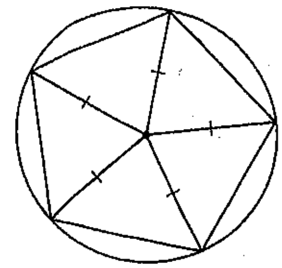
CONDENSED LESSON

Areas of Regular Polygons

In this lesson you will

- Discover the area formula for regular polygons

You can divide a regular polygon into congruent isosceles triangles by drawing segments from the center of the polygon to each vertex. The center of the polygon is actually the center of the circumscribed circle, so each of these congruent segments is called a radius of the regular polygon.



In the investigation you will divide regular polygons into triangles. Then you will write a formula for the area of any regular polygon.

Investigation: Area Formula for Regular Polygons

The **apothem** of a regular polygon is a perpendicular segment from the center of the polygon's circumscribed circle to a side of the polygon. The apothem is also the length of the segment. Follow the steps in your book to find the formula for the area of a regular n-sided polygon with sides of length s and apothem a. Your findings can be summarized in this conjecture.

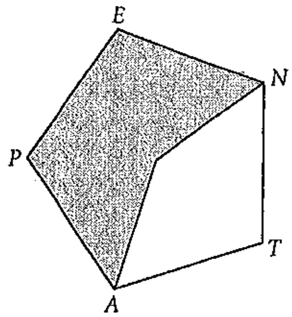
Regular Polygon Area Conjecture The area of a regular polygon is given by the formula $A = \frac{1}{2}asn$ or $A = \frac{1}{2}aP$, where A is the area, P is the perimeter, a is the apothem, s is the length of each side, and n is the number of sides. **C-79**

The examples below show you how to apply your new formulas.

EXAMPLE A The area of a regular nonagon is about 302.4 cm^2 and the apothem is about 9.6 cm. Find the approximate length of each side.

▶ ***Solution***

EXAMPLE B Find the shaded area of the regular pentagon *PENTA*. The apothem measures about 2.0 cm. Segment *PE* measures about 2.9 cm.



Solution

CONDENSED LESSON

Areas of Circles

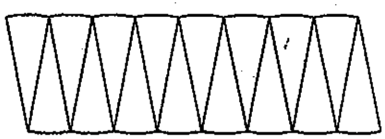
In this lesson you will

- Discover the area formula for circles

A rectangle has straight sides, while a circle is entirely curved. So, it may surprise you that you can use the area formula for rectangles to help you find the area formula for circles. In the next investigation you'll see how.

Investigation: Area Formula for Circles

Follow Steps 1–3 in your book to create a figure like the one below.



The figure resembles a parallelogram with two bumpy sides. If you cut the circle into more wedges, you could arrange these thinner wedges to look more like a rectangle. You would not lose or gain any area in this change, so the area of this new "rectangle" would be the same as the area of the original circle. If you could cut infinitely many wedges, you'd actually have a rectangle with smooth sides.

The two longer sides of the rectangle would be made up of the circumference, C, of the circle. (Each side would be half the circumference.) Consider one of these sides to be the base. Recall the formula for the circumference of a circle that you learned in Chapter 6. Now use this formula to write the length of the base of the rectangle in terms of r, the radius of the original circle.

How is the height of the rectangle related to the original circle?

Remember, the area of the rectangle is the same as the area of the original circle. Use this idea and your findings to complete this conjecture.

Circle Area Conjecture The area of a circle is given by the formula $A =$ ________________, where A is the area and r is the radius of the circle. **C-80**

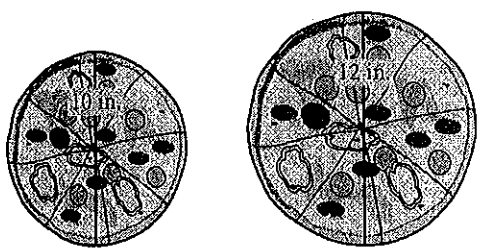
Examples A and B in your book show how to use your new conjecture. Read these examples, and then read the examples on the next page.

(continued)

EXAMPLE A The circumference of a circle is 22π ft. What is the area of the circle?

