknown values. The first part of the alphabet was used to designate known values. Many of the curves studied in classical geometry can be described as the set of points (x,y) that satisfy some equation $f(x,y)=0$. In this way certain questions in geometry can be transformed into questions about numbers and resolved by means of analytic geometry.



FINDING THE DISTANCE BETWEEN TWO POINTS

The distance, d , between two points can be found using the Pythagorean Theorem.

Example 2 p. 4

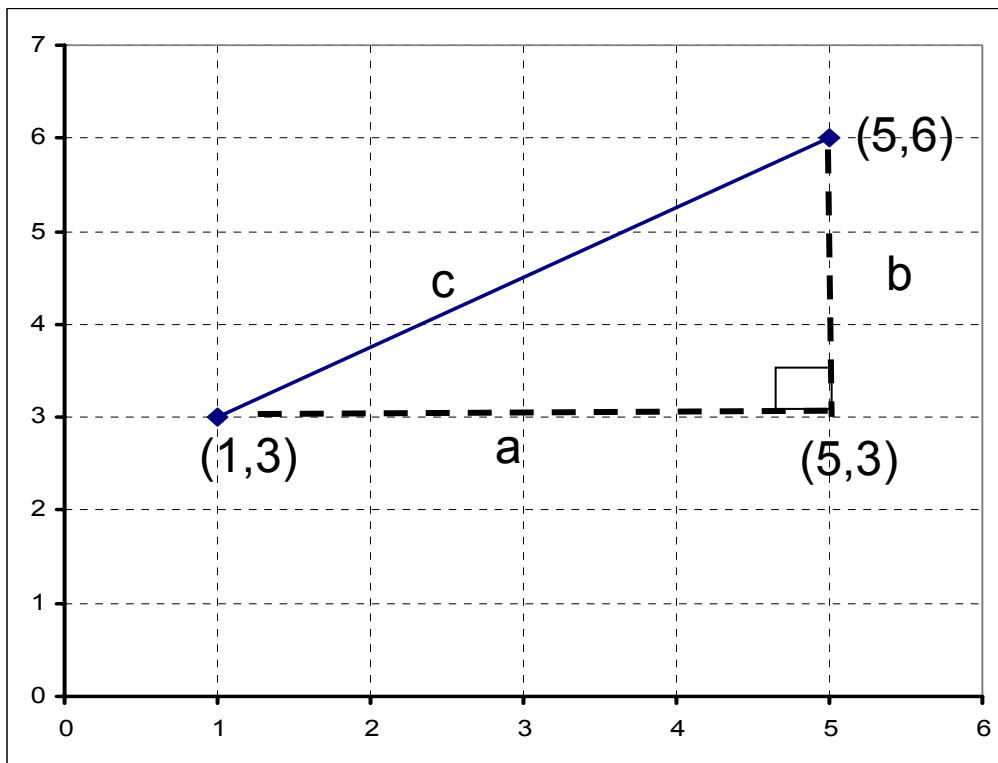
Find the distance d between points $(1,3)$ and $(5,6)$

Solution: Create a right triangle by drawing a horizontal line from $(1,3)$ to $(5,3)$ [since 5 is the x-coordinate of $(5,6)$, and a vertical line from $(5,3)$ to $(5,6)$. The line segment from $(1,3)$ to $(5,6)$ is the hypotenuse, c . The other line segments are the legs, a and b .

The distance between two points on a horizontal line is just the absolute value of the difference of their x-coordinates $(|x_2 - x_1|)$, so the length of $a = |5 - 1| = 4$. Likewise, the distance between two points on a vertical line is just the absolute value of the difference of their y-coordinates $(|y_2 - y_1|)$, so the length of $b = |6 - 3| = 3$

From the Pythagorean Theorem, $c^2 = a^2 + b^2 = 4^2 + 3^2 = 16 + 9 = 25$

$$c = \sqrt{25} = 5$$



DISTANCE FORMULA

The distance between two points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$ is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

