1.6 PROPERTIES OF FUNCTIONS

Even and Odd Functions

A function that is symmetric with the y-axis is called **even**.
In other words, a function *f* is **even** if for every number x in its domain the number –x is also in the domain and *f(-x) = f(x)*

A function that is symmetric with the origin is called **odd.**
In other words, a function f is odd if for every number x in its domain the number –x is also in the domain and *f(-x) = -f(x).*

Note a function symmetric with the x-axis is not odd. In fact, a function symmetric with the x-axis is not a function of x at all, because it does not pass the vertical line test.

Always verify your conjectures about functions with the definitions that *f(-x) = f(x)* for even functions and *f(x)= -f(x)* for odd functions.

Example 2 on p. 56
Use a graphing utility to conjecture whether the following functions are even, odd, or neither. State symmetry.

a) $f(x) = x^2-5$

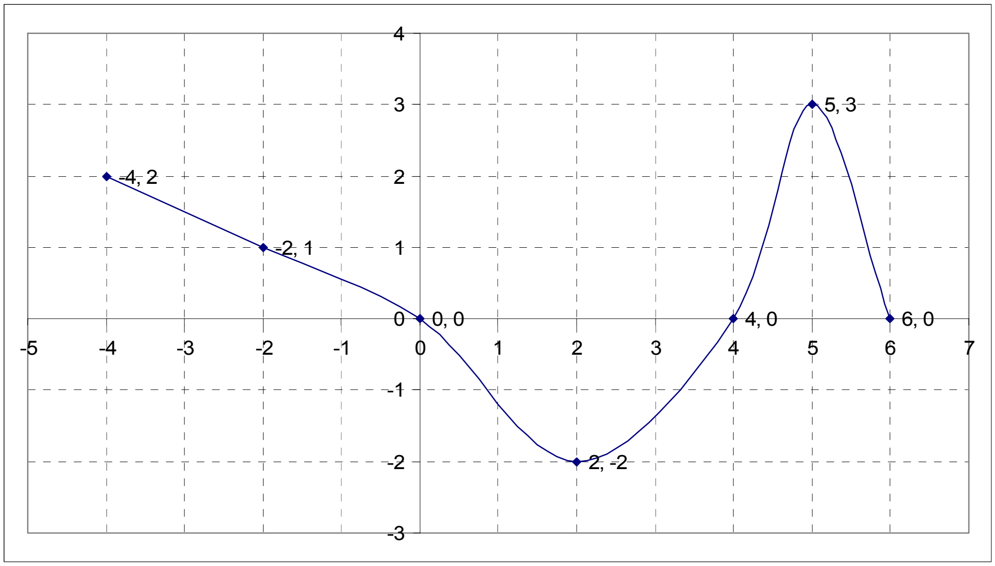
Check if even: $f(-x) = f(x)$?

$(-x)^2 – 5 = x^2 -5$ yes

Since it is an even function, it is symmetric with respect to the y-axis.

NOW YOU DO #51

1.5 Obtaining Information from the Graph of a Function
Problem 10 on p. 51



a) Find *f(0)* and *f(6)*
What is y when x is 0 and x is 6? From the data given, we see the y-coordinate at x=0 is 0, so *f(0)=0.* The y-coordinate at x=6 is also 0, so *f(6)=0.*

b) Find *f(2)* and *f(-2)*
What is y when x is 2 and x is -2? From the data given, we see the y-coordinate at x=2 is -2, so *f(2)=-2.* The y-coordinate at x=-2 is 1, so *f(-2)=1.*

c) Is f(3) positive or negative? We see that at x=3 the graph is below the x-axis (where y < 0) so f(*3)* is negative.

d) Is f(-*1)* positive or negative? We see that at x=3 the graph is below the x-axis (where y < 0) so f(*3)* is negative.

e) For what numbers x is f(x) = 0? In other words, at which values of x cross the x-axis (where y=0)? The graph crosses the x-axis at x=0,x=4, x=6.

f) For what numbers x is f(x) <0? In other words, at which values of x is the graph below the x-axis? Remember, the coordinates where y=0 are not included. The graph is < 0 only for 0 < x < 4. In interval form this is $(0,4)$.

g) What is the domain of f? Domain is the possible x values. Remember that this graph does not continue into infinity on both sides. It is only defined for the graph drawn. Therefore, we can infer that the possible x values are $-4 \le x \le 6$, or [-4,6]

h) What is the range of f? The y values range from as low as -2 to as high as 3, so range is $\{y \mid -2 \le y \le 3\}$ or [-2,3].

