3.3 -3.4 More Triangle Parts and Properties

Base and altitude

Every triangle has three **bases** (any of its sides) and three **altitudes** (heights). Every **altitude is the perpendicular segment from a vertex to its opposite side (or the extension of the opposite side)** (Figure 1).

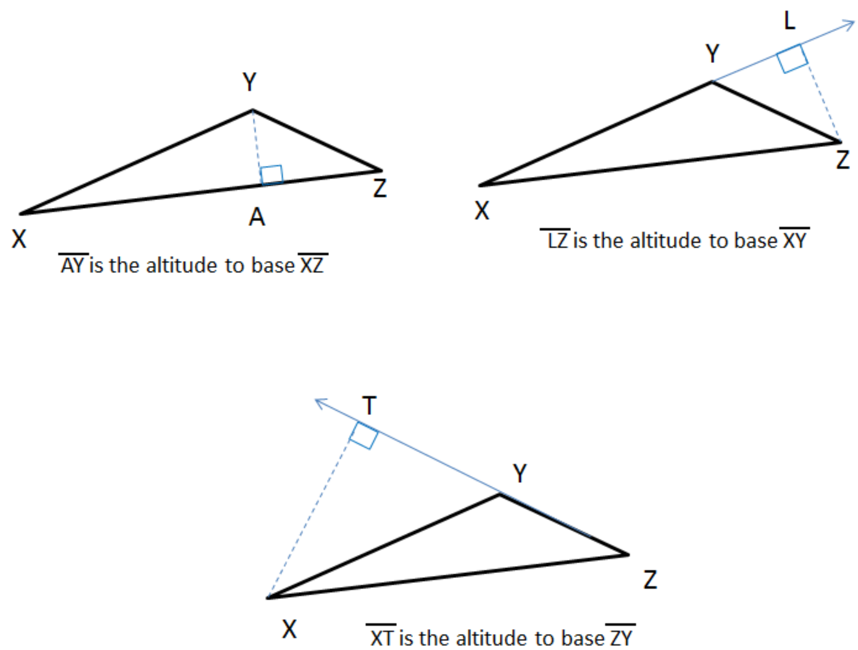


Figure 1

Altitudes can sometimes coincide with a side of the triangle or can sometimes meet an extended base outside the triangle. In Figure 2, *AC* is an altitude to base *BC* , and *BC* is an altitude to base *AC* .

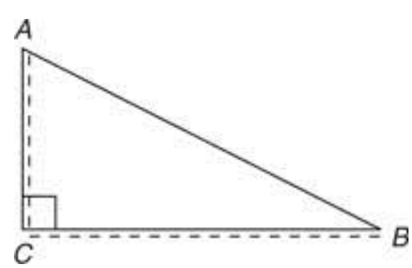


Figure 2 In a right triangle, each leg can serve as an altitude.

It is interesting to note that in any triangle, the three lines containing the altitudes meet in one point (Figure 4).

3.3 -3.4   More Triangle Parts and Properties

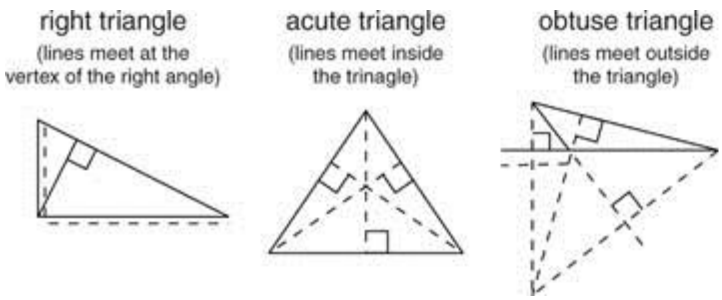


Figure 4 The three lines containing the altitudes intersect in a single point, which may or may not be inside the triangle. This point is called the orthocenter.