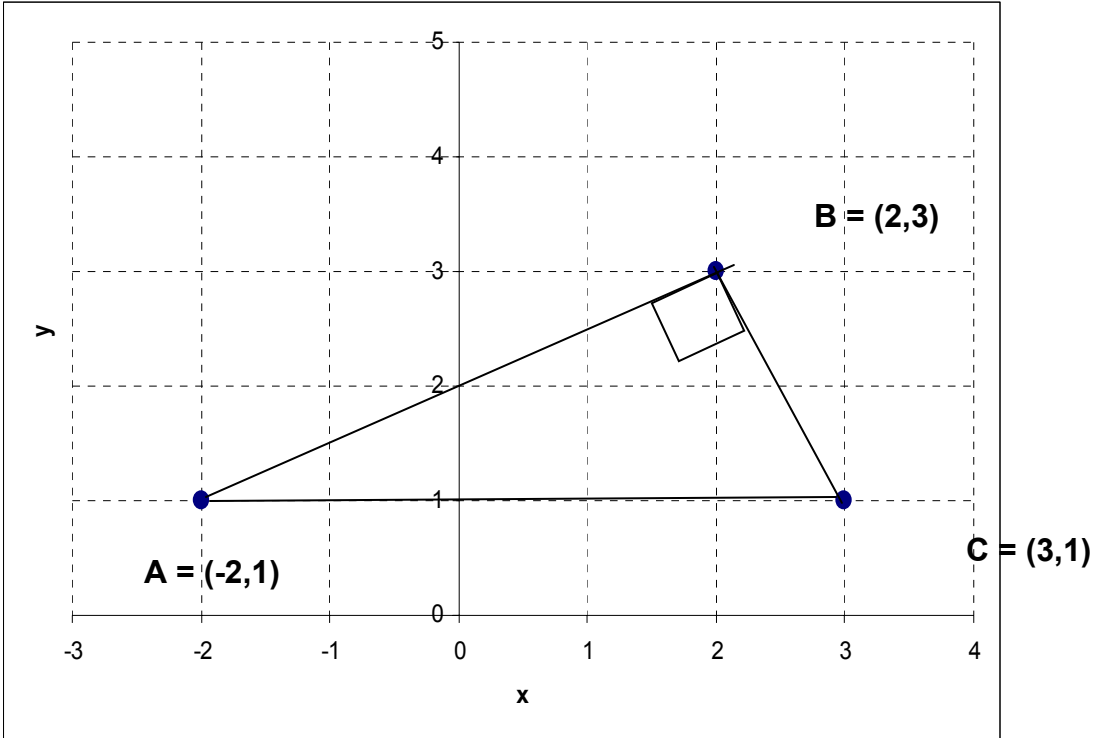
Example 3 p.6 (DONE IN CLASS)

Now do problem 31 on p. 9

Example 4 Using Algebra to Solve Geometry Problems

Consider three points $A = (-2,1)$, $B = (2,3)$, and $C = (3,1)$

a) Plot each point and form triangle ABC.



b) Find the length of the three sides of the triangle, AB, BC, and AC

$d(A,B) =$

$d(B,C) =$

$d(A,C) =$

c) The converse of the Pythagorean Theorem says that if the square of the longest side is equal to the sum of the squares of the other two sides, the triangle is a right triangle. The longest side is 5, so does

$$5^2 = (2\sqrt{5})^2 + (\sqrt{5})^2 \quad ?$$

$$25 = 4 \cdot 5 + 5 = 20 + 5 = 25$$

Yes it is a right triangle

d) Find the area of the triangle.

The area of a triangle is $\frac{1}{2}$ (base)(height)

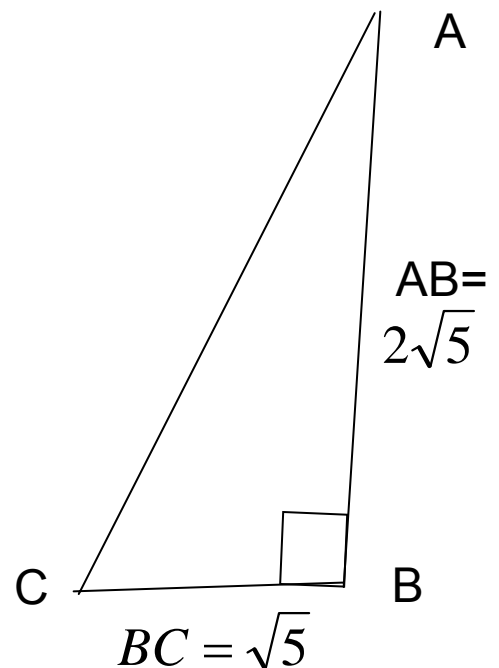
If you rotated the triangle to lay on one of it's legs instead of the hypotenuse, you can see that the base is AB and the height is BC (or vice versa).