i) What are the x-intercepts? The x-intercepts are found when y=0, which are the points {(0,0), (4,0),(6,0)}.

j) What is the y-intercept? By definition, this would not be a function if it crossed the y-axis (or any other vertical line) more than once. The only point that does this is (0,0).

k) How often does the line y=-1 intersect the graph? If we draw a horizontal line through y=-1, we'd see it intersects twice.

l) How often does he line x=1 intersect the graph? Three times.

m) For what value of x does f(x) = 3? Remember f(x) is the same as y. What is x when y=5? There's only one point on the graph that gives a y-value of 3. That is when x=5.

n) For what value of x does f(x)=-2? There's only one point on the graph that gives a y-value of -2. That is when x=2.

Example 3 on p. 48

$$f(x) = \frac{x}{x+2}$$

a) Is the point (1, ½) on the graph of *f*? Substitute 1 for x and ½ for f(x) and see if the statement is true.
Does ½ = 1/(1+2)? ½ ≠ ⅓ Therefore (1, ½) is not on the graph.

b) If x = -1, what is f(x)? f(-1) = -1/(-1+2) = -1/1 = -1
The point at x=-1 is (-1,-1).

c) If f(x) = 2, what is x? YOU DO THIS!

Example 4 on p. 49 – Average Cost Function

$$\overline{C}(x) = 0.56x^2 - 34.39x + 1212.57 + \frac{20{,}000}{x}$$

Where $\overline{C}(x)$ = average cost and x = number of computers per day.

a) Determine average cost for 30 computers.

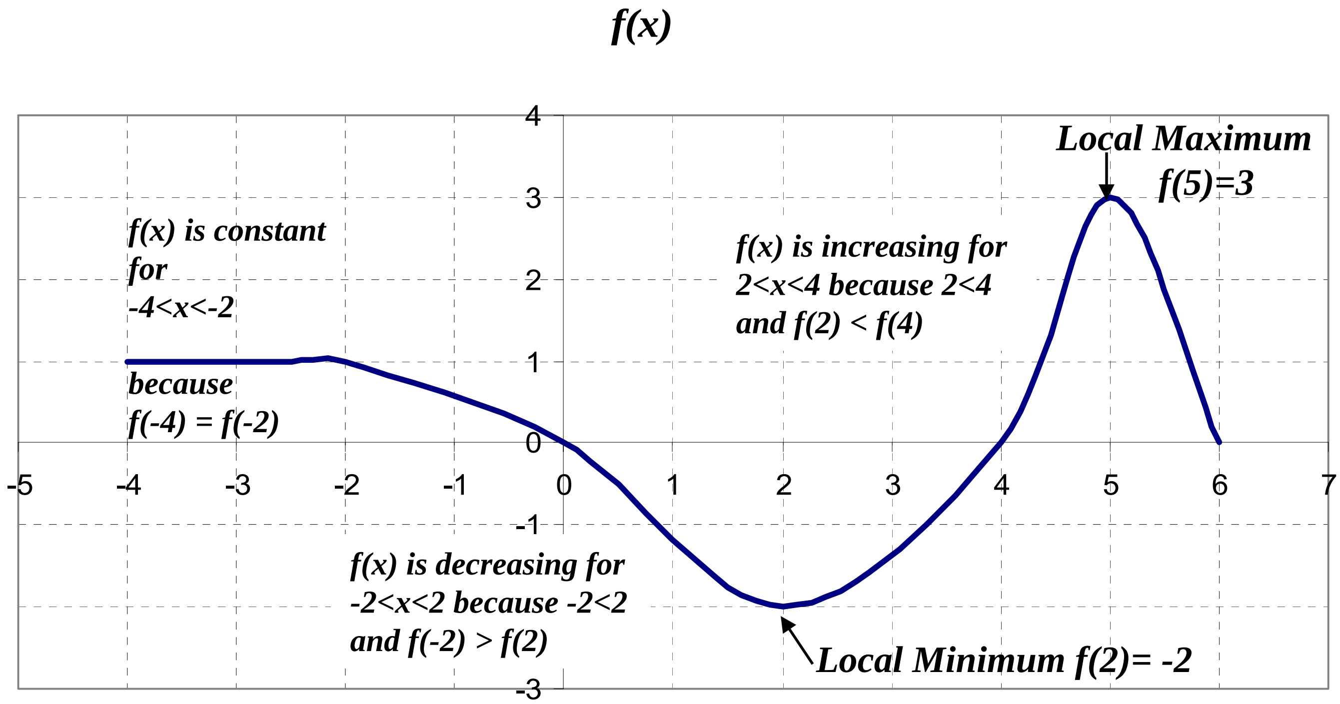
$$\overline{C}(30) = 0.56(30)^2 - 34.39(30) + 1212.57 + \frac{20{,}000}{30} = \$1351.54$$

