SYSTEMS OF LINEAR EQUATIONS

A system of linear equations is a set of two equations of lines.
A solution of a system of linear equations is the set of ordered pairs that makes each equation true. That is, the set of ordered pairs where the two lines intersect.
If the system is ________________, there is ONE SOLUTION, an ordered pair (x,y)
If the system is _______________, there is INFINITELY MANY SOLUTIONS, all (x,y)'s that make either equation true (since both equations are essentially the same in this case.
If the system is ________________, there are NO SOLUTIONS, because the two equations represent parallel lines, which never intersect.

GRAPHING METHOD.
Graph each line. This is easily done be putting them in slope-intercept form, $y = mx + b$.
The solution is the point where the two lines intersect.

SUBSTITUTION METHOD

$2x – y = 5$

$3x + y = 5$

Choose equation to isolate a variable to solve for. In this system, solving for y in the second equation makes the most sense, since y is already positive and has a coefficient of 1.

This second equation turns into $y = -3x + 5$

Now that you have an equation for y in terms of x, substitute that equation for y in the first equation in your system.

Substitute $y = -3x + 5$ in $2x – y =5$

ADDITION METHOD

$5x + 2y = -9$

$12x - 7y = 2$

Eliminate one variable by finding the LCM of the coefficients, then multiply both sides of the equations by whatever it takes to get the LCM in one equation and –LCM in the other equation. After this we can add both equations together and eliminate a variable.

Let's choose to eliminate y. The y terms are 2y and -7y. The LCM is ____

Orthocenter:

Calculate the slope of sides AB and BC of the triangle using the slope formula.

Then, using the point-slope form of an equation, and the fact that perpendicular lines have slopes that are negative reciprocals, write equations of the two altitudes to sides AB and BC -- lines perpendicular to AC and passing through B and perpendicular to BC and passing through A.

Solve the 2X2 system. The intersection of the two altitudes is the orthocenter.

Circumcenter

Using the slopes calculated above for AB and BC and the mid-points calculated for the Centroid solution, write equations of the perpendicular bisectors of AB and BC. Perpendicular to AB and passing through the mid-point of AB, then perpendicular to BC and passing through the mid-point of BC.

Solve the 2X2 system. The intersection of the perpendicular bisectors is the circumcenter (a point equidistant from the three vertices and therefore the center of a circle that passes through the three vertices of the triangle.)

http://easycalculation.com/analytical/circumcenter-triangle.php

Method to calculate the circumcenter of a triangle: Let the points of the sides be A(5,7), B(6,6) and C(2,-2). Consider the points of the sides to be (x_1,y_1) and (x_2,y_2) respectively. We need to find the equation of the perpendicular bisectors to find the points of the Circumcenter.

Step 1:
Lets calculate the midpoint of the sides AB, BC and CA which is the average of the x and y co-ordinates. Midpoint of a line in the triangle = $(x_1+x_2)/2, (y_1+y_2)/2$

Midpoint of AB =

Midpoint of BC =

Midpoint of CA =

Step 2:
Next, we need to find the slope of the sides AB, BC and CA using the point-slope formula

$(y_2-y_1)/(x_2-x_1)$.

Slope of AB =

Slope of BC =

Slope of CA =

Step 3:
Now, lets calculate the slope of the perpendicular bisector of the lines AB, BC and CA. The slope of the perpendicular bisector = -1/slope of the line.

Slope of the perpendicular bisector of AB =

Slope of the perpendicular bisector of BC =

Slope of the perpendicular bisector of CA =

Step 4:
Once we find the slope of the perpendicular lines, we have to find the equation of the perpendicular bisectors with the slope and the midpoints. Lets find the equation of the perpendicular bisector of AB with midpoints (11/2,13/2) and the slope 1. Formula to find the circumcenter equation:

$y-y_1 = m(x-x_1)$

$y-13/2 = 1(x-11/2)$

By solving the above, we get the equation _____________

Similarly, we have to find the equation of the perpendicular bisectors of the lines BE and CF.