So Area =

$$= \frac{1}{2} (2\sqrt{5})(\sqrt{5})$$

$$= \frac{1}{2} (2)(5) = 5 \text{ square units}$$

Now do problem 49 on p.9



Midpoint Formula

The midpoint $M = (x, y)$ of the line segment from $P_1 = (x_1, y_1)$ to $P_2 = (x_2, y_2)$ is

$$M = (x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Therefore, to find the midpoint of a line segment, we average the x-coordinates and the y-coordinates of the endpoints.

Example 5 (DONE IN CLASS)

Do problem 55 on p.9

Ch. 1.2 Graphing Equations

A point (x,y) are on the graph of the equation if substituting those values for x and y still render a true statement (that is, *satisfy* the equation).

Example 1

Determine if the following points are on the graph of the equation $2x-y=6$

a) $(2,3)$

$$2(2) - 3 = ? 6$$

$$4 - 3 = 1 \neq 6$$

No $(2,3)$ is not on the graph.

b) $(2,-2)$

$$2(2) - (-2) = 4 + 2 = 6$$

$$6 = 6$$

Yes $(2,-2)$ is on the graph

Now do p.22 #21

If $(4,b)$ is a point on the graph of $y = x^4 - \sqrt{x}$ what is b ?

Substitute 4 for x and b for y .

$$b = 4^4 - \sqrt{4} = 256 - 2 = 254$$

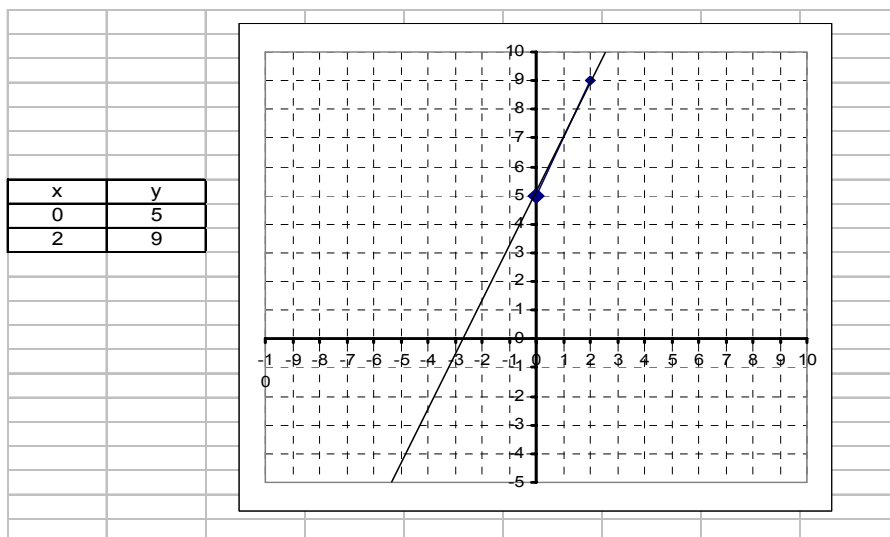
Example 2

Graph the equation $y = 2x + 5$

Because this equation is in the form $y = mx + b$, we know it's a line. To graph the equation, we need to know at least two points on the line.

When $x = 0$, $y = 2(0) + 5 = 5$

When $x = 2$, $y = 2(2) + 5 = 9$



Ch. 1.2

USING A GRAPHING CALCULATOR TO GRAPH EQUATIONS

To graph an equation in two variables, x and y , **using a graphing calculator requires that the equation be written in the form $y = \{\text{expression in } x\}$** . If the original equation is not in this form, there are ways to obtain an equivalent equation in the “ $y =$ ” form.

