Math 170 Test 2 Review **Name** ________________________________

1. Find the limit: $\lim_{x \to 0} \frac{\sin x}{x + \tan x}$

2. A particle moves according to the law of motion $s = f(t) = t^3 - 12t + 3,\ t \ge 0$, where t is measured in seconds and s is in feet.

a. Find the velocity at time t.

b. When is the particle moving in the positive direction?

c. Find the total distance traveled during the first 8 seconds.

d. Find the acceleration at the time(s) when the velocity is zero.

3. Find the equation of the tangent line to the curve $y = x \cos x$ at the point $(\pi, -\pi)$.

In problems 4-5, find the derivative and simplify.

4. $f(x) = (3x-2)^{10}(5x^2 - x + 1)^{12}$

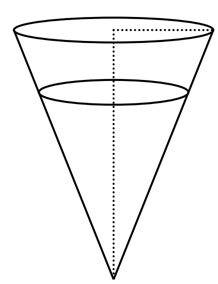
5. $f(x) = \sec^3(x^5)$

6. Find the equation of the tangent line to the curve $\frac{x^2}{16} - \frac{y^2}{9} = 1$ at the point $\left(-5, \frac{9}{4}\right)$.

7. For what values of x does the graph of $f(x) = x + 2\sin x$ have a horizontal tangent line?

8. Find $\frac{d^2y}{dx^2}$ by implicit differentiation and simplify. $\sqrt{x} + \sqrt{y} = 1$

9. A water tank has the shape of an inverted circular cone with base radius of 3 meters and height of 6 meters. If the water is being pumped into the tank at a rate of 2 m^3/min, find the rate at which the water level is rising when the water is 4 meters deep. $\left(V = \frac{1}{3}\pi r^2 h\right)$



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10. Two cars start moving from the same point. One travels south at 60 mph and the other travels west at 25 mph. At what rate is the distance between the two cars increasing two hours later?

11. Let $f(x) = \sqrt{1-x}$.

a. Find the linear approximation of $f(x)$ at $a = 0$.

b. Use your approximation to approximate $\sqrt{0.9}$ and $\sqrt{0.99}$.

12. The circumference of a sphere was measured to be 84 cm with a possible error in measurement of 0.2 cm.

a. Use differentials to estimate the maximum error in the calculated surface area. $(A = 4\pi r^2)$

b. What is the relative error?

13. Find the critical numbers of the function $f(t) = t + \sin t$.

14. Show that 5 is a critical number of the function $f(x) = 2 + (x-5)^3$ but $f(x)$ does not have a local extreme value at 5.

15. Find the absolute maximum and absolute minimum values of the function $f(x)$ on the given interval.
$f(x) = x^2 + \frac{2}{x}$, [0.5, 2]

16. Show that the equation $3x - 2 + \cos\left(\frac{\pi x}{2}\right) = 0$ has exactly one real root.

17. If $f(1) = 10$ and $f'(x) \geq 2$ for $1 \leq x \leq 4$, how small can $f(4)$ possibly be?

18. Let $f(x) = 2x^3 - 3x^2 - 12x$.

a. Find the intervals where the function is increasing and decreasing and the local extrema values.

b. Find the intervals where it is concave up and concave down and the inflection points.

c. Use part (a) and (b) to sketch a graph of $f(x)$.

19. Find $f'(x)$, $f''(x)$, all asymptotes of $f(x)$ and graph: $f(x) = \dfrac{x^2}{x^2 - 9}$

20. Evaluate the limit: $\lim\limits_{x \to -\infty} \left(x + \sqrt{x^2 + x + 1} \right)$

21. Evaluate the limit: $\lim\limits_{x \to \infty} \dfrac{(1 - x)(2 + x)}{(1 + 2x)(2 - 3x)}$

Math 170 Test 2 Review Answers

1. $\frac{1}{2}$

2. **a.** $3t^2 - 12$ **b.** $t > 2$ **c.** 448 ft **d.** 12 ft/sec^2

3. $y + \pi = -(x - \pi)$ or $y = -x$

4. $6(3x-2)^9(5x^2 - x + 1)^{11}(85x^2 - 51x + 9)$

5. $15x^4 \sec^3(x^5)\tan(x^5)$

6. $y - \frac{9}{4} = -\frac{5}{4}(x+5)$ or $y = -\frac{5}{4}x - 4$

7. $x = \frac{2\pi}{3} + 2\pi n$ or $x = \frac{4\pi}{3} + 2\pi n$, where n is an integer.

8. $\frac{\sqrt{x} + \sqrt{y}}{2x\sqrt{x}}$ or $\frac{1}{2x\sqrt{x}}$

9. $\frac{1}{2\pi}$ m/min

10. 65 mph

11. **a.** $L(x) = -\frac{1}{2}x + 1$ **b.** $\sqrt{0.9} \approx L(0.1) = .95$ and $\sqrt{0.99} \approx L(0.01) = .995$

12. **a.** $dA = \frac{168}{5\pi} \approx 10.695$ **b.** $\frac{1}{210} \approx 0.00476$ or 0.476%

13. $t = \pi + 2n\pi$ or $t = \pi(2n+1)$

14. $f'(x) = 3(x-5)^2 \geq 0$ for all x. Since $f'(x)$ doesn't change sign there are no extreme values.

15. Absolute maximum value = 5 when x = 2
Absolute minimum value = 3 when x = 1

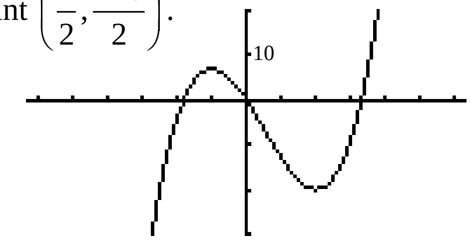
16. If $f(x) = 3x - 2 + \cos\left(\frac{\pi x}{2}\right)$ then $f(0) = -1$ and $f(1) = 1$ so the Intermediate Value Theorem says there must be at least one root. Assume there is another root. Then Rolle's Theorem says there must be a point c where $f'(c) = 0$. However $f'(x) = 3 - \frac{\pi}{2}\sin\left(\frac{\pi x}{2}\right) > 0$ for all x. Therefore there is no more than one root and there must be exactly one root.

17. $f'(x) = \frac{f(b) - f(a)}{b - a}$ so $\frac{f(4) - f(1)}{4 - 1} \geq 2 \Rightarrow \frac{f(4) - 10}{3} \geq 2 \Rightarrow f(4) \geq 16$. Therefore the minimum value of $f(4)$ is 16.

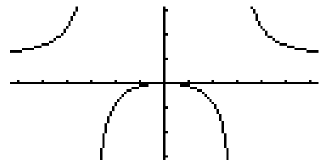
18. **a.** Increasing on $(-\infty,-1)\cup(2,\infty)$, decreasing on $(-1, 2)$. Local maximum value = 7 at $x = -1$ and local minimum value = -20 at $x = 2$.

b. Concave up on $\left(\frac{1}{2},\infty\right)$, concave down on $\left(-\infty,\frac{1}{2}\right)$, inflection point $\left(\frac{1}{2},\frac{-13}{2}\right)$.

c. See graph $\rightarrow$



19. $f'(x)=\dfrac{-18x}{(x^2-9)^2}$, $f''(x)=\dfrac{54(x^2+3)}{(x^2-9)^3}$, $x = 3$, $x = -3$, $y = 1$



20. $-\dfrac{1}{2}$

21. $\dfrac{1}{6}$