Median

A **median** in a triangle is the line segment drawn from a vertex to the midpoint of its opposite side. Every triangle has three medians. In Figure 5, *E* is the midpoint of *BC* . Therefore, *BE* = *EC*. *AE* is a median of Δ *ABC.*

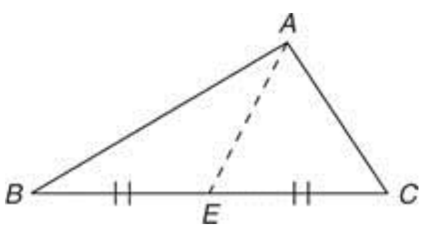


Figure 5 A median of a triangle.

In every triangle, the three medians meet in one point inside the triangle (Figure 6).

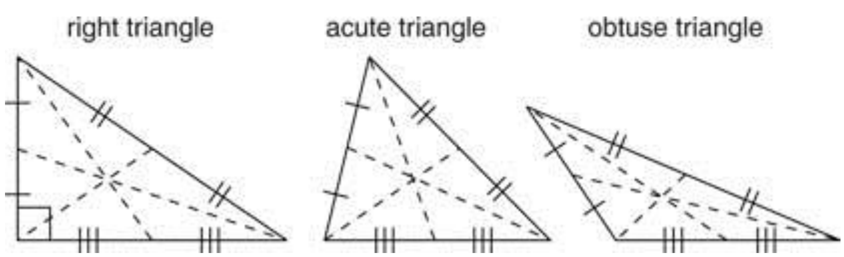


Figure 6 The three medians meet in a single point inside the triangle. This point is called the Centroid.

Angle bisector

An **angle bisector** in a triangle is a segment drawn from a vertex that bisects (cuts in half) that vertex angle. Every triangle has three angle bisectors. In Figure , is an angle bisector in Δ *ABC.*

3.3 -3.4   More Triangle Parts and Properties

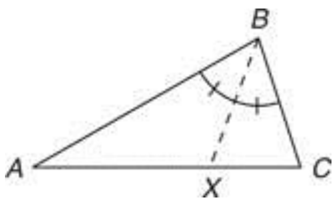


Figure 7 An angle bisector.

In every triangle, the three angle bisectors meet in one point inside the triangle (Figure 8).

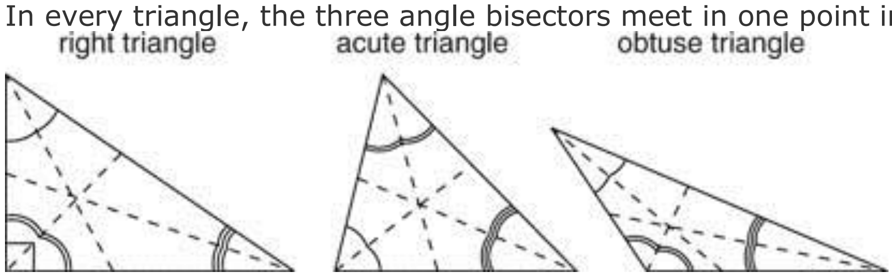
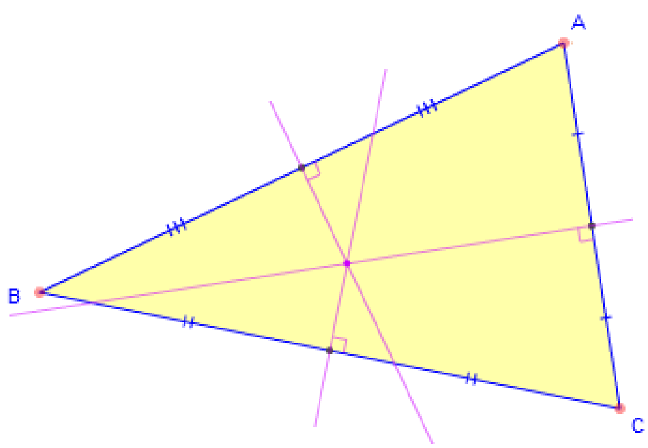


Figure 8 The three angle bisectors meet in a single point inside the triangle. This point is called the Incenter.

Perpendicular bisector

A **perpendicular bisector** in a triangle is a segment drawn from a midpoint of a side that is perpendicular to that site. The perpendicular bisectors of a triangle meet at a single point, called the Circumcenter.