Procedures “Allowed” in order to maintain equivalent equations:

1. You can switch sides of the equation and it remains the same: Ex: $3x + 5 = y$ is the same as $y = 3x + 5$
2. You can simplify the equation by combining like terms, eliminating parentheses, and so on:
Ex: $2y + 2 + 6 = 2x + 5(x + 1)$ is the same as
 $2y + 8 = 7x + 5$
3. You can add or subtract the same expression on both sides of the equation.
Ex: $y + 3x - 5 = 4$ is the same as
 $y + 3x - 5 + 5 = 4 + 5$, which can be simplified to
 $y + 3x = 9$
4. You can multiply or divide both sides of the equation by the same nonzero expression:
Ex:
 $3y = 6 - 2x$ can be multiplied on both sides by $1/3$
 $\frac{1}{3} \bullet 3y = \frac{1}{3}(6 - 2x)$ which can be simplified to
 $y = 2 - \frac{2}{3}x$

SETTING THE STANDARD VIEWING WINDOW

The viewing window is the portion of the coordinate plane visible on your graphing calculator screen. It is defined by **Xmin**, **Xmax**, **Ymin**, and **Ymax**. **Xscl** and **Yscl** define the distance between tick marks on the x-axis and y-axis, respectively.

To display the current window variable values, press the **WINDOW** button.

The standard values for the viewing window are

Xmin = -10

Xmax = 10

Xscl = 1

Ymin = -10

Ymax = 10

Yscl = 1

These standard values can be automatically set by pressing the **ZOOM** button and then selecting **6:Zstandard**.

Example 5 p. 15

Graph the equation: $6x^2 + 3y = 36$

Step 1: Use Equivalent Equations Procedures to solve for y in terms of x.

Subtract $6x^2$ from both sides:

$$3y = -6x^2 + 36$$

Divide both sides by 3:

$$y = -2x^2 + 12$$

What is the y-intercept of this equation?

$$y=12$$

So we should reset the **Ymax** in the viewing window to be at least 12.

To graph this, press the **Y=** button and type in the expression.

Press

Then press

To reproduce this graph manually on your own graph paper, it is helpful to create a table of x and y values for the equation. This can be done with the Table function in your calculator.

Press to display the TABLE SETUP screen.

Press to set **TblStart** = -10

Press to set **ΔTbl** = 1.

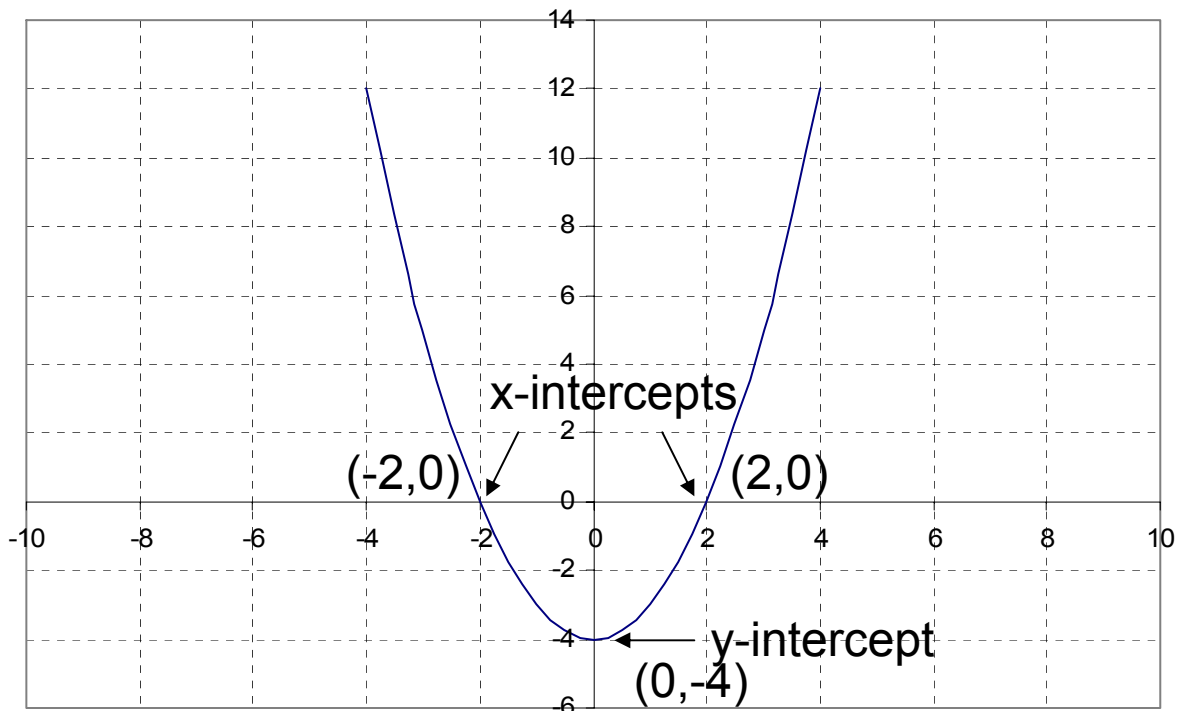
Press

To automatically generate independent and dependent values.

