3.4 Combining Like Terms

Term

A ***term*** is a single number or variable, or it can be a product of a number (called its ***coefficient***) and one or more variables.
Examples of terms:

$$-3, \quad x, \quad 2a, \quad -3xy, \quad 4a^2bc, \quad 25, \quad -2xyz$$

$$\underset{\textbf{\textit{coefficient}}}{\boxed{-2}}\underset{\textbf{\textit{variable part}}}{\boxed{xyz}}$$

Note: An algebraic expression is made of one or more terms. The terms in an algebraic expression are connected with either addition or subtraction.

Example 1: Identify the terms in the algebraic expression, $3x^2+5xy+9y^2+12$. For each term, identify its coefficient and variable part.

Solution:

$3x^2+5xy+9y^2+12$ (4 terms)		
Term	**Coefficient**	**Variable Part**
$3x^2$	3	x^2
$5xy$	5	xy
$9y^2$	9	y^2
12	12	none

You Try It 1: How many terms are in the algebraic expression, $3x^2+2xy-3y^2$?

Example 2: Identify the terms in the algebraic expression, $a^3-3a^2b+3ab^2-b^3$. For each term, identify its coefficient and variable part.

Solution:

$a^3-3a^2b+3ab^2-b^3$ (4 terms)		
Term	**Coefficient**	**Variable Part**
a^3	1	a^3
$-3a^2b$	–3	a^2b
$3ab^2$	3	ab^2
$-b^3$	–1	b^3

You Try It 2: How many terms are in the algebraic expression, $11-a^2-2ab+3b^2$?

Like Terms

Terms are ***like terms***, if their variable parts match identically (all variables and corresponding exponents must match).

Terms are ***unlike terms***, if their variable parts do not match identically (the variables or corresponding exponents are different).

Examples of like terms:

$3x$ and $-5x$, -9 and 25, $-2ab$ and $-7ab$, x^2y and $10x^2y$

Examples of unlike terms:

$3x$ and $3y$, 17 and x, $-ab$ and a^2b, x^2y^2 and $10x^2y$

Example 3: Classify each of the following pairs as either *like terms* or *unlike terms*.

a) $3x$ and $-7x$
b) $2y$ and $3y^2$
c) $-3t$ and $5u$
d) $-4a^3$ and $3a^3$

Solution: a) $3x$ and $-7x$
The variable parts are both x, therefore these are $\boxed{\text{like terms}}$.

b) $2y$ and $3y^2$
The variable parts are y and y^2. Even though the variables match, the exponents do not. Since these do not match identically, these are $\boxed{\text{unlike terms}}$.

c) $-3t$ and $5u$
The variable parts are t and u. Since these do not match, these are $\boxed{\text{unlike terms}}$.

d) $-4a^3$ and $3a^3$
The variable parts are both a^3, therefore these are $\boxed{\text{like terms}}$.

You Try It 3: Are $-3xy$ and $11xy$ *like* or *unlike* terms?

The Distributive Property

Let *a*, *b*, and *c* be any whole numbers. Then,

$$a(b+c)=ab+ac \quad \text{and} \quad a(b-c)=ab-ac$$

Note: Using the commutative property, we can rewrite the above as,

$$(b+c)a=ba+ca \quad \text{and} \quad (b-c)a=ba-ca$$

Since the two sides of each equation are equal, we can switch them and get,

$$ba+ca=(b+c)a \quad \text{and} \quad ba-ca=(b-c)a \quad \text{(This is know as factoring)}$$

We will use this to combine like terms.

Example 4: Use the distributive property and the note above to combine like terms (if possible).

a) $-5x^2-9x^2$
b) $-5ab+7ab$
c) $4y^3-7y^2$
d) $3xy^2-7xy^2$

Solution: a)

$$-5x^2-9x^2=\underbrace{(-5-9)}_{-14}x^2$$
$$=\boxed{-14x^2}$$

b)

$$-5ab+7ab=\underbrace{(-5+7)}_{2}ab$$
$$=\boxed{2ab}$$

c) $4y^3-7y^2$, these are not like terms since the exponents do not match. Since they are not like terms we cannot combine them. $\boxed{\text{Not Like Terms}}$

d)

$$3xy^2-7xy^2=\underbrace{(3-7)}_{-4}xy^2$$
$$=\boxed{-4xy^2}$$

You Try It 4: Simplify. $-8z-11z$

Combining Like Terms

The easier way of combining like terms is to combine the coefficients and carry the variable part. Remember that when combining like terms the variable part does not change.

Example 5: Combine like terms.

a) $-9y-8y$

b) $-3y^5+4y^5$

c) $-3u^2+2u^2$

Solution: a) $\underline{-}9y\,\underline{-}8y=\boxed{-17y}$

b) $\underline{-}3y^5\,\underline{+}4y^5=\boxed{1y^5 \text{ or } y^5}$

c) $\underline{-}3u^2\,\underline{+}2u^2=\boxed{-1u^2 \text{ or } -u^2}$

You Try It 5: Combine like terms. $-3x^2-4x^2$

Note:
The direction, **Simplify**, means to write the expression in its most compact form (using the fewest symbols possible). When simplifying an expression, you must always combine like terms, if possible.

Example 6: Simplify. $2x+3y-5x+8y$

Solution: $2x+3y-5x+8y=\boxed{-3x+11y}$ Combine the x terms together and combine the y terms together.

You Try It 6: Simplify. $-3a+4b-7a-9b$

Simplify

1) **Distribute.** If the expression contains parentheses, distribute first to get rid of the parentheses.
2) **Combine Like Terms**, if possible.

Example 7: Simplify. $-2x-3-(3x+4)$

Solution: $-2x-3-(3x+4)=-2x-3-3x-4$ Distribute first to get rid of parentheses.