Solution Use the circumference formula to find the radius. Then use the area formula to find the area.

EXAMPLE B At Maria's Pizzeria, a pepperoni pizza with diameter 10 inches costs \$8, and a pepperoni pizza with diameter 12 inches costs \$10. Which size is a better buy?



\$8 \$10

Solution Find the area of each pizza, and then find the price per square inch.

10-inch pizza | **12-inch pizza**

CONDENSED LESSON

Any Way You Slice It

In this lesson you will

- Learn how to find the area of a sector, a segment, and an annulus of a circle

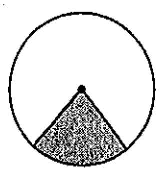
In Lesson 8.5, you discovered the formula for calculating the area of a circle. In this lesson you'll learn how to find the areas of three types of sections of a circle.

A **sector of a circle** is the region between two radii and an arc of the circle.

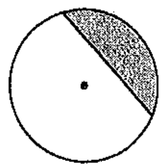
A **segment of a circle** is the region between a chord and an arc of the circle.

An **annulus** is the region between two concentric circles.

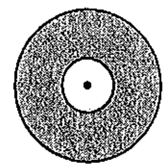
Examples of the three types of sections are pictured below.



Sector of a circle

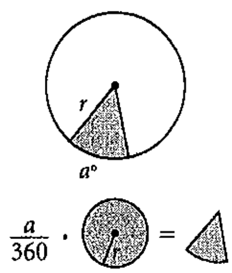


Segment of a circle

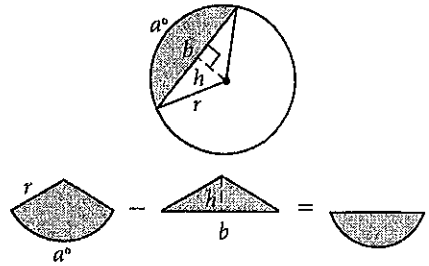


Annulus

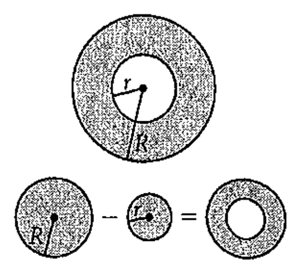
The "picture equations" below show you how to calculate the area of each type of section.



$\frac{a}{360} \cdot \pi r^2 = A_{\text{sector}}$



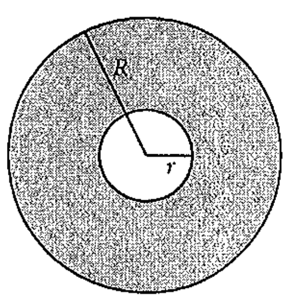
$\frac{a}{360}\pi r^2 - \frac{1}{2}bh = A_{\text{segment}}$



$\pi R^2 - \pi r^2 = A_{\text{annulus}}$

Read the examples in your book carefully. Then read the examples below.

EXAMPLE A $R = 9$ cm and $r = 3$ cm. Find the area of the annulus.

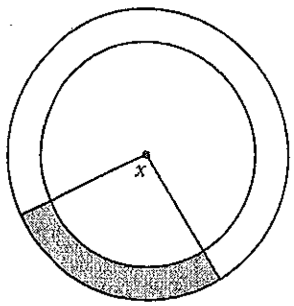


(continued)

Solution

EXAMPLE B

The shaded area is 21π cm^2. The radius of the large circle is 12 cm, and the radius of the small circle is 9 cm. Find x, the measure of the central angle.



Solution

First, find the area of the whole annulus.

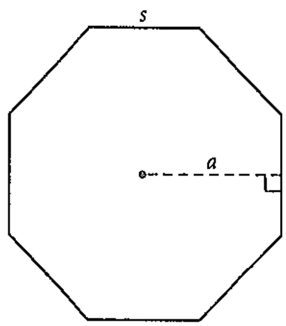
• Areas of Regular Polygons

Name ______________________________ Period __________ Date ____________

In Exercises 1–3, the polygons are regular.