d) Graph the function C= $\overline{C}(x)$, $0<x\le 80$. This means you should set your WINDOW for Xmin = 0 and Xmax = 80. Since $\overline{C}(30)$ = \$1351.54, you should set your Ymax to at least 2000. Let $Y_1 = \overline{C}(x)$

e) Create a TABLE with TblStart=1 and ΔTbl = 1. Which x minimizes the average cost. (i.e., which value of x gives the smallest value of $\overline{C}(x)$?

1.6 PROPERTIES OF FUNCTIONS

When is a function increasing, decreasing, or constant?



A function f is **increasing** on a open interval I if, for any choice of x_1 and x_2 in I, with $x_1 < x_2$, we have $f(x_1) < f(x_2)$.

A function f is **decreasing** on a open interval I if, for any choice of x_1 and x_2 in I, with $x_1 < x_2$, we have $f(x_1) > f(x_2)$.

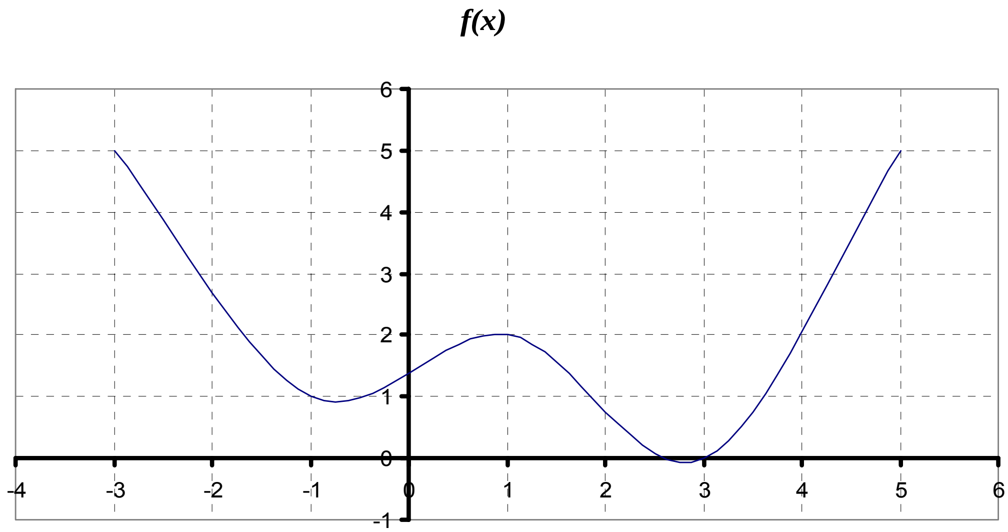
A function f is **constant** on a open interval I if, for any choice of x_1 and x_2 in I, we have $f(x_1) = f(x_2)$. *(The values f(x) are equal for all choices of x in I.)*

A function f has a ***local minimum of f*** at x=c if there is an open interval I containing c so that for all $x \neq c$ in I, $f(x) > f(c)$. We call $f(c)$ a local maximum of f.

A function f has a ***local maximum of f*** at x=c if there is an open interval I containing c so that for all $x \neq c$ in I, $f(x) < f(c)$. We call $f(c)$ a local maximum of f.

These are called "local" because there may be other points outside of the interval I that are less than the local minimum or greater than the local maximum.

Example 4 p.59



a) At what number(s) does *f* have a local maximum?
   There is an interval -1<x<3 where all f(x) are less than *f(1),* so f has a local maximum at 1.

b) What are the local maxima? The local maximum is *f*(1) = 2.

c) At what number(s) does *f* have a local minimum?
   There are two intervals with local mimimums. The interval x<1 has a local minimum at x=-1 and the interval x>1 has a local mimum at x=2.

d) What are the local minima? The local minima ar *f*(-1) = 1 and *f*(3)=0.

e) List the intervals on which *f* is increasing and on which *f* is decreasing.
   f is increasing at -1<x<1 and at x>3 →(-1,1) and $(3,\infty)$
   f is decreasing at x<-1 and at 1<x<3 →$(-\infty,-1)$ and (1,3)

NO DO PROBLEM #37 on p.67
The numbers at which *f* has a local maximum are the *x*-coordinates.
The local maxima are the *y*-coordinates.
For the graph at the bottom of p.67, list the numbers at which *f* has a local maximum.
What are these local maxima?

