

1.7 Graphing Techniques

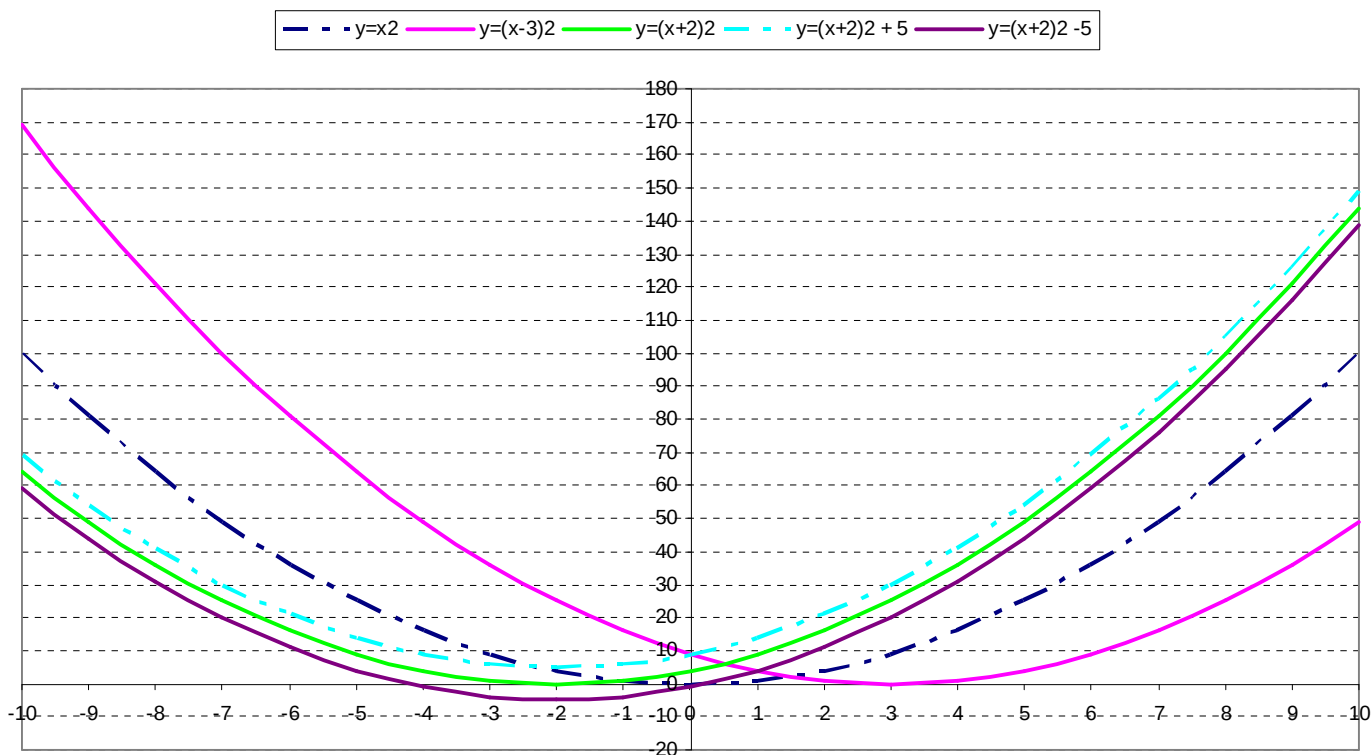
Vertical and Horizontal Shifts

If the argument x of a function is replaced by $x-h$, $h>0$, the graph of the new function $y = f(x-h)$ is the graph of f shifted **horizontally to the RIGHT** by h units.

If the argument x of a function is replaced by $x+h$, $h>0$, the graph of the new function $y = f(x+h)$ is the graph of f shifted **horizontally to the LEFT** by h units.

If a constant k ($k>0$) is added to a function, so the new function is $y = f(x) + k$, then the graph of f is shifted vertically UP by k units.