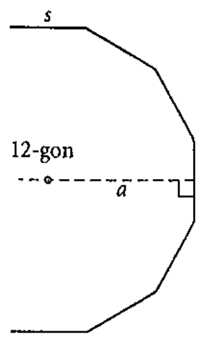
1. $s = 12$ cm
$a \approx 14.5$ cm

$A \approx$ _____



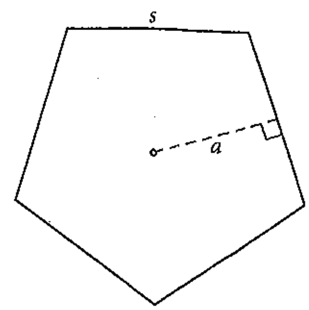
2. $s = 4.2$ cm
$A \approx 197\ \text{cm}^2$

$a \approx$ _____



3. $a = 6$ cm
$A \approx 130.8\ \text{cm}^2$

$p \approx$ _____

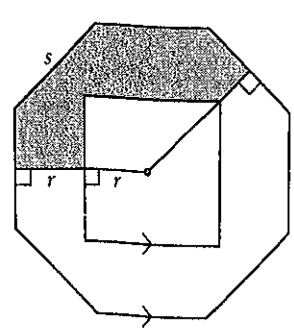


4. In a regular n-gon, $s = 4.8$ cm, $a \approx 7.4$ cm, and $A \approx 177.6\ \text{cm}^2$. Find n.

5. Draw a regular pentagon so that it has perimeter 20 cm. Use the Regular Polygon Area Conjecture and a centimeter ruler to find its approximate area.

6. Use a compass and straightedge to construct a regular octagon and its apothem. Use a centimeter ruler to measure its side length and apothem, and use the Regular Polygon Area Conjecture to find its approximate area.

7. Find the area of the shaded region between the square and the regular octagon. $s \approx 5$ cm. $r = 3$ cm.



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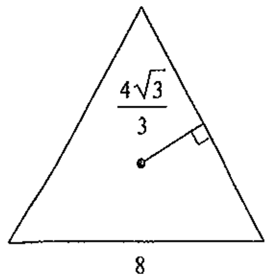
Area of Regular Polygons

Date________________ Period____

Find the area of each regular polygon. Leave your answer in simplest form.

1)



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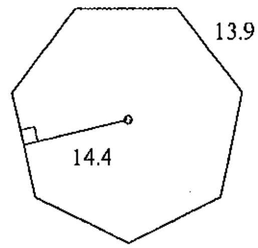
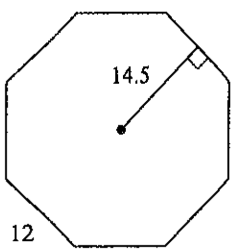
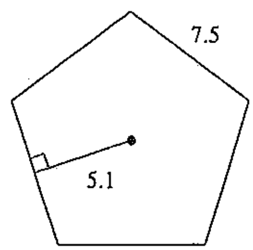


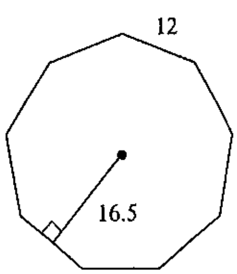
3)



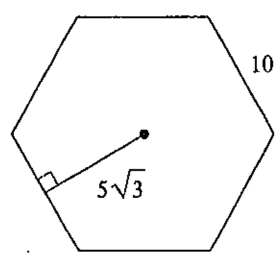
4)



5)



6)



7) pentagon
apothem = 7.3
side = 10.6

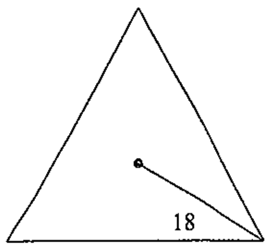
8) triangle
apothem = 14
side = $28\sqrt{3}$

9) 7-gon
apothem = 21.8
side = 21

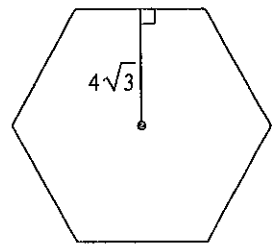
10) octagon
apothem = 14.1
side = 11.7

Use what you know about special right triangles to find the area of each regular polygon. Leave your answer in simplest form.