For BC with midpoints (4,2) and slope -1/2,

$y-2 = -1/2(x-4)$

By solving the above, we get the equation ____________

For CA with midpoints (7/2,5/2) and slope -1/3

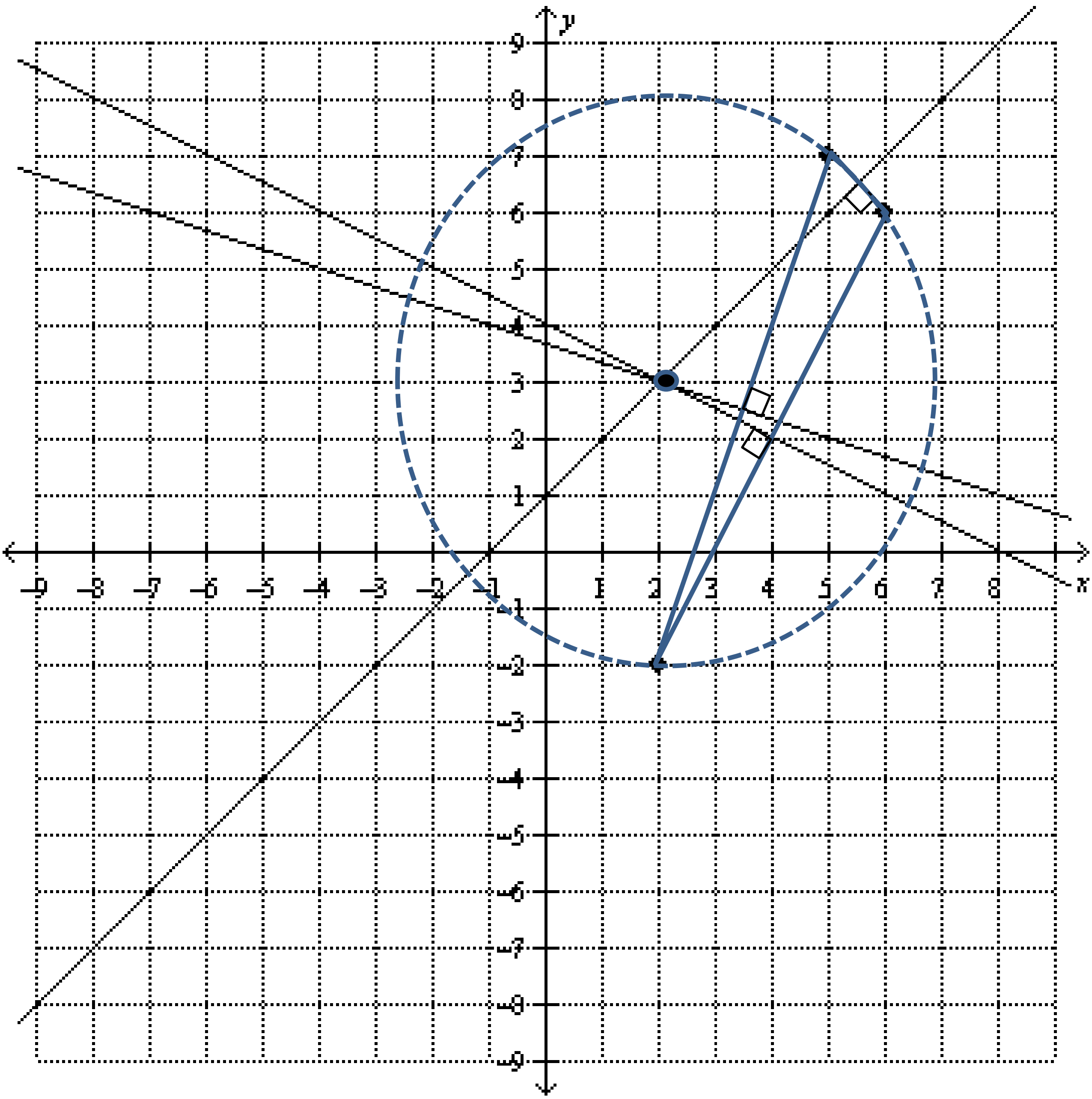
$y-5/2 = -1/3(x-7/2)$

By solving the above, we get the equation ____________

Step 5:
Find the value of x and y by solving any 2 of the above 3 equations.

In this example, the values of x any y are (2,3) which are the coordinates of the Circumcenter.