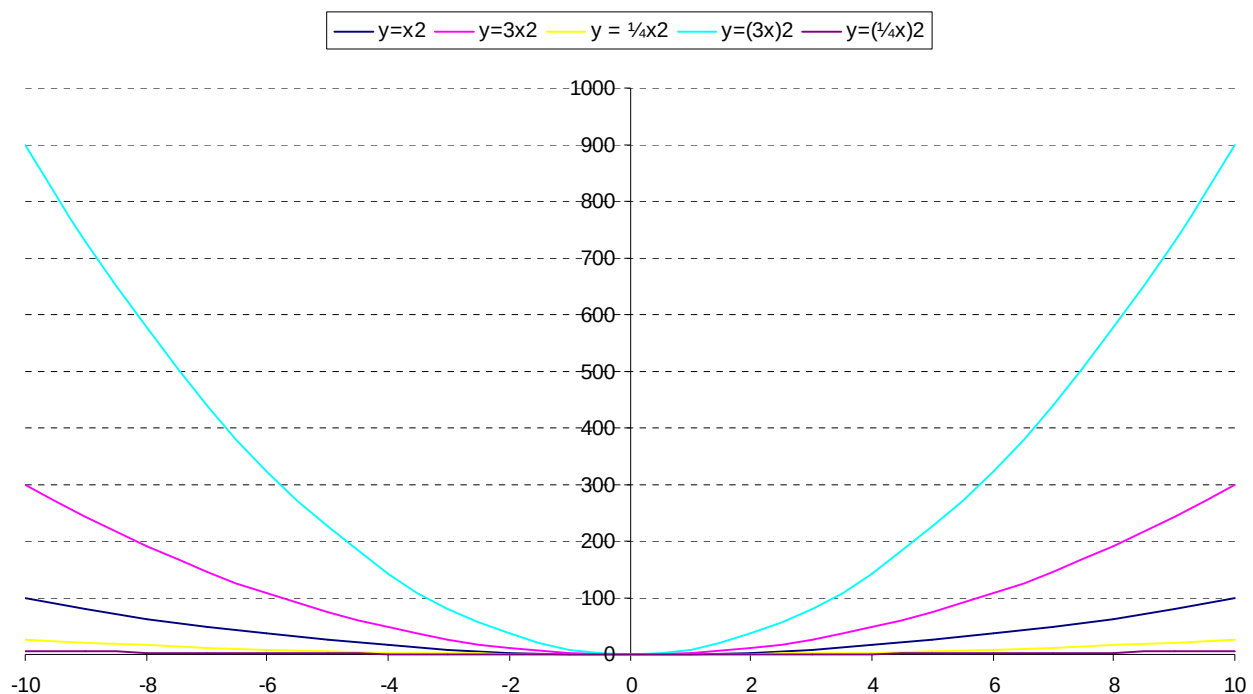
If a constant k ($k>0$) is subtracted from a function, so the new function is $y = f(x) - k$, then the graph of f is shifted **vertically DOWN** by k units.



Stretches and Compressions

$y = af(x)$, $a > 0$ This function multiplies each y-coordinate of $f(x)$ by a . The graph of f is **stretched vertically** if $a > 1$.
The graph of f is **compressed vertically** if $0 < a < 1$.

