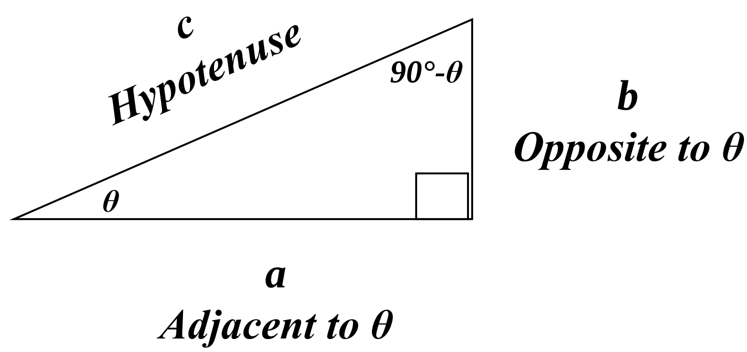
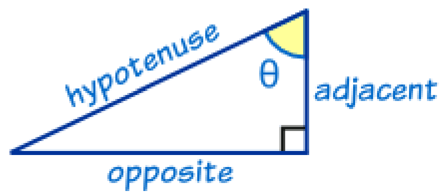
Right Triangle Trigonometry

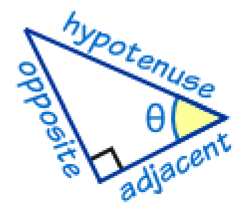
also see http://www.mathsisfun.com/sine-cosine-tangent.html

*θ is an **acute** angle because it is less than 90 degrees.*

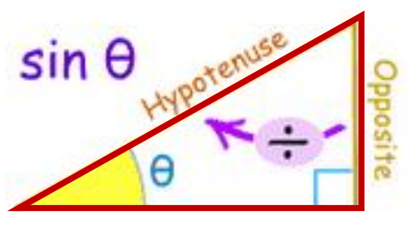


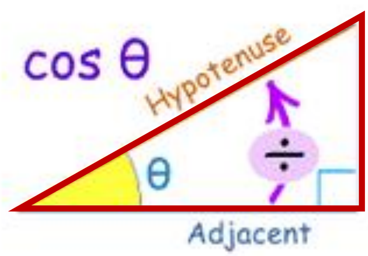
Adjacent is always next to the angle
And Opposite is opposite the angle

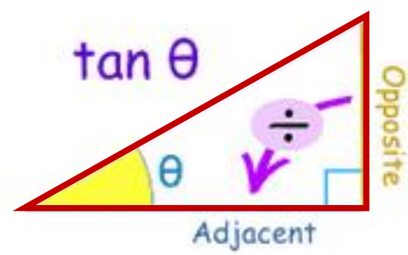




These ratios are the same for any right triangle with acute angle θ. They are







$\sin(\theta) = O/H$ **$\cos(\theta) = A/H$,** **$\tan(\theta) = O/A$)**

FUNCTION NAME	ABBREV.	VALUE
Sine of θ	$\sin(\theta)$	*b/c=**o**pposite/**h**ypotenuse*
Cosine of θ	$\cos(\theta)$	*a/c=**a**djacent/**h**ypotenuse*
Tangent of θ	$\tan(\theta)$	*b/a=**o**pposite/**a**djacent*
Cosecant of θ	$\csc(\theta)$	*c/b=hypotenuse/opposite*
Secant of θ	$\sec(\theta)$	*c/a=hypotenuse/adjacent*
Cotangent of θ	$\cot(\theta)$	*a/b=adjacent/opposite*

Notice these functions are the reciprocals of sine, cosine, & tangent, respectively.

Remember SOH-CAH-TOA!

In other words:

$\csc(\theta)=1/\sin(\theta)$, $\sec(\theta)=1/\cos(\theta)$, $\cot(\theta)=1/\tan(\theta)$

Example:

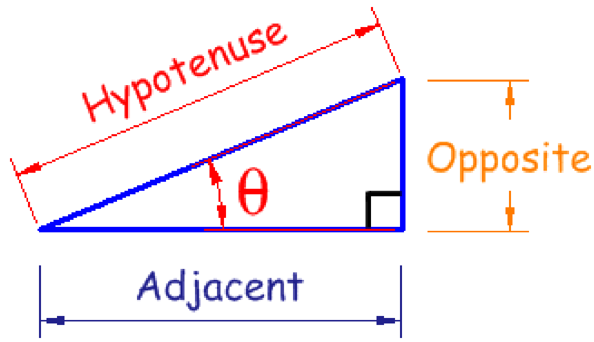
Sine Function

The Sine of angle θ is:

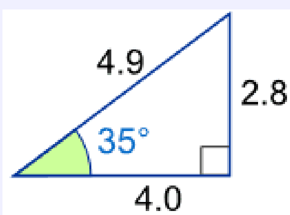
- the **length of the side Opposite** angle θ
- divided by the **length of the Hypotenuse**

Or more simply:

$$\sin(\theta) = \text{Opposite} / \text{Hypotenuse}$$



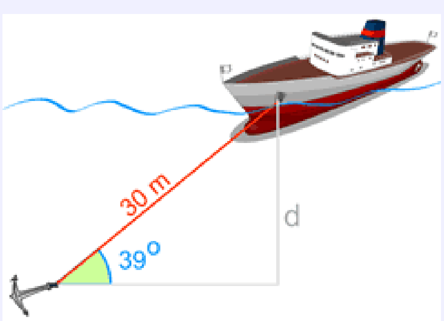
Example: What is the sine of 35°?



Using this triangle (lengths are only to one decimal place):

$\sin(35°) = \text{Opposite} / \text{Hypotenuse} = 2.8/4.9 = \mathbf{0.57...}$

The Sine Function can help us solve things like this:



Example: Use the sine function to find "d"

We know

* The angle the cable makes with the seabed is 39°

* The cable's length is 30 m.

And we want to know "d" (the distance down).

Start with:	$\sin 39° = \text{opposite/hypotenuse} = d/30$
Swap Sides:	$d/30 = \sin 39°$
Use a calculator to find sin 39°:	$d/30 = 0.6293...$
Multiply both sides by 30:	$d = 0.6293... \times 30 = \mathbf{18.88}$ to 2 decimal places.

The depth "d" is **18.88 m**

http://www.mathsisfun.com/algebra/trig-inverse-sin-cos-tan.html

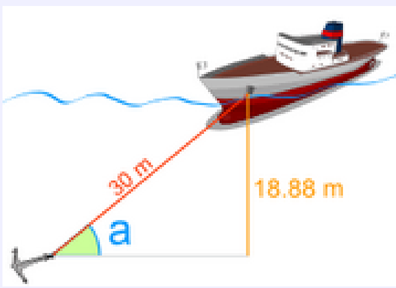
Inverse Sine

But what if it is the **angle** we don't know?

This is where "Inverse Sine" comes in.

It answers the question "what **angle** has sine equal to opposite/hypotenuse?"

The symbol for inverse sine is $\sin^{-1}$



Example: Find the angle "a"

We know

* The distance down is 18.88 m.

* The cable's length is 30 m.

And we want to know the angle "a"

Start with: sin a° = opposite/hypotenuse = 18.88/30
Calculate 18.88/30: sin a° = 0.6293...

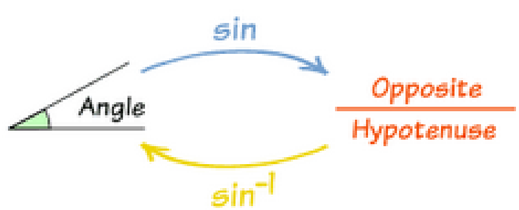
What **angle** has sine equal to 0.6293...?
The **Inverse Sine** will tell us.

Inverse Sine: $a° = \sin^{-1}(0.6293...)$

Use a calculator to find $\sin^{-1}(0.6293...)$: a° = **39.0°** (to 1 decimal place)

The angle "a" is **39.0°**

They Are Like Forward and Backwards!



- The Sine function sin takes an **angle** and gives us the **ratio** "opposite/hypotenuse"
- Inverse Sine $\sin^{-1}$ takes the **ratio** "opposite/hypotenuse" and gives us the **angle.**

Example:

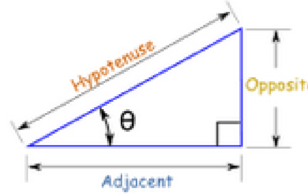
Sine Function: sin(**30°**) = **0.5**
Inverse Sine: $\sin^{-1}$(**0.5**) = **30°**

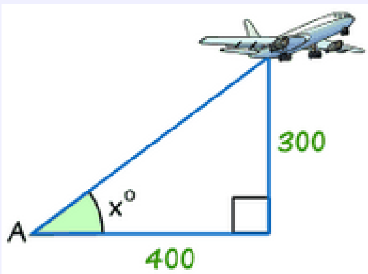
The Tangent of angle **θ** is:

$$\tan(\theta) = \text{Opposite} / \text{Adjacent}$$

So Inverse Tangent is :

$$\tan^{-1} (\text{Opposite} / \text{Adjacent}) = \theta$$





Example: Find the size of angle x°

tan x° = Opposite / Adjacent

tan x° = 300/400 = 0.75

x° = $\mathbf{tan^{-1}}$ (0.75) = **36.9°** (correct to 1 decimal place)

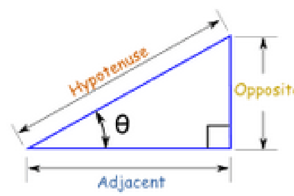
Summary

The Sine of angle **θ** is:

$$\sin(\theta) = \text{Opposite} / \text{Hypotenuse}$$

And Inverse Sine is :

$$\sin^{-1} (\text{Opposite} / \text{Hypotenuse}) = \theta$$



What About "cos" and "tan" ... ?