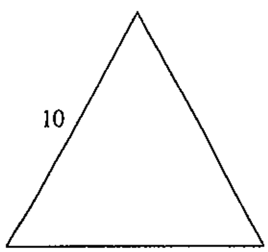
11)



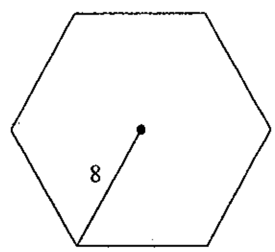
12)



13)



14)



15) quadrilateral
radius = $16\sqrt{2}$

16) hexagon
side = $\dfrac{16\sqrt{3}}{3}$

Critical thinking questions:

17) Find the perimeter of a regular hexagon that has an area of $54\sqrt{3}$ units².

18) Can a regular octagon have an area of 10 units²?

• Areas of Circles

Name ______________________ Period __________ Date ______________

In Exercises 1–4, write your answers in terms of π.

1. If $r = 9$ cm, $A =$ _____.

2. If $d = 6.4$ cm, $A =$ _____.

3. If $A = 529\pi$ cm^2, $r =$ _____.

4. If $C = 36\pi$ cm, $A =$ _____.

In Exercises 5–8, round your answers to the nearest 0.01 unit.

5. If $r = 7.8$ cm, $A \approx$ _____.

6. If $A = 136.46$, $C \approx$ _____.

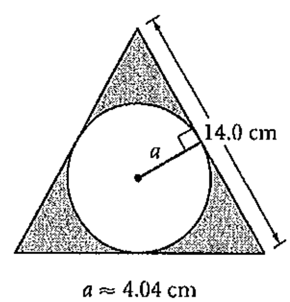
7. If $d = 3.12$, $A \approx$ _____.

8. If $C = 7.85$, $A \approx$ _____.

For Exercises 9 and 10, refer to the figure of a circle inscribed in an equilateral triangle. Round your answers to the nearest 0.1 unit.

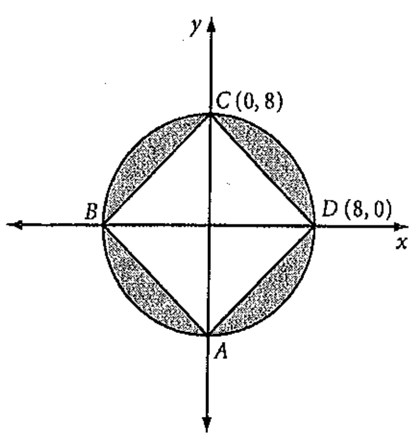
9. Find the area of the inscribed circle.

10. Find the area of the shaded region.

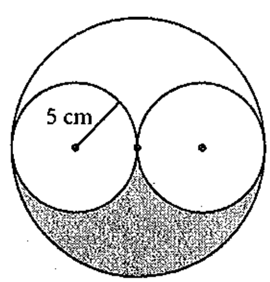


In Exercises 11 and 12, find the area of the shaded region. Write your answers in terms of π.

11. *ABCD* is a square.



12. The three circles are tangent.

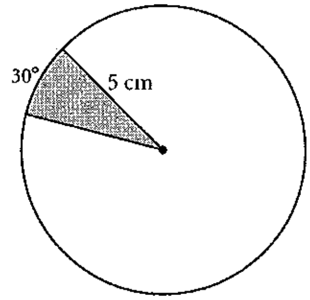


• Any Way You Slice It

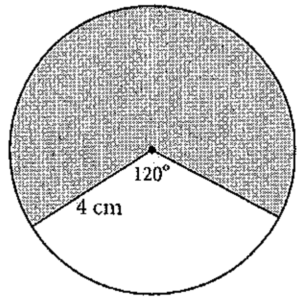
Name ______________________ Period ________ Date ____________

In Exercises 1–6, find the area of the shaded region. Write your answers in terms of π and rounded to the nearest 0.01 cm^2.