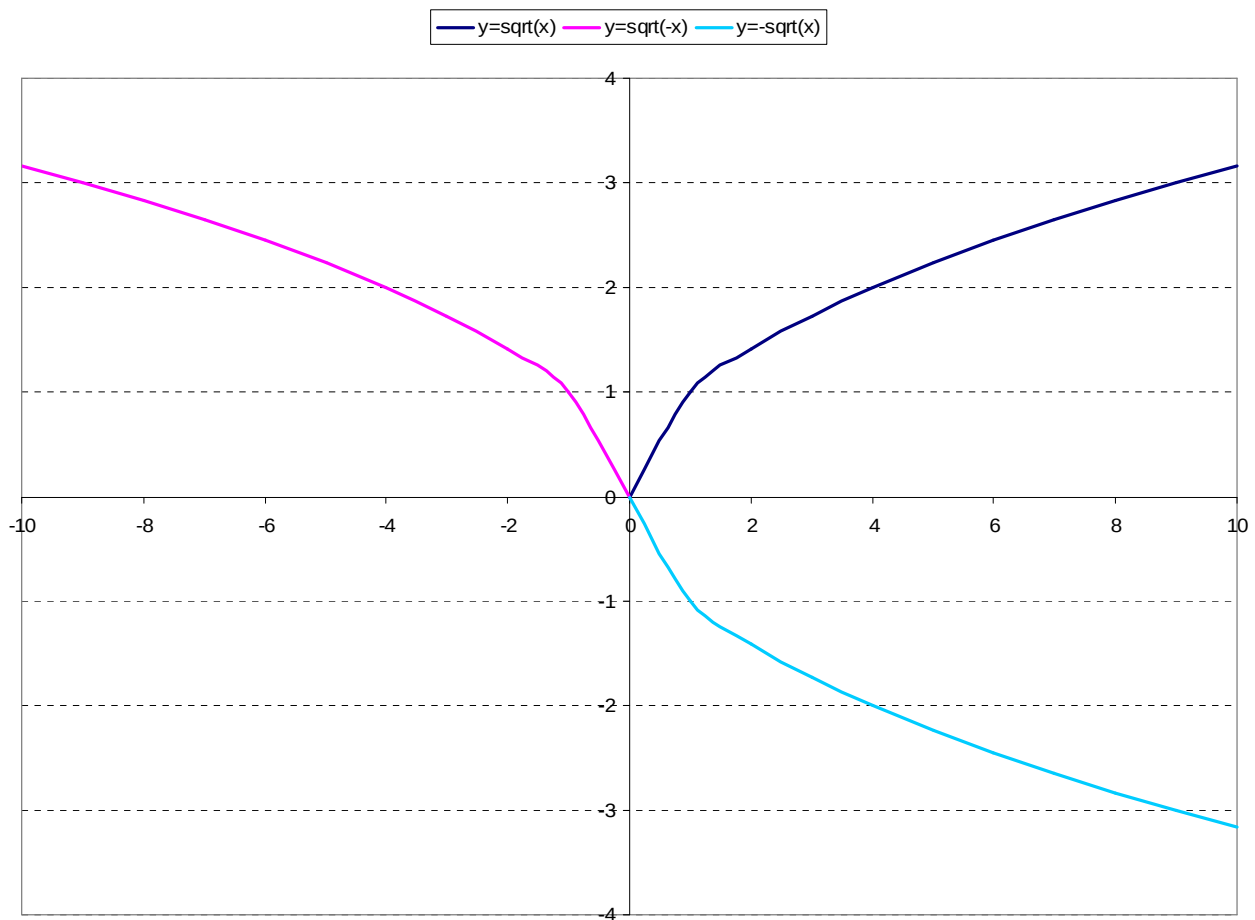
$y = f(ax)$, $a > 0$ This function multiplies each x-coordinate of $f(x)$ by $1/a$. The graph of f is **stretched horizontally** if $0 < a < 1$.
The graph of f is **compressed horizontally** if $a > 1$.



Reflections

When the right side of the function $y = f(x)$ is multiplied by -1 , the graph of the **new function $y = -f(x)$ is the reflection about the x-axis of the graph of the function $f(x)$.**

When each x-coordinate in $f(x)$ multiplied by -1 , the graph of **the new function $y = f(-x)$ is the reflection about the y-axis of the graph of the function $f(x)$.**