Circumcenter – Intersection of the perpendicular bisectors



Centroid – intersection of the medians

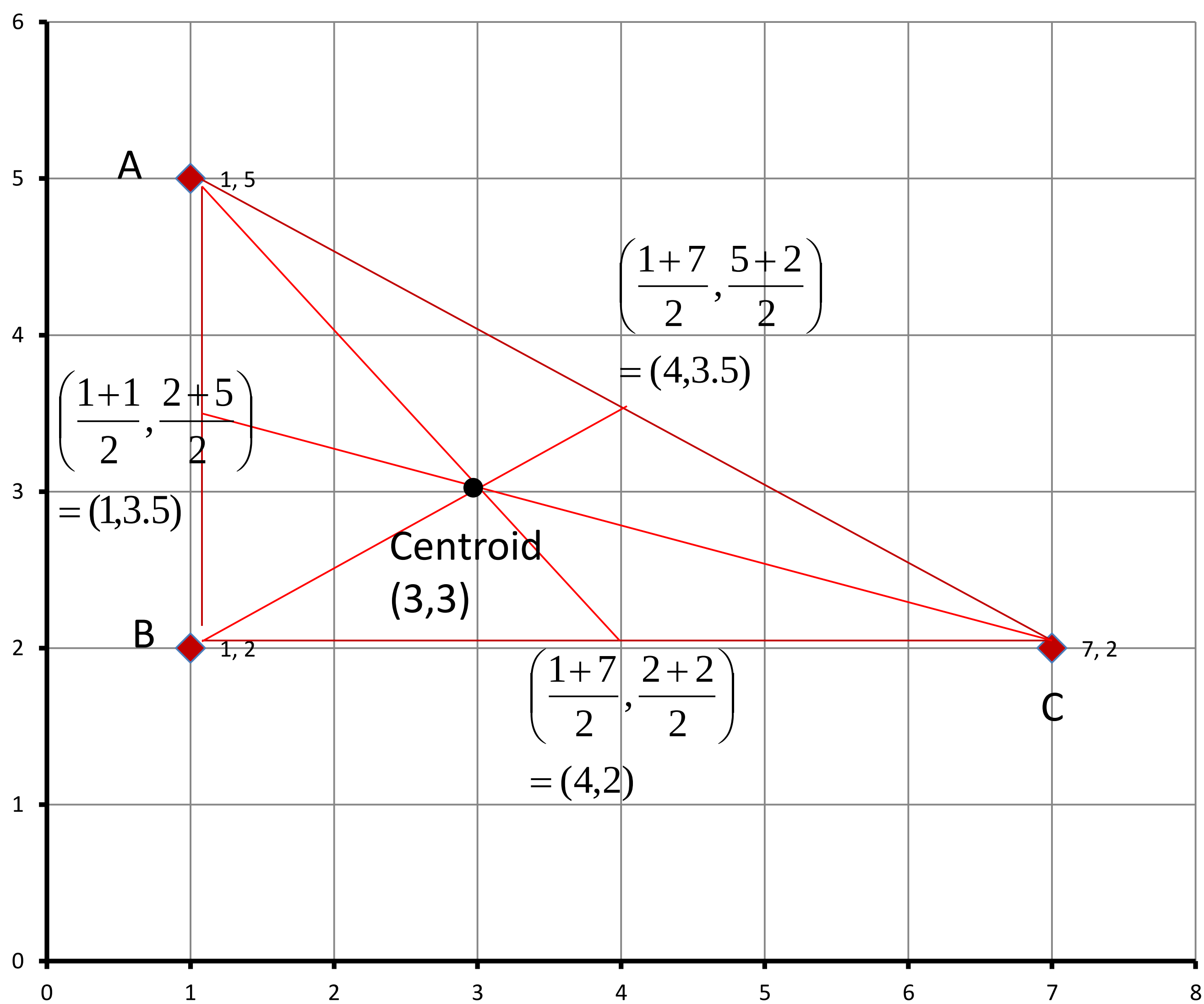
Long way: Find the midpoint of each side, then find the slope between the midpoint and the opposite vertex. Now use the point-slope formula using this slope and the vertex used to find the linear equation for the median. The intersection of all the medians is the centroid.

Shortcut to finding the coordinates:

Just find the average of all the vertex x-coordinates and vertex y-coordinates!

A (1,5) B(1,2) C(7,2)

Centroid: $$\left(\frac{1+1+7}{3},\frac{5+2+2}{3}\right)=\left(\frac{9}{3},\frac{9}{3}\right)=(3,3)$$



Worksheet
Centroid, Circumcenter, Orthocenter

Find the coordinates of the centroid given the vertices of the following triangles.

1. P(-6, 9), Q(6, 1), R(-6, -7)

2. A(12, 18), B(18, 6), C(3, 12)

3. P(-12, 6), Q(4, 0), R(-8, -6)

Find the coordinates of the circumcenter given the vertices of the following triangles.

1. D(5, 1), E(-2, 0), F(4, 8)

2. J(-2, 0), K(2, 8), L(7, 3)

3. A(-90, 28), B(0, -35), C(125, 20) .

Find the coordinates of the orthocenter given the vertices of the following triangles.

1. D(-2, 5), E(5, 1), F(-4, -5)

2. P(-1, 5), Q(7, 2), R(-1, -4)

3. P(-3, 4), Q(10, -3), R(3, -2)