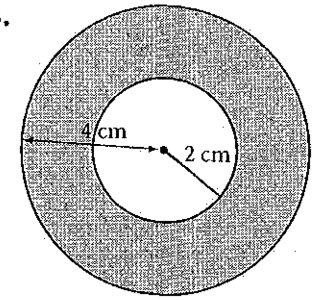
1.



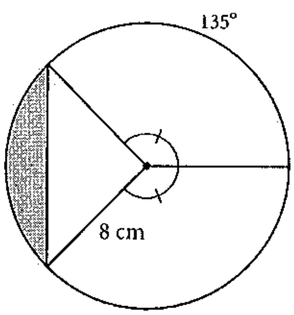
2.



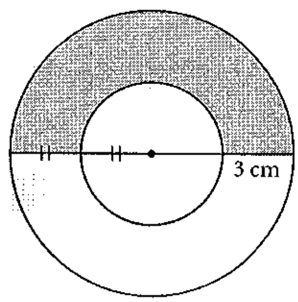
3.



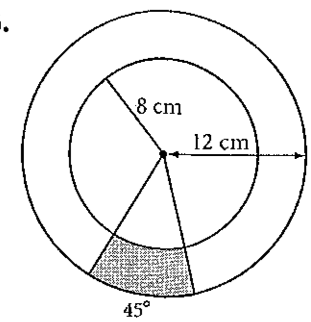
4.



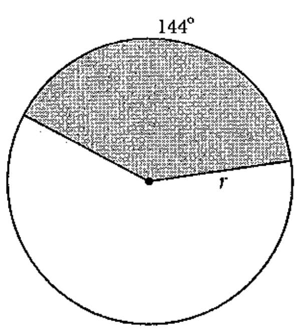
5.



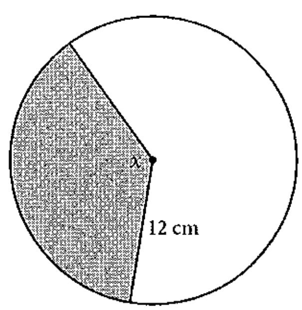
6.



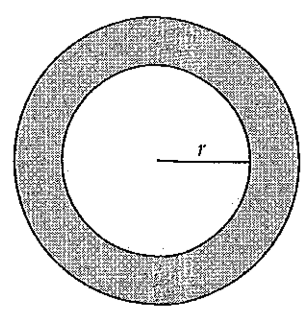
7. Shaded area is 40π cm^2. Find r.



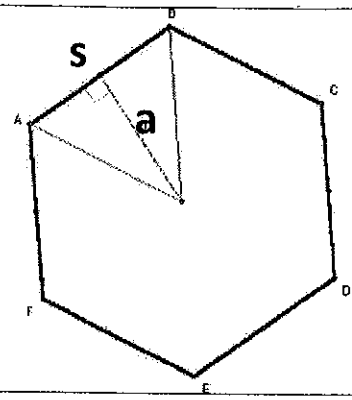
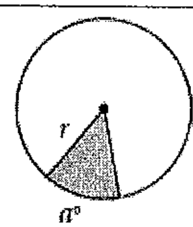
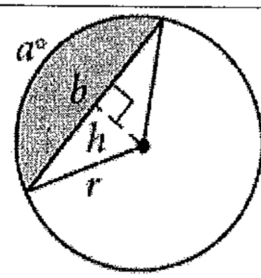
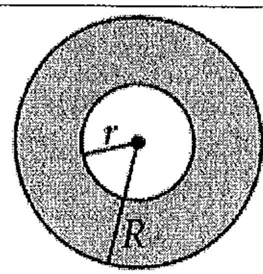
8. Shaded area is 54π cm^2. Find x.



9. Shaded area is 51π cm^2. The diameter of the larger circle is 20 cm. Find r.



More Area Formula's (8.3-8.5)

n-sided Regular Polygon		$\frac{1}{2} asn$
Circle		πr^2
Sector		$\frac{a}{360} \cdot \pi r^2$
Segment		$\frac{a}{360} \pi r^2 - \frac{1}{2} bh$
Annulus		$\pi R^2 - \pi r^2$