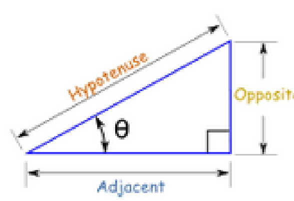
Exactly the same idea.

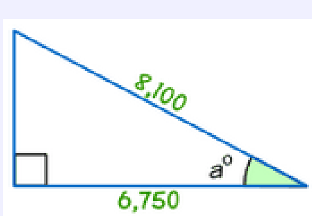
The Cosine of angle **θ** is:

$$\cos(\theta) = \text{Adjacent} / \text{Hypotenuse}$$

And Inverse Cosine is :

$$\cos^{-1} (\text{Adjacent} / \text{Hypotenuse}) = \theta$$





Example: Find the size of angle a°

cos a° = Adjacent / Hypotenuse

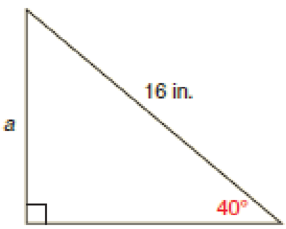
cos a° = 6,750/8,100 = 0.8333...

a° = $\mathbf{cos^{-1}}$ (0.8333...) = **33.6°** (to 1 decimal place)

Chapter 11 REVIEW EXERCISES

In Exercises 1 to 4, state the ratio needed, and use it to find the measure of the indicated line segment to the nearest tenth of a unit.

1.



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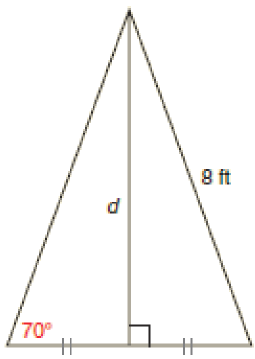

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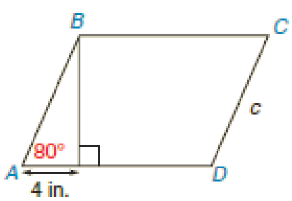
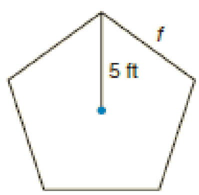


▱ABCD

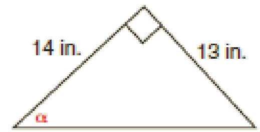
4.



Regular pentagon
with radius = 5 ft

In Exercises 5 to 8, state the ratio needed, and use it to find the measure of the indicated angle to the nearest degree.

5.



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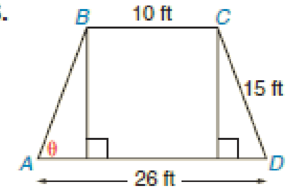
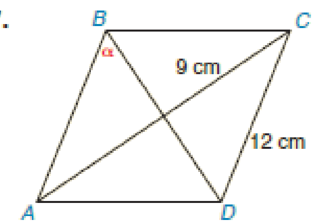


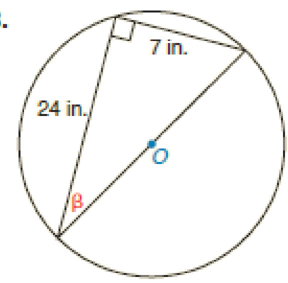
Isosceles trapezoid *ABCD*

7.



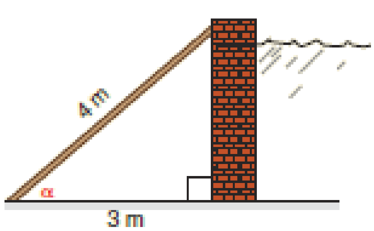
Rhombus *ABCD*

8.