Example 5 on p.59 Using a graphing utility

a) Use a graphing utility to graph $f(x) = 6x^3 - 12x + 5$ for $-2<x<2$ to approximate the local maximum

Start off with setting the standard viewing window.

ZOOM , then select 6:ZStandard

Then press WINDOW and modify the viewing window so that Xmin=-2 and Xmax =2

Press Y=, then fill in with the function 6X^3-12X+5, then press GRAPH

You will be able to see the function better by fitting the window using Zoomfit

Press ZOOM, then select 0:ZoomFit

To calculate the maximum, press 2nd [CALC], then select 4:maximum

It wants to know the interval that you are searching for a local maximum for. Choose a leftbound and and right bound. You can type a value directly in the calculator or use the arrow buttons to surround the “protrusion” where the local maximum is. The “Guess?” is any point within the interval you have defined.

You should get x=-.816498 y=11.531973

Let $x = -.82$, $f(-.82) = 11.532$ = local maximum

To calculate the minimum, press 2nd [CALC], then select 3:minimum and follow the same instructions as above, except now you are looking for an “intrusion” where the local minimum is.

You should get x=.816498 y=-1.531973

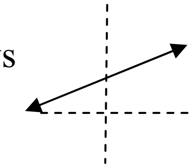
If you use the TABLE to show how f is increasing up to the point when x=-.82 and then decreases after that until you get to x=.82, at which point it increases again.

Press 2nd [TBLSET]

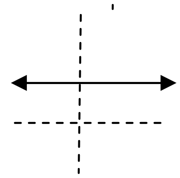
Let’s choose TblStart = -1 and ΔTbl = .02. Select Auto for both Independent and Dependent Variables then press 2nd [TABLE]

LIBRARY OF FUNCTIONS

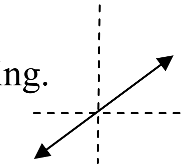
Linear Function: ***f(x)* = m*x* + b** where m and b are real numbers.
This is a nonvertical line with slope m and y-intercept b. Function is always increasing if m>0 and always decreasing if m<0.
Domain is all real numbers, Range is all real numbers.



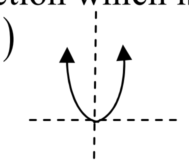
Constant Function: ***f(x)* = b** where b is a real number.
This is a horizontal line going through the y-intercept=b.
Domain is all real numbers. Range is the single number, b.



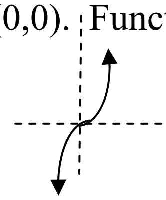
Identity Function: ***f(x)* = *x***
This is a line with slope = 1 and y-intercept = 0. Function is always increasing.
Domain is all real numbers. Range is all real numbers.



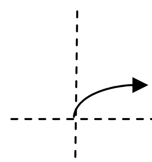
Square Function: ***f(x)* = *x*²**
This is an upward parabola with vertex at the origin. It is an even function which is decreasing for the interval $(-\infty,0)$ and increasing on the interval $(0,\infty)$
Domain is all real numbesrs. Range is {y|y>0}



Cube Function: ***f(x)* = *x*³**
This is an odd function whose y-intercept is at (0,0). Function is always increasing.
Domain is all real numbers.
Range is all real numbers.

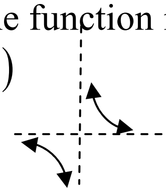


Square Root Function: $f(x) = \sqrt{x}$

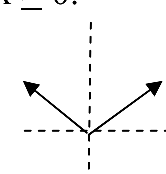


This is a function that is increasing on the interval x> 0.
Domain is {x|x>0}
Range is {y|y>0}

Reciprocal Function: ***f(x)* = 1/*x***
This is an odd function with no intercepts. The function is undefined at x=0 and is decreasing on the intervals $(-\infty,0)$ and $(0,\infty)$
Domain is {x|x≠0}
Range is {y|y≠0}



Absolute Value Function: ***f(x)* = |*x*|**
This function is the line f(x) = -x for x ≤ 0 and then changes to f(x) = x for x ≥ 0.
This is an even function that is decreasing for x<0 and increasing for x>0.
Domain is all real numbers.
Range is {y|y ≥ 0}



HOMEWORK
Ch. 1.5 p.51 #9-18 ETP, p.10 #21-27 ETP
Ch. 1.6 p.66 #29-37 EOO**
p.67#41-61 EOO**, p.68#71,75,81-84
EXTRA CREDIT:
Ch.1 Review p.107 #1-29 EOO**
p.108 #33-57 EOO
p.109 #61