Press to display the table screen.

INTERCEPTS

The points, if any, at which a graph crosses or touches the coordinate axes are called the **intercepts**. The x-coordinate of a point at which the graph crosses or touches the x-axis is an **x-intercept**. The y-coordinate of a point at which the graph crosses or touches the y-axis is a **y-intercept**. Because the x-intercepts of the graph of an equation are those x-values for which $y=0$, they are also called the **zeros** (or **roots**) of the equation.



PROCEDURE FOR FINDING INTERCEPTS

1. To find the x-intercept(s), if any, of the graph of an equation, let $y=0$ in the equation and solve for x .
2. To find the y-intercept(s), if any, of the graph of an equation let $x=0$ in the equation and solve for y .

x-intercepts of $y=x^2 - 4$

Set $y=0$

$$0 = x^2 - 4$$

$$4 = x^2$$

$$x^2 = 4$$

$$x = \pm 2$$

And remember $y=0$, so

x-intercepts are

$(-2,0)$ and $(2,0)$

y-intercepts of $y=x^2 - 4$

Set $x=0$

$$y = 0^2 - 4$$

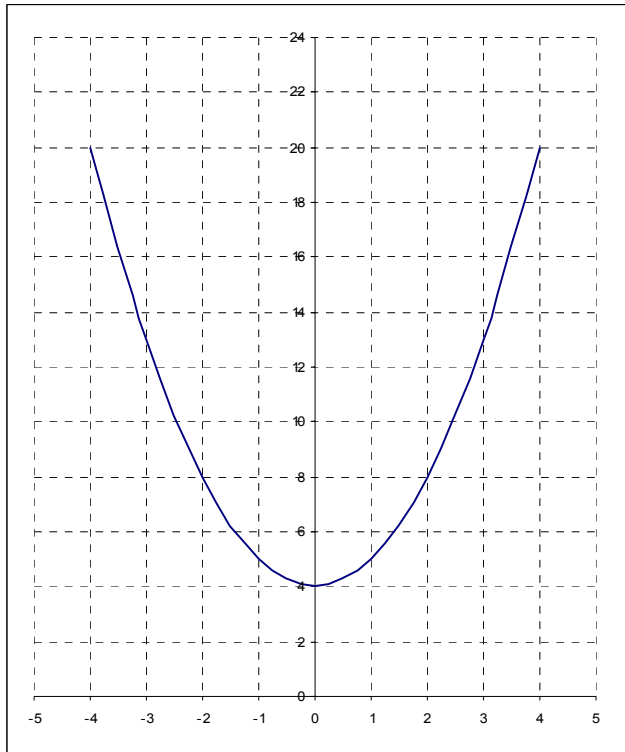
$$y = -4$$

And remember $x=0$, so

y-intercept is

$(0,-4)$

What about $y = x^2 + 4$?



We can see that the y-intercept is (0,4), but what are the x-intercepts for this graph?

Set $y=0$ to find x-intercepts.

$$0 = x^2 + 4$$

$$-4 = x^2$$

$$x^2 = -4$$

$$x = \pm\sqrt{-4}$$

This is not a real number, so there are no x-intercepts for this equation.

Now do #43 on p.23

Testing an Equation for Symmetry (p.17)

Knowing an equation's symmetry can help you know how to graph it.

A graph can be **symmetric with respect to the x-axis**, for every point (x,y) on the graph, the point $(x, -y)$ is also on the graph.

If you fold the graph at the x-axis graphs line up to each other.

A graph can be **symmetric with respect to the y-axis**, for every point (x,y) on the graph, the point $(-x, y)$ is also on the graph.

If you fold the graph at the y-axis graphs line up to each other.

A graph can be **symmetric with respect to the origin**, for every point (x,y) on the graph, the point $(-x, -y)$ is also on the graph.

If you fold the graph at x-axis and then at the y-axis graphs line up to each other.

