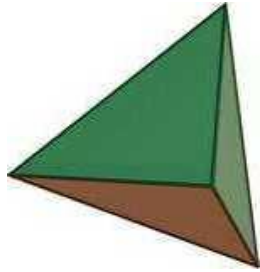


Solid Geometry

Solid Geometry is the geometry of three-dimensional space, the kind of space we live in ...

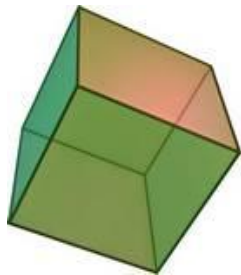
Platonic Solids

Below are the five platonic solids (or *regular polyhedra*). For each solid there is a printable net. These nets can be printed onto a piece of card. [You can then make your own platonic solids](#). Cut them out and tape the edges together.



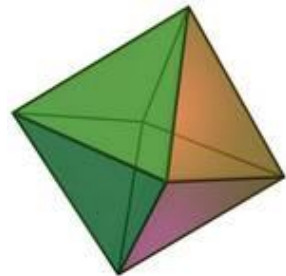
Tetrahedron

- 4 Faces
- 4 Vertices
- 6 Edges



Cube

- 6 Faces
- 8 Vertices
- 12 Edges



Octahedron

- 8 Faces
- 6 Vertices
- 12 Edges



Dodecahedron

- 12 Faces
- 20 Vertices
- 30 Edges



Icosahedron

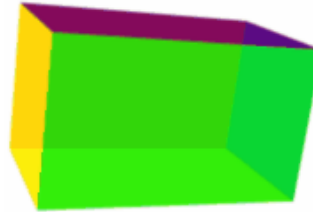
- 20 Faces
- 12 Vertices
- 30 Edges

Prisms

A prism is officially a [polyhedron](#), which means all sides should be flat. No curved sides.

So the cross section will be a polygon (a straight-edged figure). For example, if the cross section was a circle then it would be a [cylinder](#), not a prism. See next page.

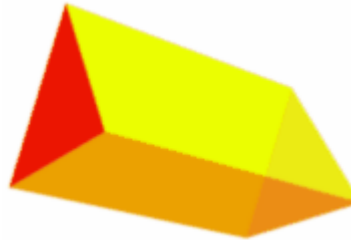
Rectangular Prism:



Cross-Section:



Triangular Prism:



Cross-Section:



Irregular Prisms

All the previous examples are **Regular** Prisms, because the cross section is regular (in other words it is a shape with equal edge lengths)

Here is an example of an **Irregular Prism**:

Irregular Pentagonal Prism:



Cross-Section:



(It is "irregular" because the Pentagon is not "regular" in shape)

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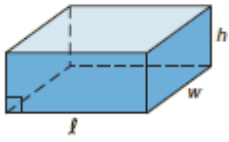
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SOLIDS (SPACE FIGURES):

L = Lateral Area; T (or S) = Total (Surface) Area; V = Volume

Parallelepiped (box):



$$T = 2\ell w + 2\ell h + 2wh$$
$$V = \ell wh$$

Right Prism:



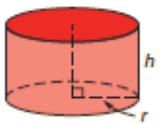
$$L = hP$$
$$T = L + 2B$$
$$V = Bh$$

Regular Pyramid:



$$L = \frac{1}{2}\ell P$$
$$\ell^2 = a^2 + h^2$$
$$T = L + B$$
$$V = \frac{1}{3}Bh$$

Right Circular Cylinder:



$$L = 2\pi rh$$
$$T = 2\pi rh + 2\pi r^2$$
$$V = \pi r^2 h$$

Right Circular Cone:



$$L = \pi r\ell$$
$$\ell^2 = r^2 + h^2$$
$$T = \pi r\ell + \pi r^2$$
$$V = \frac{1}{3}\pi r^2 h$$