3.3 -3.4   More Triangle Parts and Properties

In general, altitudes, medians, perpendicular bisectors and angle bisectors are different segments (or lines, as in perp. bisectors). In certain triangles, though, they can be the same segments. In Figure 9, the altitude drawn from the vertex angle of an isosceles triangle can be proven to be a median as well as an angle bisector and a perpendicular bisector.

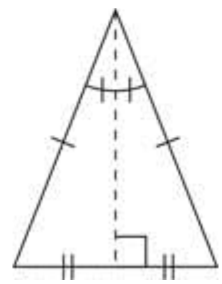


Figure 9 The altitude drawn from the vertex angle of an isosceles triangle.

Example 1: Based on the markings in Figure 10, name:

an altitude of Δ *QRS* ________________________________

a median of Δ *QRS* ___________________________________

an angle bisector of Δ *QRS* _______________________________

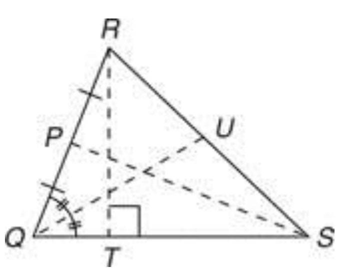


Figure 10 Finding an altitude, a median, and an angle bisector.

3.3 -3.4   More Triangle Parts and Properties

Special Features of Isosceles Triangles

Isosceles triangles are special and because of that there are unique relationships that involve their internal line segments. Consider isosceles triangle *ABC* in Figure 1.

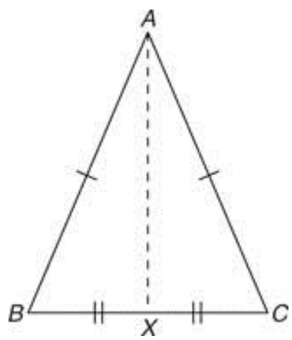


Figure 1 *An isosceles triangle with a median.*

With a median drawn from the vertex to the base, *BC* ,

it can be proven that Δ *BAX* ≅ Δ *CAX*, which leads to several important theorems.

Theorem: If two sides of a triangle are equal, then the angles opposite those sides are also equal.	
Theorem: If *a triangle is equilateral, then it is also equiangular.*	
Theorem: If two angles of a triangle are equal, then the sides opposite these angles are also equal.	
Theorem: If a triangle is equiangular, then it is also equilateral.	

3.3 -3.4 More Triangle Parts and Properties

Example 1: *Figure has Δ QRS with QR = QS. If m ∠ Q = 50°, find m ∠ R and m ∠ S.*

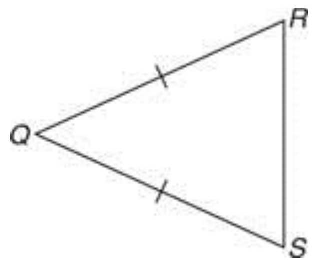


Figure 2 An isosceles triangle with a specified vertex angle.

Because m ∠ Q + m ∠ R + m ∠ S = 180°, and because QR = QS implies that m ∠ R = m ∠ S,

$$m\angle Q + m\angle R + m\angle R = 180^\circ$$
$$50^\circ + 2m\angle R = 180^\circ$$
$$2m\angle R = 130^\circ$$
$$m\angle R = 65^\circ \text{ and } m\angle S = 65^\circ$$

Example 2: *Figure 3 has Δ ABC with m ∠ A = m ∠ B = m ∠ C, and AB = 6. Find BC and AC.*

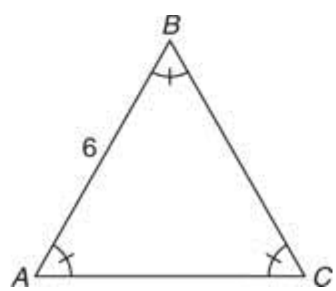


Figure 3 *An equiangular triangle with a specified side.*

Because the triangle is equiangular, it is also equilateral. Therefore, BC = AC = 6.