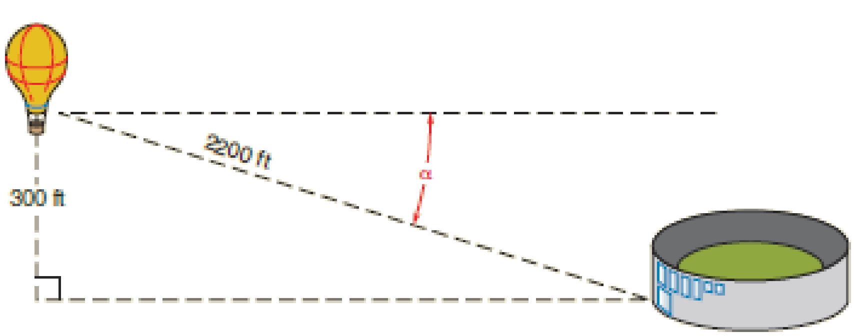


Circle *O*

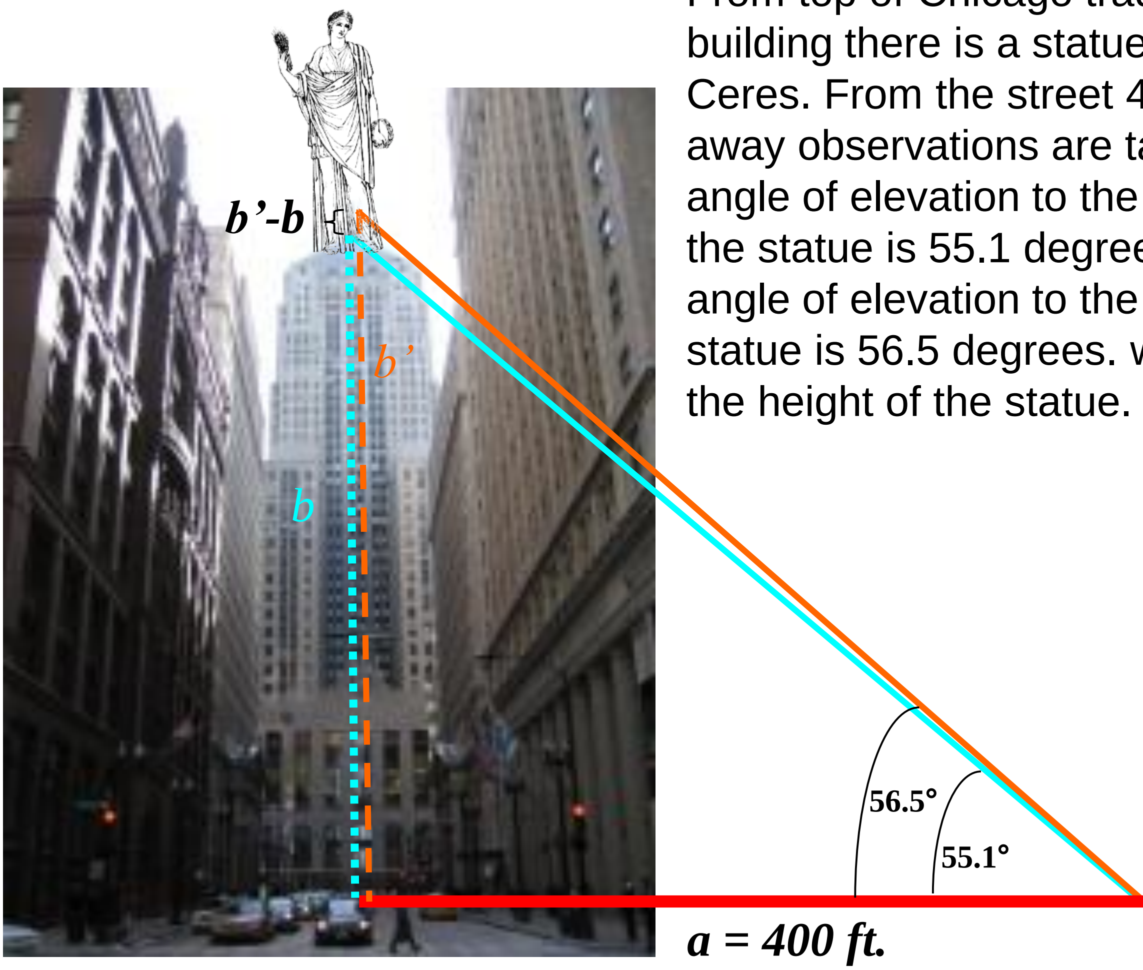
23. A 4-m beam is used to brace a wall. If the bottom of the beam is 3 m from the base of the wall, what is the angle of elevation to the top of the wall?



24. The basket of a hot-air balloon is 300 ft high. The pilot of the balloon observes a stadium 2200 ft away. What is the measure of the angle of depression?



From top of Chicago trade building there is a statue of Ceres. From the street 400 feet away observations are taken. The angle of elevation to the base of the statue is 55.1 degrees and the angle of elevation to the top of the statue is 56.5 degrees. what is the height of the statue.



$$\tan 55.1° = \frac{b}{400} \quad b = 400 \tan 55.1° \approx 573$$

$$\tan 56.5° = \frac{b'}{400} \quad b' = 400 \tan 56.5° \approx 604$$

Height of statute is approx. $604 - 573 = 31$ feet