Sphere:



$$S = 4\pi r^2$$
$$V = \frac{4}{3}\pi r^3$$

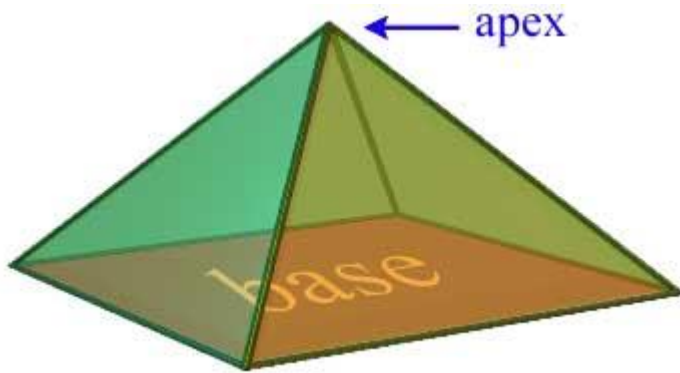
Miscellaneous:

Euler's Equation: $V + F = E + 2$

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Pyramids - A pyramid is made by connecting a base and an apex

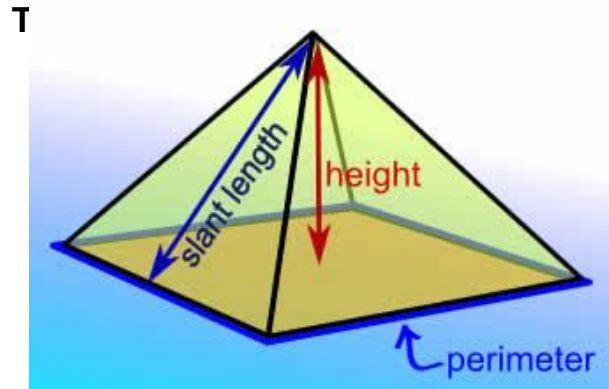


When all side faces are the same:

- $[\text{Base Area}] + \frac{1}{2} \times \text{Perimeter} \times [\text{Slant Length}]$

When side faces are different:

- $[\text{Base Area}] + [\text{Lateral Area}]$



Notes On Surface Area

The Surface Area has two parts: the area of the base (the **Base Area**), and the area of the side faces (the **Lateral Area**).

For **Base Area** :

It depends on the shape, there are different formulas for triangle, square, etc.

For **Lateral Area** :

When all the side faces are the same:

- Just multiply the perimeter by the "slant length", l , and divide by 2. This is because the side faces are always triangles and the triangle formula is "base times height divided by 2"

$$\bullet = \frac{lP}{2}$$

But if the side faces are different (such as an "irregular" pyramid) then add up the area of each triangular shape to find the total lateral area.

Solid Geometry

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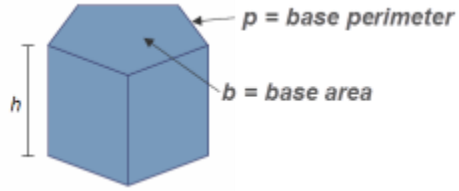
<http://www.mathopenref.com/prism.html> <http://www.mathopenref.com/prismarea.html>

<http://www.mathsisfun.com/geometry/prisms.html>

Surface area of a prism

Surface **Area = $2b + ph$**

b = area of a base
 p = perimeter of a base
 h = height of the prism



A right prism is composed of a set of flat surfaces.

- The two bases are congruent polygons.
- The lateral faces (or sides) are rectangles.

An oblique prism is composed of a set of flat surfaces.

- The two bases are congruent polygons.
- The lateral faces (or sides) are parallelograms.

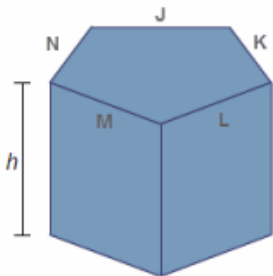
The total surface area is the sum of these.

Bases

Each base is a polygon. In the figure above it is a regular pentagon, but it can be any regular or irregular polygon. To find the area of the base polygons, see [Area of a regular polygon](#) and [Area of an irregular polygon](#). Since there are two bases, this is doubled and accounts for the "2b" term in the equation above.

Lateral faces

Each lateral face (side) of a right prism is a rectangle. One side is the height of the prism, the other the length of that side of the base.