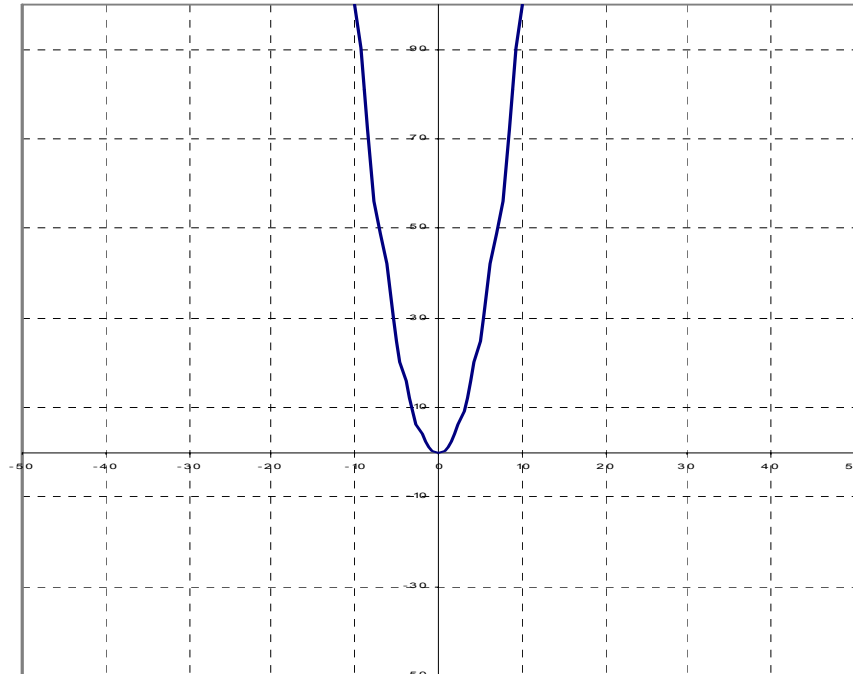
This is the graph of $y = x^2$

It is symmetric to the y-axis.

To check, plug in $(-x,y)$ and see if you get an equivalent equation.

$$y = (-x)^2 = x^2$$

Therefore it is symmetric to the y-axis



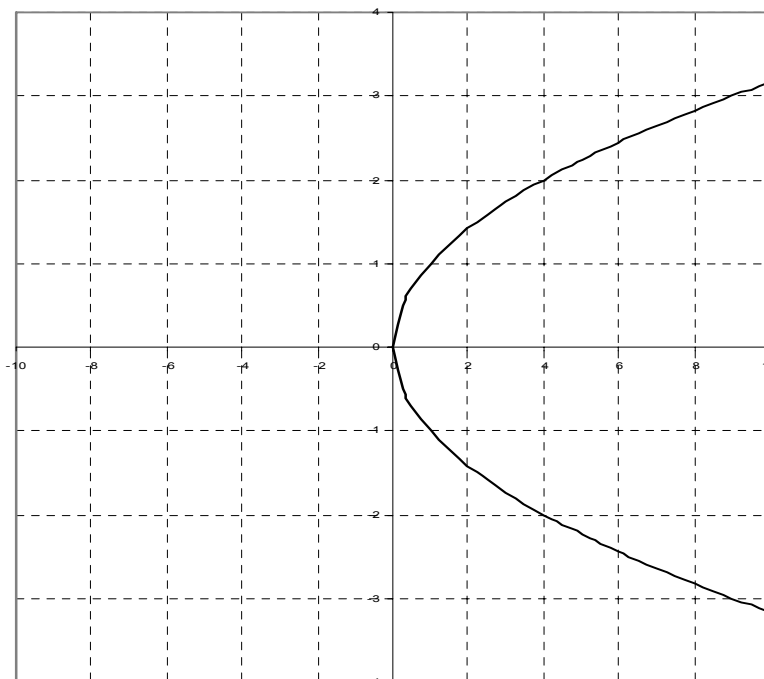
This is the graph of $x = y^2$

It is symmetric to the x-axis

To check, plug in $(x,-y)$ and see if you get an equivalent equation.