Example 5 Combining Graphing Procedures

Graph the function: $f(x) = \frac{3}{x-2} + 1$

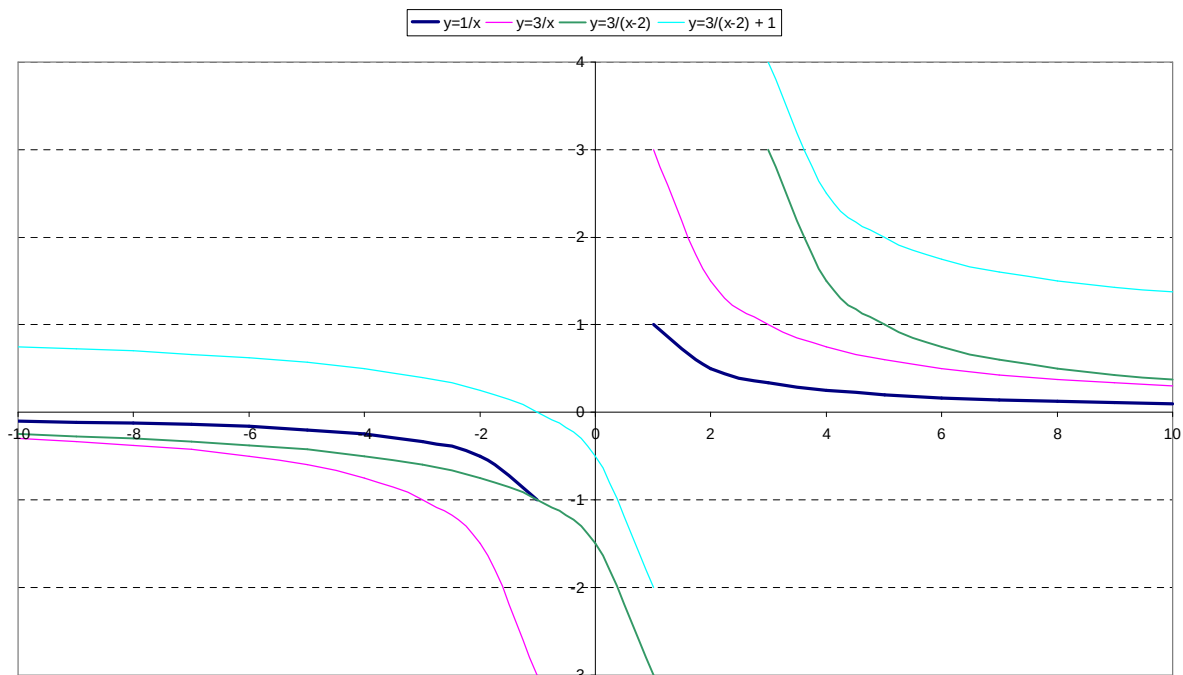
Step 1: From the Library of Functions, which one does this most look like? $y = 1/x$

Step 2: We did we do to $1/x$ to get it to be $f(x) = \frac{3}{x-2} + 1$?

We multiplied $1/x$ by 3 (*multiplying $f(x)$ by 3 means we stretch the graph vertically since $3 > 1$*)

Step 3: What else did we do? We subtracted 2 from x . This means we shift the graph of f to the RIGHT by 2 units.

Step 4: What else did we do? We added 1 to $f(x)$. This means we shift the graph of f UP by 2 units.



Example 6

Graph the function: $f(x) = \sqrt{1-x} + 2$

Step 1: What function from the Library of Functions does this look like? $f(x) = \sqrt{x}$

Step 2: What did we do to $f(x) = \sqrt{x}$ to make it look like

$$f(x) = \sqrt{1-x} + 2 \quad ?$$

Let's re-write the function so x is in the front: $f(x) = \sqrt{-x+1} + 2$

Now we can see that x was made into $-x$ and then 1 was added to it.

Replacing x by $-x$ reflects the graph of $f(x) = \sqrt{x}$ about the _____-axis.

Adding 1 to $-x$ shifts the graph _____ to the _____ by _____ units.

Adding 2 to $f(x)$ shifts the graph _____ by _____ units.

NOW YOU DO #57 on p.79

1.8 COMPOSITE FUNCTIONS

A **composite function** is the combination of one function with another. The function resulting from “f composed with g” is defined by:

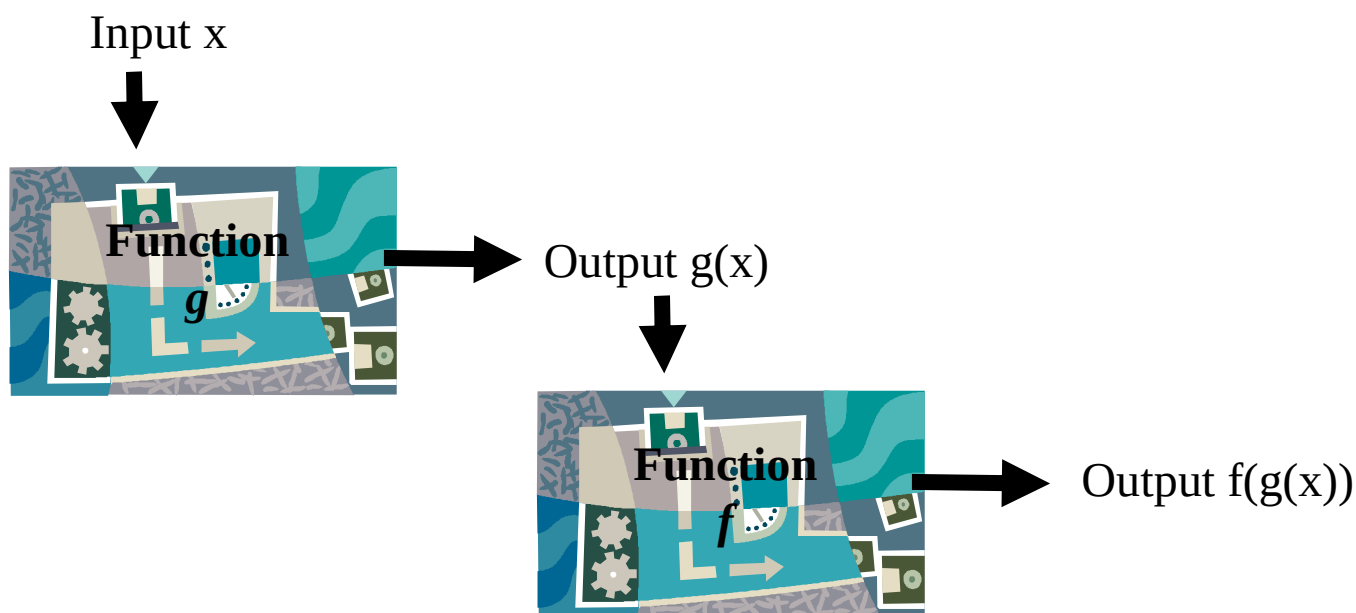
$$(f \circ g)(x) = f(g(x))$$

This means that $g(x)$ becomes the new argument of $f(x)$.

The domain of $f(g(x))$ is the set of all numbers x in the domain of g such that $g(x)$ is in the domain of f .

Recall that the domain of the function is all possible inputs and the range of a function is all possible outputs.

In the composite function, $(f \circ g)(x)$, the range of $g(x)$ now must be within the domain of $f(x)$.



In class, we'll do #10 on p.87

Example 4 Finding a Composite Function and its Domain

$$f(x) = \frac{1}{x+2} \quad \text{and} \quad g(x) = \frac{4}{x-1}$$

a) Find: $f \circ g$

Step 1: What are the domains of $f(x)$ and $g(x)$?

The domain of $f(x)$ is $\{x|x \neq -2\}$ and the domain of $g(x)$ is $\{x|x \neq 1\}$

Since the range of $g(x)$ must be in the domain of $f(x)$, we do not want the output of $g(x)$ to be -2. What is x if $g(x) = -2$?

$$g(x) = \frac{4}{x-1} = -2$$

$$-2(x-1) = 4$$

$$-2x + 2 = 4$$

$$-2x = 2$$

$$x = -1$$

Therefore, $x = -1$ is also not in the domain of $f \circ g$

Step 2: What is $f(g(x))$? Plug in $g(x)$ for x in $f(x)$.

$$f(g(x)) = \frac{1}{\left(\frac{4}{x-1}\right) + 2} = \left(\frac{1}{\left(\frac{4}{x-1}\right) + 2}\right) \frac{(x-1)}{(x-1)} = \frac{x-1}{4+2(x-1)} = \frac{x-1}{4+2x-2} = \frac{x-1}{2x+2} = \frac{x-1}{2(x+1)}$$

Now do #35 on p.88

HOMEWORK

Ch. 1.7 p.78-79 #21-63 ETP

Ch. 1.8 p. 87-88 #9-45 ETP, #65,66