$=-2x-3-3x-4$ Combine like terms. (x terms together & # terms together)

$=\boxed{-5x-7}$

You Try It 7: Simplify. $-9a-4-(4a-8)$

Example 8: Simplify. $2(5-3x)-4(x+3)$

Solution: $2(5-3x)-4(x+3)=10-6x-4x-12$ Distribute first to get rid of parentheses.

$=10-6x-4x-12$ Combine like terms. (# terms together & x terms together)

$=\boxed{-2-10x}$

You Try It 8: Simplify. $-2(3a-4)-2(5-a)$

Example 9: Simplify. $-8(3x^2y-9xy)-8(-7x^2y-8xy)$

Solution: $-8(3x^2y-9xy)-8(-7x^2y-8xy)=-24x^2y+72xy+56x^2y+64xy$ Distribute.

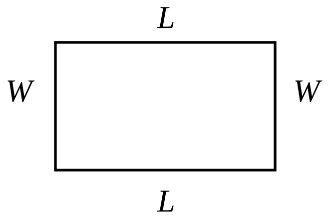
$=-24x^2y+72xy+56x^2y+64xy$ Combine like terms.

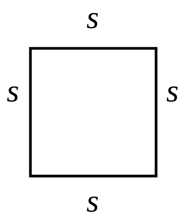
$=\boxed{32x^2y+136xy}$

Note: Remember you cannot combine x^2y & xy because the exponents do not match identically.

You Try It 9: Simplify. $(a^2-2ab)-2(3ab+a^2)$

Example 10: Find the perimeter, P, of the (a) rectangle and (b) square pictured below. Simplify your answers as much as possible.





Solution: Remember, the perimeter of a figure is the sum of the lengths of all its sides.

a) The perimeter of a rectangle is,

Perimeter of a rectangle $= L + W + L + W$ Combine like terms.

$= 2L + 2W$

Hence we get, $\boxed{\text{Perimeter of a rectangle} = 2L + 2W}$

b) The perimeter of a square is,

Perimeter of a square $= s + s + s + s$ Combine like terms.

$= 4s$

Hence we get, $\boxed{\text{Perimeter of a square} = 4s}$

You Try It 10: A regular hexagon has six equal sides, each with length x. Find its perimeter in terms of x.

Perimeter Formulas

Perimeter of a Rectangle $= 2L + 2W$ (where L = length and W = width)

Perimeter of a Square $= 4s$ (where s = the length of a side)

Example 11: Find the perimeter of the rectangle with length $(3W-4)$ feet and width W feet.

Solution: Using the new perimeter formula for a rectangle, we have

Perimeter of the rectangle $= 2L + 2W$

$= 2(\qquad) + 2W$ Now replace L with $3W-4$.

$= 2(3W-4) + 2W$ Distribute.

$= 6W - 8 + 2W$ Combine like terms.

$= 8W - 8$

So, $\boxed{P = (8W-8)\text{ feet}}$

You Try It 11: Find the perimeter of the rectangle with length $(2W+7)$ feet and width W feet.

Example 12: Find the perimeter of the rectangle with width $(3-2L)$ meters and length L meters.

Solution: Using the new perimeter formula for a rectangle, we have

Perimeter of the rectangle $= 2L + 2W$

$= 2L + 2(\qquad)$ Now replace W with $3-2L$.

$= 2L + 2(3-2L)$ Distribute.

$= 2L + 6 - 4L$ Combine like terms.

$= 6 - 2L$

So, $\boxed{P = (6-2L)\text{ meters}}$

You Try It 12: Find the perimeter of the rectangle with width $(4-3L)$ meters and length L meters.

Example 13: The length of a rectangle, *L*, is three feet longer than twice its width. Find the perimeter, *P*, of the rectangle in terms of its width, *W*.

Solution: $\underbrace{\text{The length of a rectangle}}_{L}\ \underset{=}{\text{is}}\ \underbrace{\text{three feet}}_{3}\ \underbrace{\text{longer than}}_{+}\ \underset{2\bullet}{\text{twice}}\ \underbrace{\text{its width}}_{W}.$

So, we get $L = 3 + 2W$

Using the new perimeter formula for a rectangle, we have

Perimeter of the rectangle $= 2L + 2W$

$= 2(\qquad) + 2W$ Now replace *L* with $3 + 2W$.

$= 2(3 + 2W) + 2W$ Distribute.

$= 6 + 4W + 2W$ Combine like terms.

$= 6 + 6W$

So, $\boxed{P = 6 + 6W}$

You Try It 13: The length of a rectangle, *L*, is 5 meters longer than twice its width, *W*. Find the perimeter, *P*, of the rectangle in terms of its width, *W*.

Example 14: The width of a rectangle, *W*, is two feet less than its length. Find the perimeter, *P*, of the rectangle in terms of its length, *L*.

Solution: $\underbrace{\text{The width of a rectangle}}_{W}\ \underset{=}{\text{is}}\ \underbrace{\text{two feet}}_{2}\ \underbrace{\text{less than}}_{-\ \text{(backwards minus)}}\ \underbrace{\text{its length}}_{L}.$

So, we get $W = L - 2$

Using the new perimeter formula for a rectangle, we have

Perimeter of the rectangle $= 2L + 2W$

$= 2L + 2(\qquad)$ Now replace *W* with $L - 2$.

$= 2L + 2(L - 2)$ Distribute.

$= 2L + 2L - 4$ Combine like terms.

$= 4L - 4$

So, $\boxed{P = 4L - 4}$

You Try It 14: The width of a rectangle, *W*, is 5 feet less than twice its length, *L*. Find the perimeter, *P*, of the rectangle in terms of its length, *L*.