$$x = (-y)^2 = y^2$$

Therefore it is symmetric to the y-axis



This is the graph of $y = x^3$

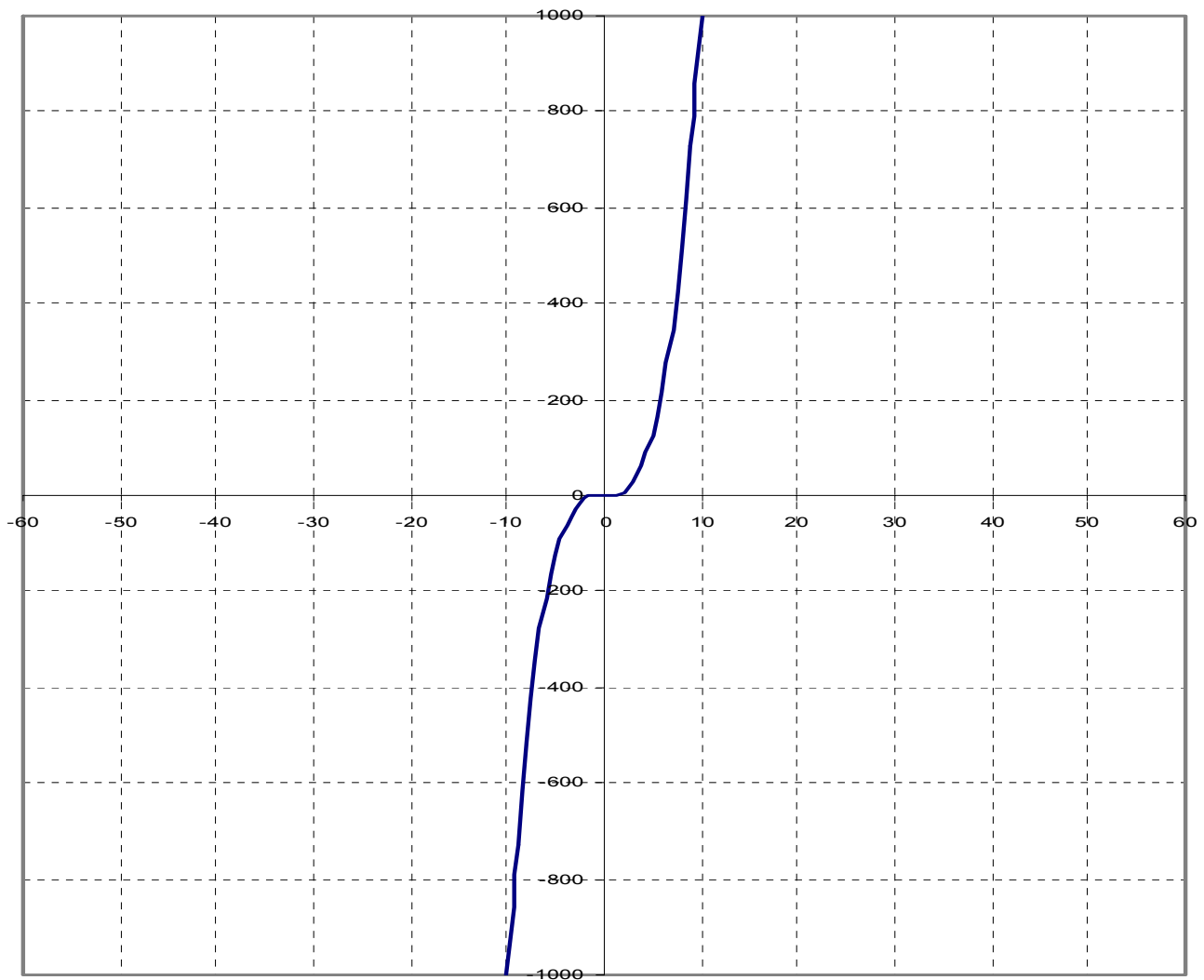
It is symmetric to the origin.

To check, plug in $(-x, -y)$ and see if you get an equivalent equation.

$-y = (-x)^3 = -x^3$ [since the exponent, 3, is an odd number]

Multiply both sides by -1 and you get $y = x^3$.

Therefore it is symmetric to the origin.



Test for Symmetry with respect to:

x-axis – Replace y by $-y$ and see if an equivalent equation results.

y-axis – Replace x by $-x$ and see if an equivalent equation results.

Origin – Replace x by $-x$ AND y by $-y$ and see if an equivalent equation results.

Example 10 on p.18

(Done in class)

Now you do #63 on p.23

HOMEWORK

A.2 p. 585 #9, 17,43

Ch. 1.1 p.8-9 #3-15,33-60 ETP*

Ch. 1.2 p.21-23 #12-45 ETP*

* Every Third Problem (ex. 3,6,9,12, etc..)