Therefore, the front left face of the prism above is its height times width or $Face\ area = hm$
 The total area of the faces is therefore $area\ of\ all\ faces = hm + hn + hj + hk + hl$ If we factor out the 'h' term from the expression we get $area\ of\ all\ faces = h(m + n + j + k + l)$ Note that the expression in the parentheses is the perimeter (p) of the base, hence we can write the final area formula

$$area = 2b + ph$$

as area of two bases area of all faces

Regular prisms

If the prism is regular, the bases are regular polygons. and so the perimeter is 'ns' where s is the side length and n is the number of sides. In this case the surface area formula simplifies to

$$area = 2b + nsh$$

b = area of a base
 n = number of sides of a base
 s = length of sides of a base

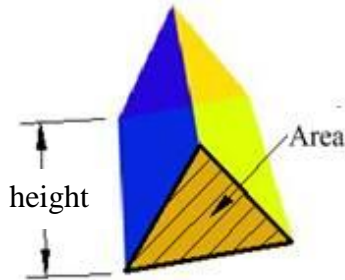
Solid Geometry

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h = height of the prism

Volume of a Prism

The Volume of a prism is simply the area of one end times the length of the prism



Volume = BASE \times HEIGHT

Base = Area of a cross-section of the prism.

Height = altitude of prism if prism is lying on a base.

Example: What is the volume of a prism whose ends are 25 in^2 and which has a height of 12 in.:

Answer: Volume = $25 \text{ in}^2 \times 12 \text{ in} = 300 \text{ in}^3$

Base Areas

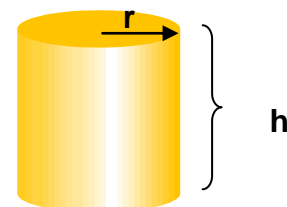
	<p><u>Triangle</u> Area = $\frac{1}{2}b \times h$ b = base h = vertical height</p>			<p><u>Square</u> Area = a^2 a = length of side</p>
	<p><u>Rectangle</u> Area = $l \times w$ b = length w = width</p>			<p><u>Parallelogram</u> Area = $l \times h$ l = length h = height</p>
	<p><u>Trapezoid (US)</u> <u>Trapezium (UK)</u> Area = $\frac{1}{2}(a+b)h$ h = vertical height</p>			<p><u>Circle</u> Area = πr^2 Circumference = $2\pi r$ r = radius</p>

If the base of a prism is a circle, it is not called a prism, it is a CYLINDER (Prisms do not have curved sides)

Volume of a Cylinder

Just multiply the area of the circle by the height of the cylinder:

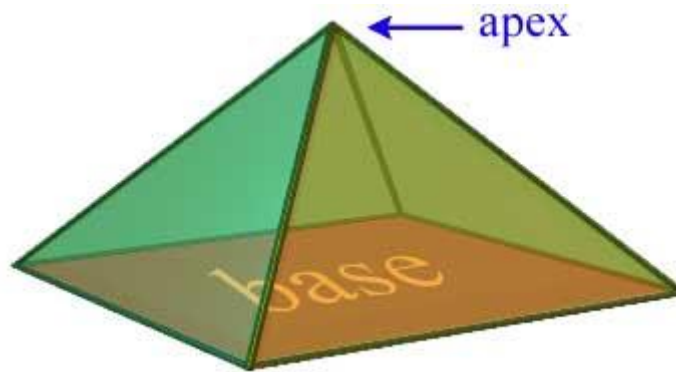
- Area of the circle: $\pi \times r^2$
- Height: h
- Volume = Area \times Height = $\pi \times r^2 \times h$



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Parts of a Pyramid



A pyramid is made by connecting a **base** to an **apex**

Pyramids and Cones

There are many types of Pyramids, and they are named after the shape of their base.

	Pyramid	Base	Volume
Triangular Pyramid:			<p style="text-align: center;">The Volumes of Pyramids and Cones</p> <p style="text-align: center;">$\frac{1}{3} \times [\text{Base Area}] \times \text{Height}$</p>
Square Pyramid:			
Pentagonal Pyramid:			
	Cone		
			<p>A Cone has a flat base and a flat top The base is the same as the top, and also in-between. It has one curved side. Because it has a curved surface it is not a polyhedron.</p>

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